

**PHYSICS 410**

**THE 2D ISING MODEL**

```
function [E M] = ising2d(n, nt, T)
% ising2d: Simulates 2d Ising model with periodic boundary
% conditions, J = 1.
%
% Inputs
%
%     n:          Lattice size
%     nt:         Number of time steps
%     T:          Vector of temperatures [nT]
%
% Outputs
%
%     E:          Normalized energy [nT x nt]
%     M:          Normalized magnetization [nT x nt]
nT = length(T);
M = zeros(nT,nt);
E = zeros(nT,nt);
lat = randi([0 1],n,n) * 2 - 1;
trace = 100;

for iT = 1 : length(T)
    for it = 1 : nt
```

```
if trace & mod(it,trace) == 0
    fprintf('ising2d: Step %d of %d\n', it, nt);
end
for ii = 1 : n
    for jj = 1 : n
        % Handle periodic boundary conditions ...
        if ii == 1
            iiw = n;
        else
            iiw = ii - 1;
        end
        if ii == n
            iie = 1;
        else
            iie = ii + 1;
        end
        if jj == 1
            jjn = n;
        else
            jjn = jj - 1;
        end
        if jj == n
```

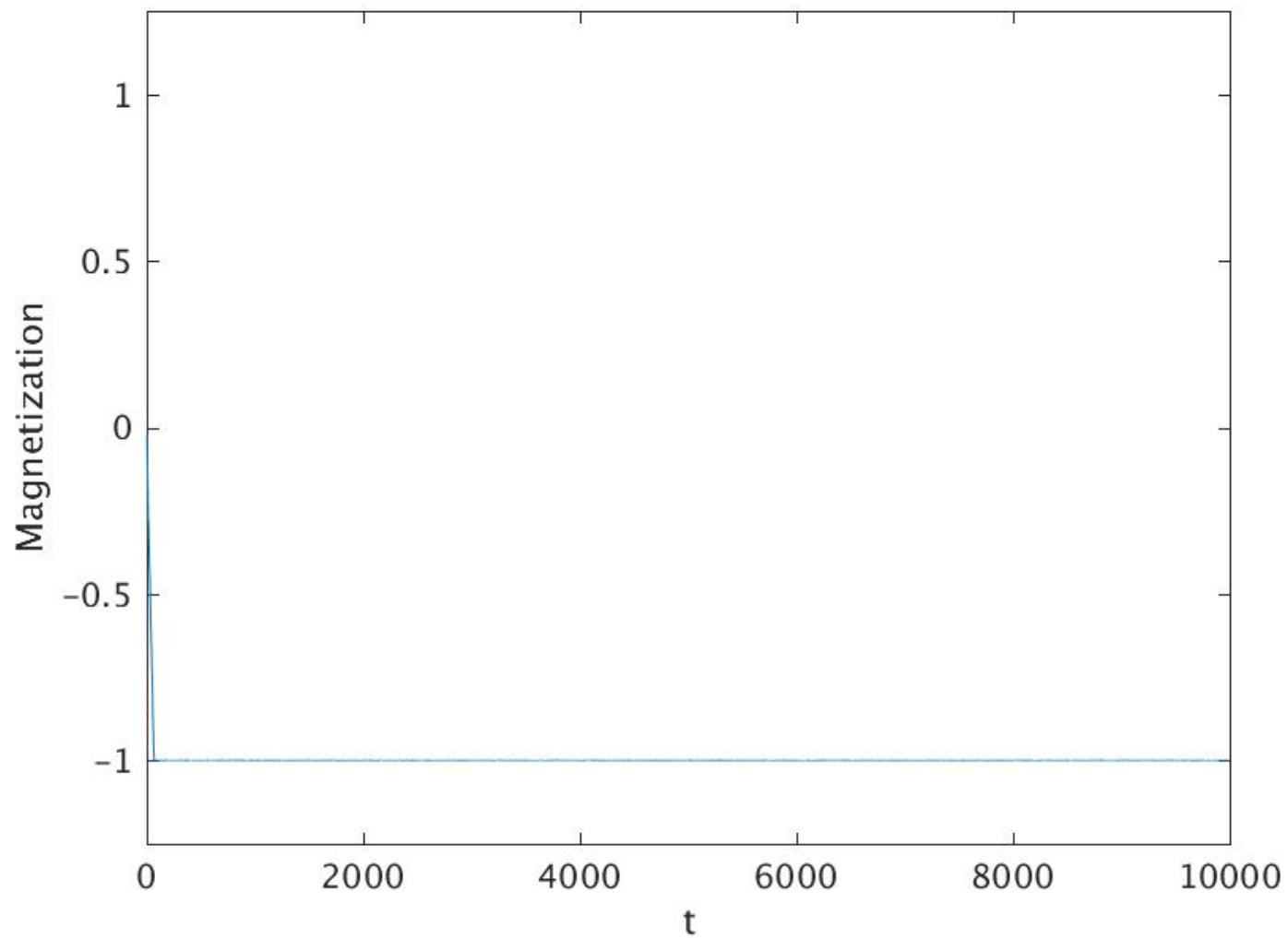
```

        jjS = 1;
    else
        jjS = jj + 1;
    end
    % Compute spin-flip energy cost ...
    Eloc = -lat(ii,jj) * (lat(iiw,jj) + lat(iie,jj) + ...
                           lat(ii,jjS) + lat(ii,jjn));
    E(iT,it) = E(iT,it) + Eloc;
    dE = -2 * Eloc;
    % Monte Carlo step ...
    if dE <= 0
        lat(ii,jj) = -lat(ii,jj);
    elseif rand(1,1) <= exp(-dE/T(iT))
        lat(ii,jj) = -lat(ii,jj);
    end
end
% Compute normalized energy and magnetization ...
E(iT,it) = E(iT,it) / (2 * n^2);
M(iT,it) = sum(sum(lat)) / n^2;
end
end
end

