GRTensorII Version 1.58 (R3)

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*Latest version available from: http://astro.queensu.ca/~gr \
tensor/*

To initiate help type ?grtensor

Defaults read from /usr/local/maple/lib/grtensor.ini

Computation of the Ricci tensor for the general static spherically symmetric metric, Wald (6.1.5)

> makeg(schwarz);

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Makeg 2.0: GRTensor metric/basis entry utility

To quit makeg, type ’exit’ at any prompt.

Do you wish to enter a 1) metric [g(dn,dn)],
                       2) line element [ds],
                       3) non-holonomic basis [e1...e4]
, or
                       4) NP tetrad [l,n,m,mbar]?
```

1

Enter coordinates as a LIST (e.g. [r,theta,phi,t]):

[t,r,theta,phi]

Is the metric 1) Diagonal, or
2) Symmetric?

1

Enter g[t,t]

-f(r)

Enter g[r,r]

h(r)

Enter g[theta,theta]

r^2

Enter g[phi,phi]

r^2 * sin(theta)^2

If there are any complex valued coordinates, constants or functions
for this spacetime, please enter them as a SET (eg. { z, psi }).

Complex quantities [default={}]:

{}

The values you have entered are:

$$Coordinates = [t, r, \theta, \phi]$$

Metric:

$$g_{a\,b} = \begin{bmatrix} -\mathrm{f}(r) & 0 & 0 & 0 \\ 0 & \mathrm{h}(r) & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin(\theta)^2 \end{bmatrix}$$

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You may choose to 0) Use the metric WITHOUT saving it,
                  1) Save the metric as it is,
                  2) Correct an element of the metric,
                  3) Re-enter the metric,
                  4) Add/change constraint equations,
                  5) Add a text description, or
                  6) Abandon this metric and return to
Maple.
```

0

```
Calculating ds for schwarz ... Done. (0.000000 sec.)
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$$Default\ spacetime = schwarz$$

$$For\ the\ schwarz\ spacetime:$$

$$Coordinates$$

$$\mathrm{x}(up)$$

$$x^a = [\,t \quad r \quad \theta \quad \phi\,]$$

$$Line\ element$$

$$ds^2 = -\mathrm{f}(r)\ d\,t^2 + \mathrm{h}(r)\ d\,r^2 + r^2\ d\,\theta^2 + r^2 \sin(\theta)^2\ d\,\phi^2$$

```
makeg completed.
```

Use the "alias" function to make functional dependence "implicit" in input and output

> alias(f=f(r),h=h(r));

$$I, f, h$$

> grcalc(R(dn,dn));

```
Calculating detg for schwarz ... Done. (0.000000 sec.)
Calculating g(up,up) for schwarz ... Done. (0.020000 se
c.)
Calculating g(dn,dn,pdn) for schwarz ... Done. (0.01000
0 sec.)
Calculating Chr(dn,dn,dn) for schwarz ... Done. (0.0200
00 sec.)
Calculating Chr(dn,dn,up) for schwarz ... Done. (0.0200
00 sec.)
Calculating R(dn,dn) for schwarz ... Done. (0.030000 se
c.)
```

$$CPU\ Time\ = .100$$

> grdisplay(R(dn,dn));

For the schwarz spacetime:

Covariant Ricci

$$\mathrm{R}(dn, dn)$$

$$R_{a\,b} =$$

$$\left[\frac{1}{4}\left(-\left(\frac{\partial}{\partial r} f\right)\left(\frac{\partial}{\partial r} h\right) f\, r + 2\left(\frac{\partial^2}{\partial r^2} f\right) h\, f\, r - \left(\frac{\partial}{\partial r} f\right)^2 h\, r + 4\left(\frac{\partial}{\partial r} f\right) h\, f\right) \Big/ (h^2 f\, r),\, 0,\, 0,\, 0\right]$$

$$\left[0,\, -\frac{1}{4}\left(-\left(\frac{\partial}{\partial r} f\right)^2 h\, r + 2\left(\frac{\partial^2}{\partial r^2} f\right) h\, f\, r - \left(\frac{\partial}{\partial r} f\right)\left(\frac{\partial}{\partial r} h\right) f\, r - 4\left(\frac{\partial}{\partial r} h\right) f^2\right) \Big/ (f^2 h\, r),\, 0,\, 0\right]$$

$$\left[0,\, 0,\, \frac{1}{2}\,\frac{-r\left(\frac{\partial}{\partial r} f\right) h + r\left(\frac{\partial}{\partial r} h\right) f + 2\, h^2 f - 2\, h\, f}{h^2 f},\, 0\right]$$

$$\left[0,\, 0,\, 0,\, \frac{1}{2}\,\frac{\sin(\theta)^2\left(-r\left(\frac{\partial}{\partial r} f\right) h + r\left(\frac{\partial}{\partial r} h\right) f + 2\, h^2 f - 2\, h\, f\right)}{h^2 f}\right]$$

Select specific tensor components using the "grcomponent" function

> R00 := grcomponent(R(dn,dn) , [t,t]);

$$R00 := \frac{1}{4}\left(-\left(\frac{\partial}{\partial r} f\right)\left(\frac{\partial}{\partial r} h\right) f\, r + 2\left(\frac{\partial^2}{\partial r^2} f\right) h\, f\, r - \left(\frac{\partial}{\partial r} f\right)^2 h\, r + 4\left(\frac{\partial}{\partial r} f\right) h\, f\right) \Big/ (h^2\, f\, r)$$

> R11 := grcomponent(R(dn,dn) , [r,r]);

$$R11 := -\frac{1}{4}\left(-\left(\frac{\partial}{\partial r} f\right)^2 h\, r + 2\left(\frac{\partial^2}{\partial r^2} f\right) h\, f\, r - \left(\frac{\partial}{\partial r} f\right)\left(\frac{\partial}{\partial r} h\right) f\, r - 4\left(\frac{\partial}{\partial r} h\right) f^2\right) \Big/ (f^2\, h\, r)$$

> R22 := grcomponent(R(dn,dn) , [theta,theta]);

$$R22 := \frac{1}{2}\,\frac{-r\left(\frac{\partial}{\partial r} f\right) h + r\left(\frac{\partial}{\partial r} h\right) f + 2\, h^2 f - 2\, h\, f}{h^2\, f}$$

> R33 := grcomponent(R(dn,dn) , [phi,phi]);

$$R33 := \frac{1}{2}\,\frac{\sin(\theta)^2\left(-r\left(\frac{\partial}{\partial r} f\right) h + r\left(\frac{\partial}{\partial r} h\right) f + 2\, h^2 f - 2\, h\, f\right)}{h^2\, f}$$

> R33 / R22;

$$\sin(\theta)^2$$