```

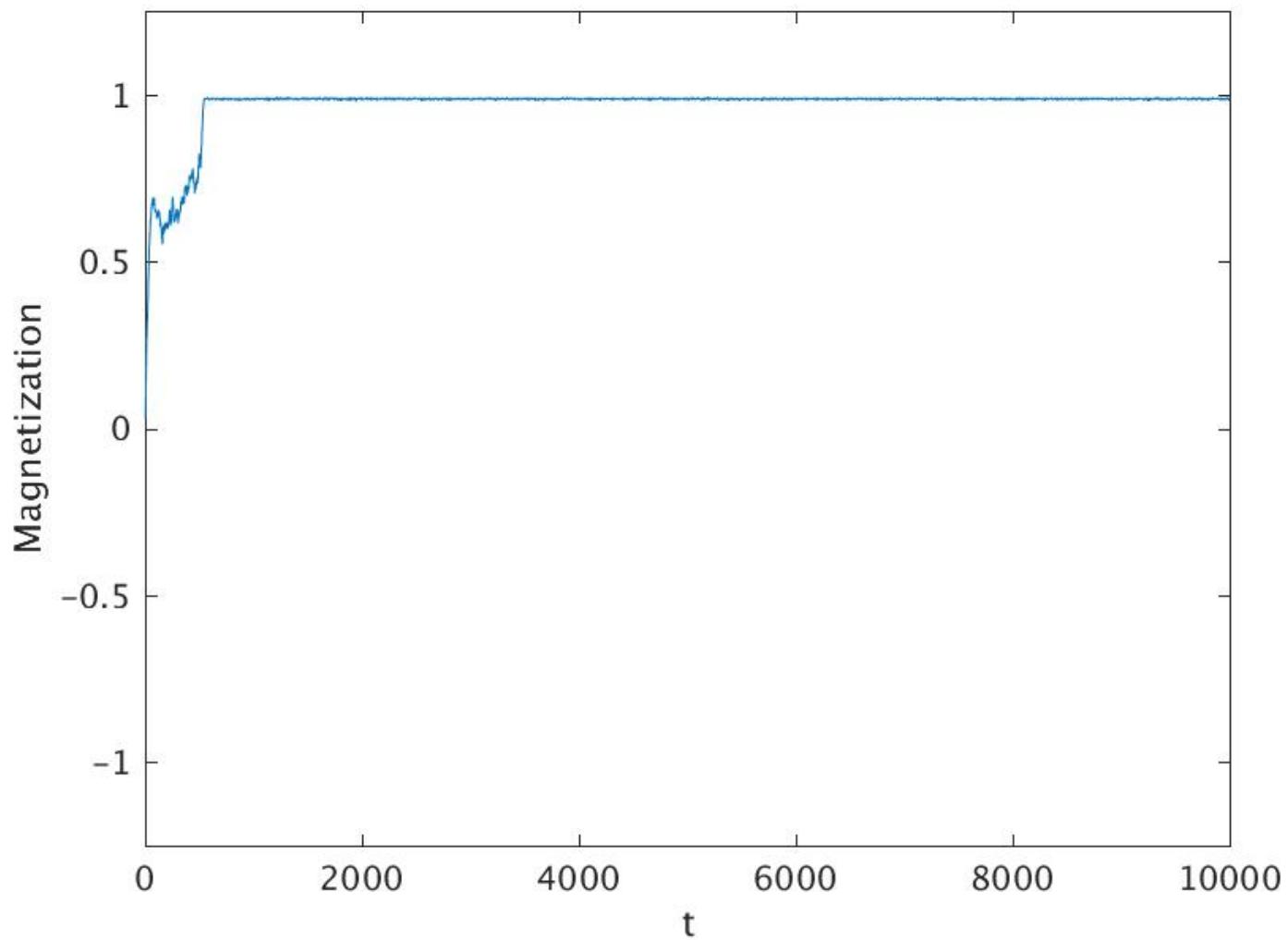
- Magnetization per spin vs discrete time

**Ising model: 100 x 100 lattice:  $T = 1$**



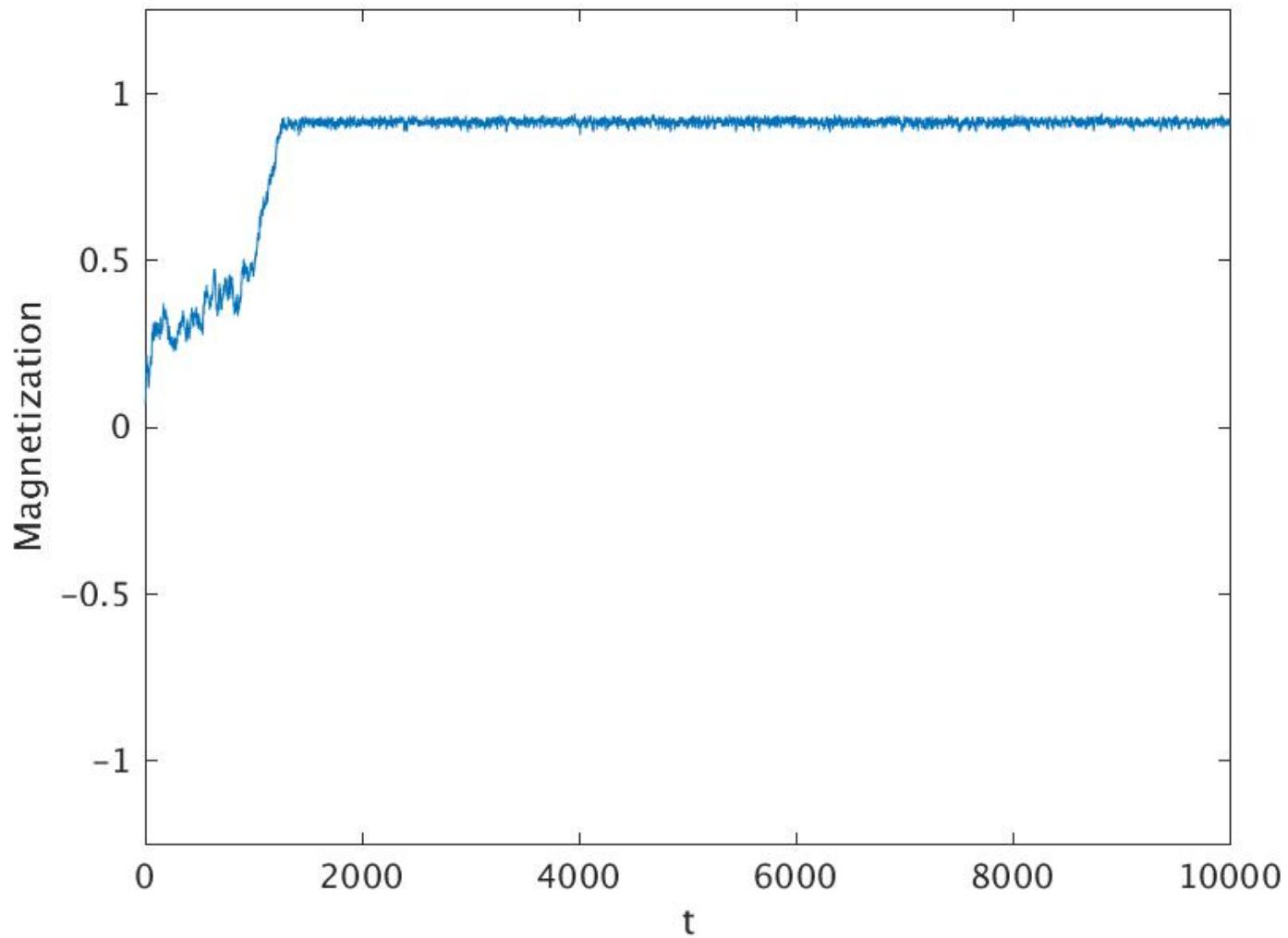
- Magnetization per spin vs discrete time

**Ising model: 100 x 100 lattice:  $T = 1.5$**



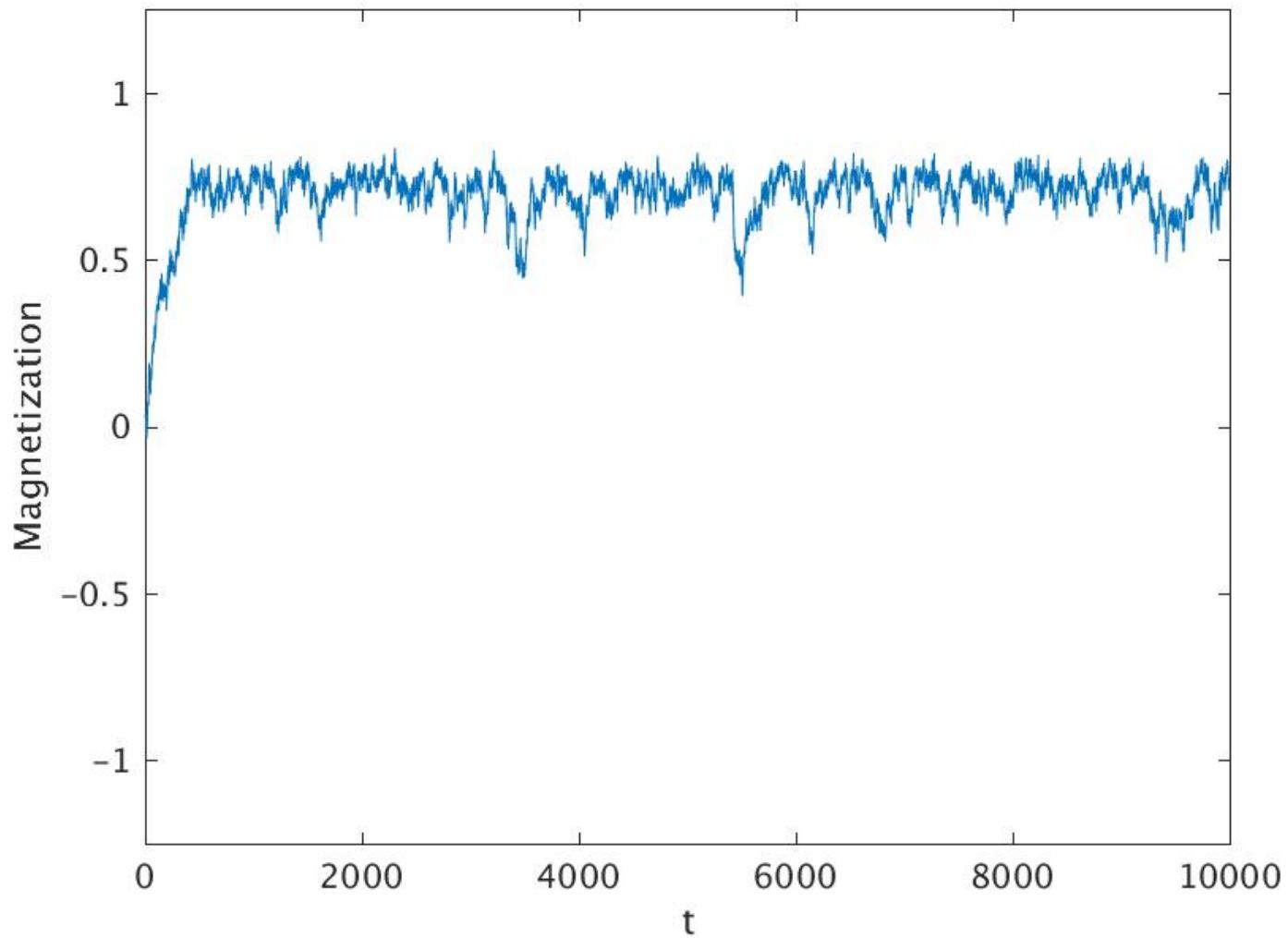
- Magnetization per spin vs discrete time

**Ising model: 100 x 100 lattice: T = 2**



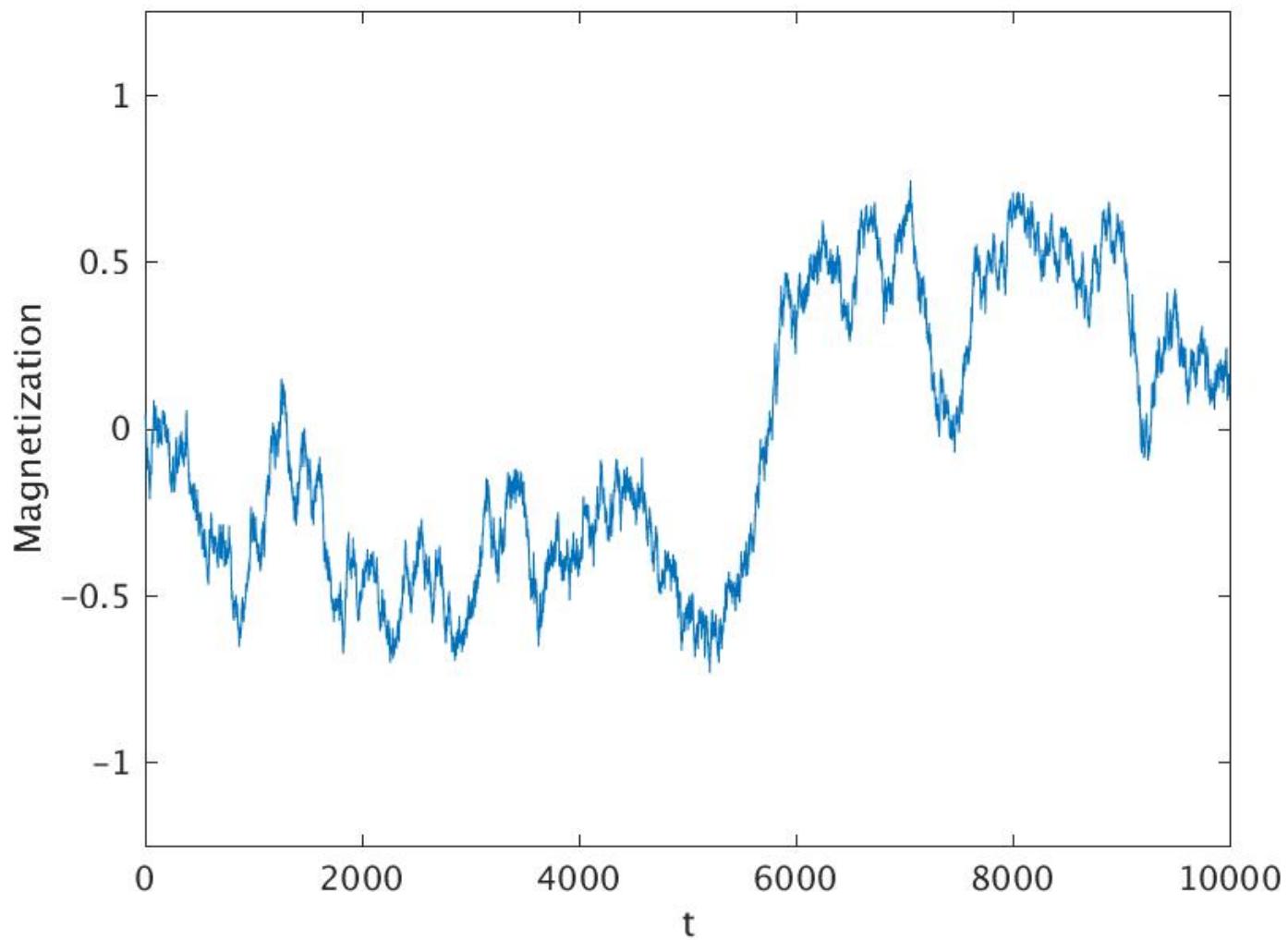
- Magnetization per spin vs discrete time

**Ising model: 100 x 100 lattice:  $T = 2.25$**



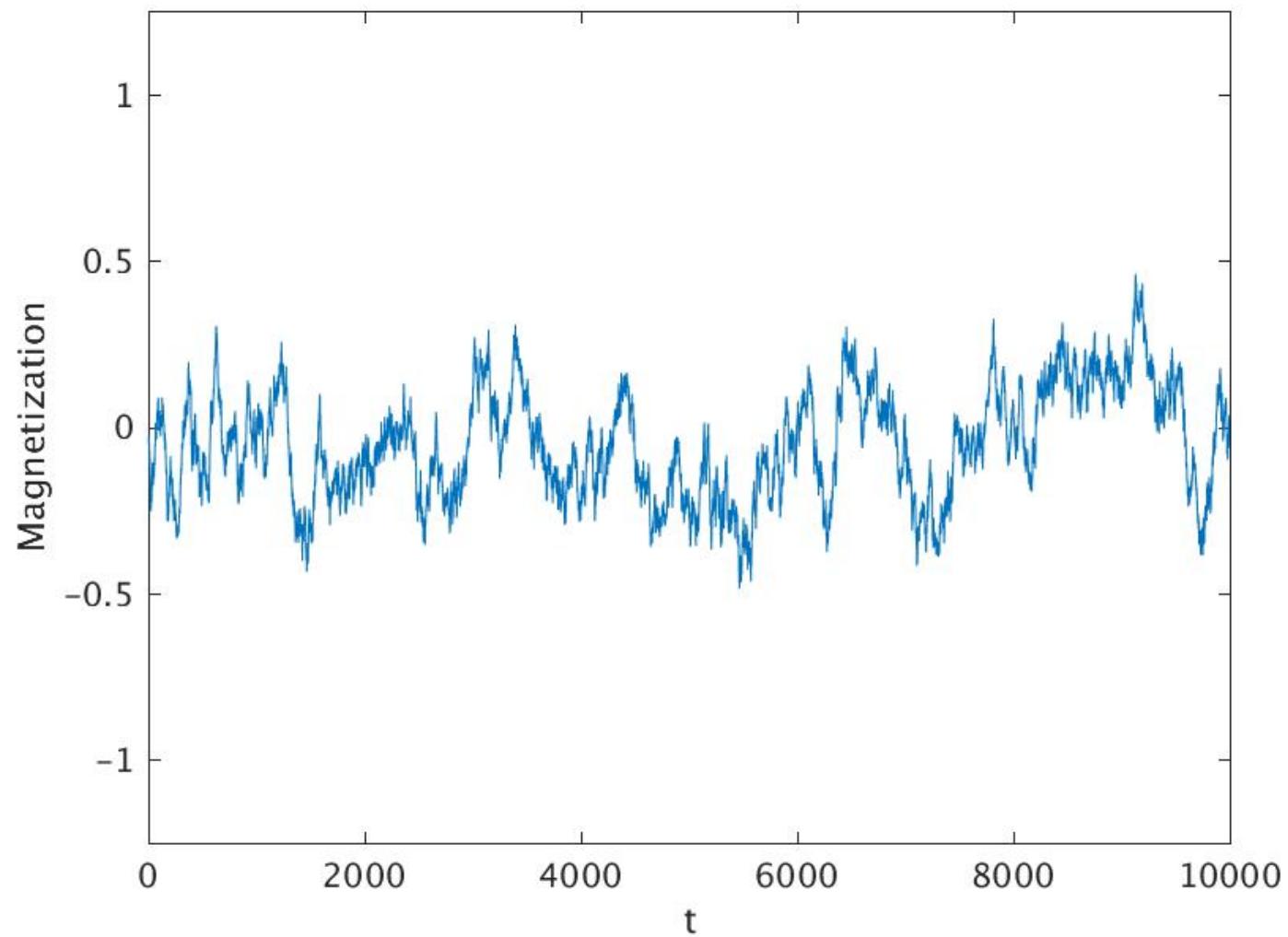
- Magnetization per spin vs discrete time

**Ising model: 100 x 100 lattice: T = 2.3**



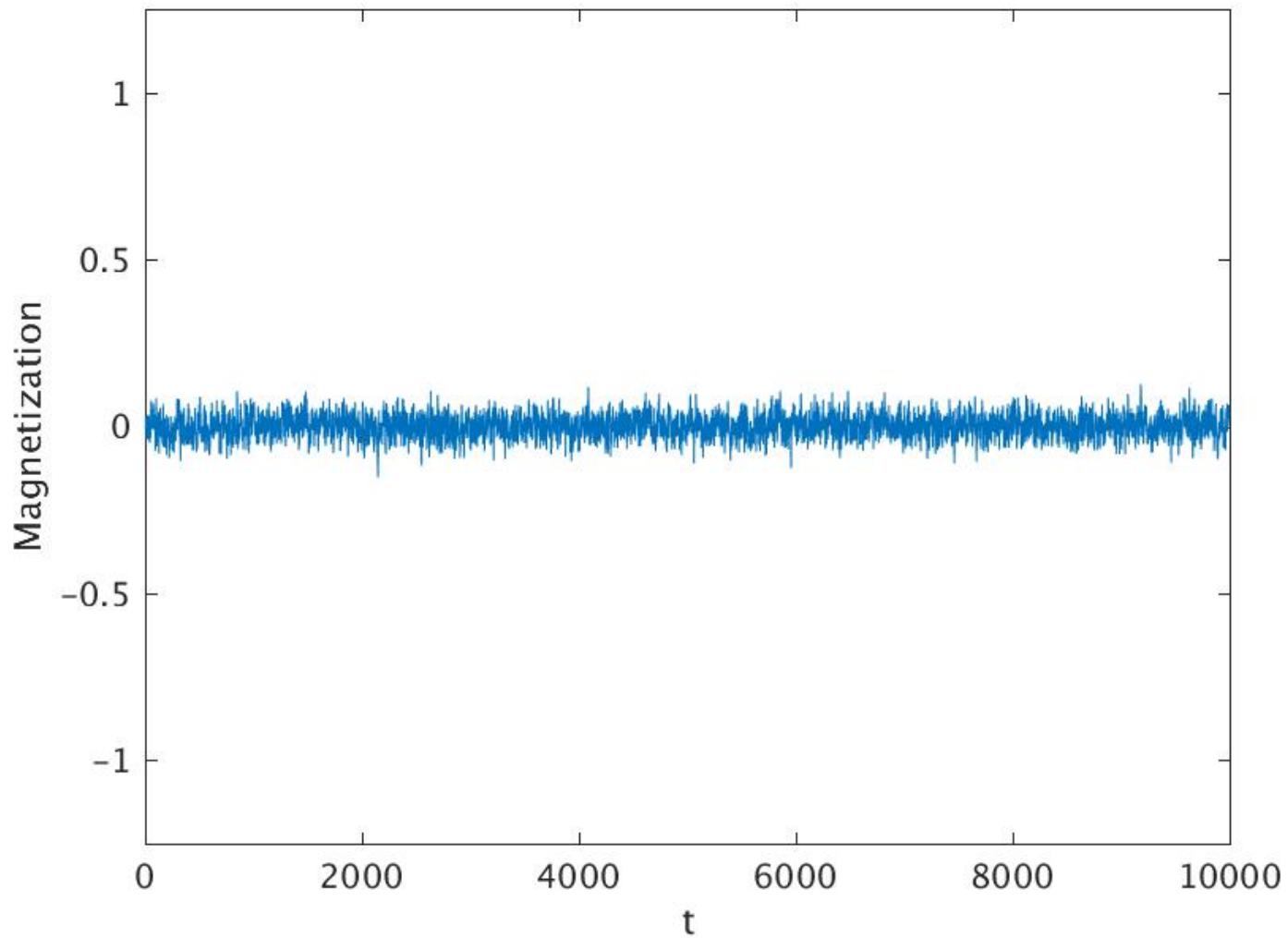
- Magnetization per spin vs discrete time

**Ising model: 100 x 100 lattice: T = 2.35**



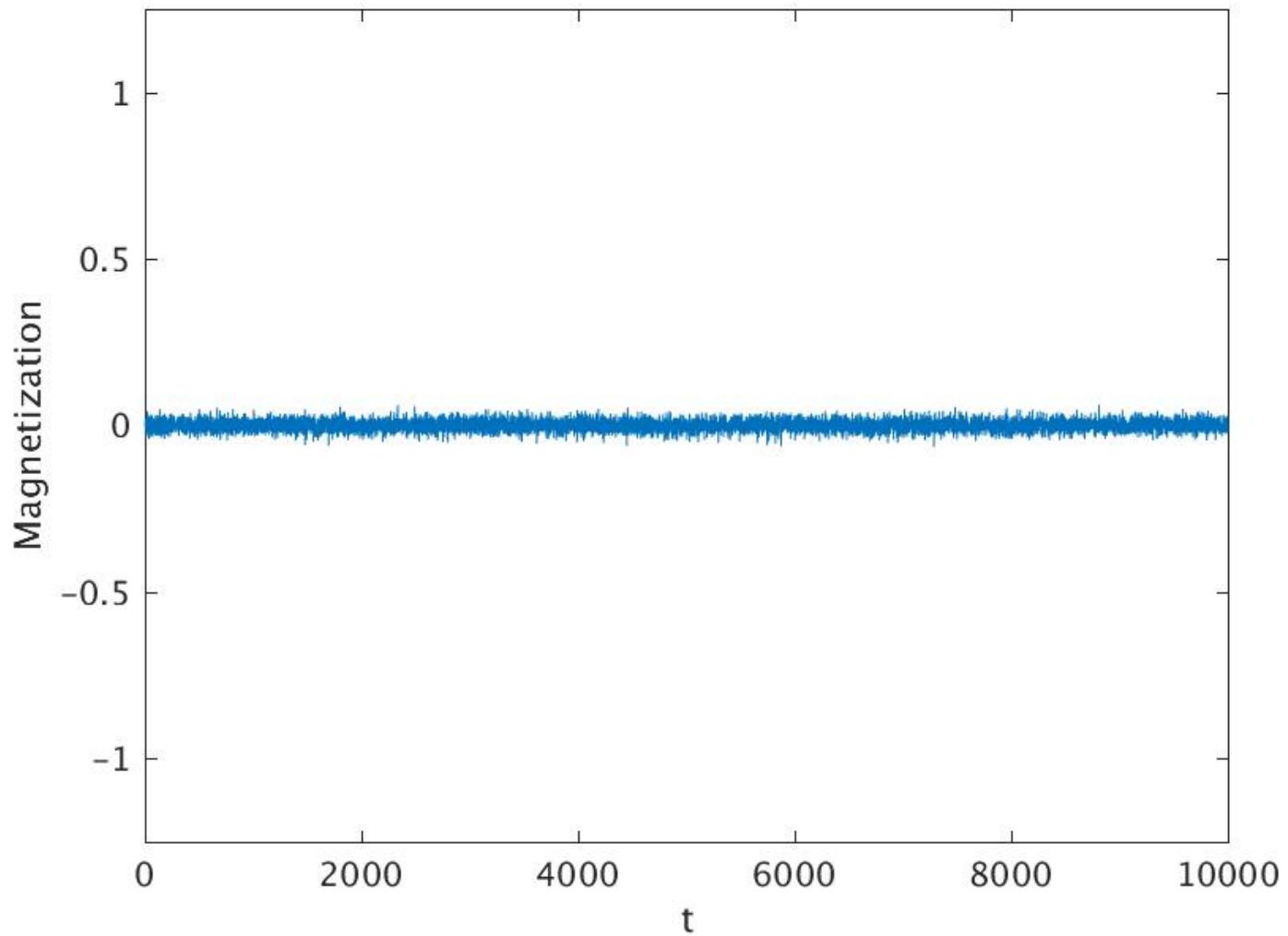
- Magnetization per spin vs discrete time

**Ising model: 100 x 100 lattice: T = 3**



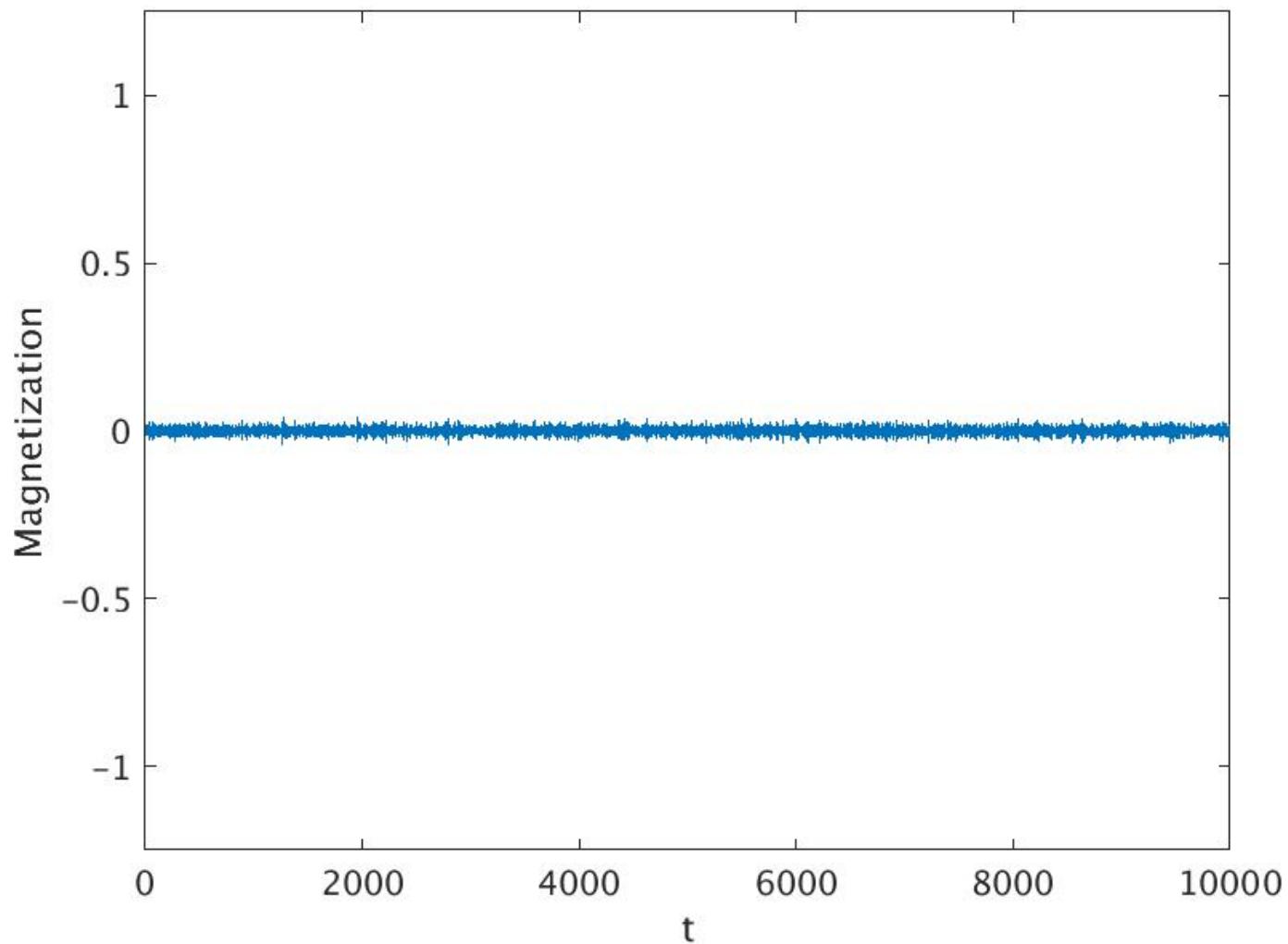
- Magnetization per spin vs discrete time

**Ising model: 100 x 100 lattice: T = 5**

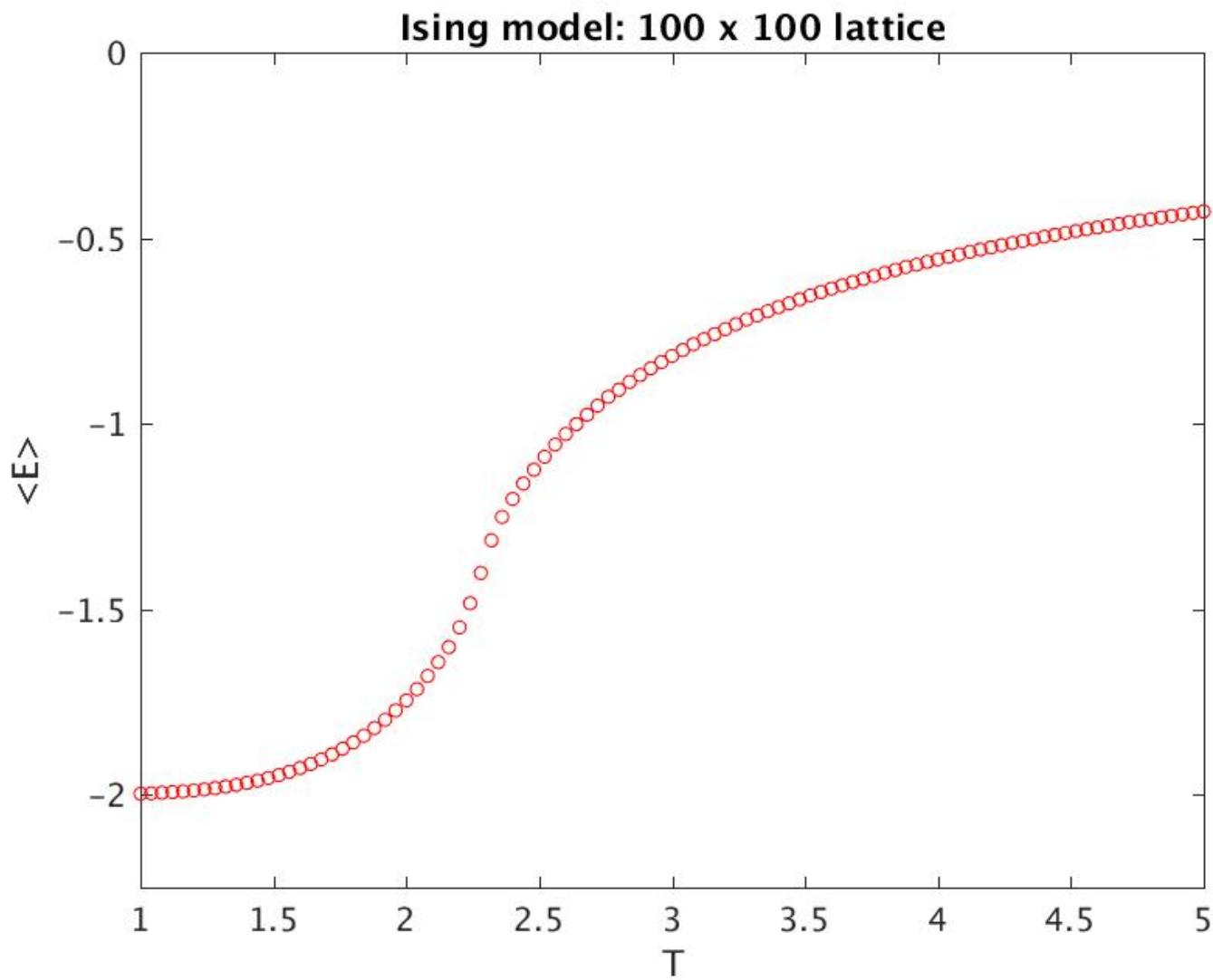


- Magnetization per spin vs discrete time

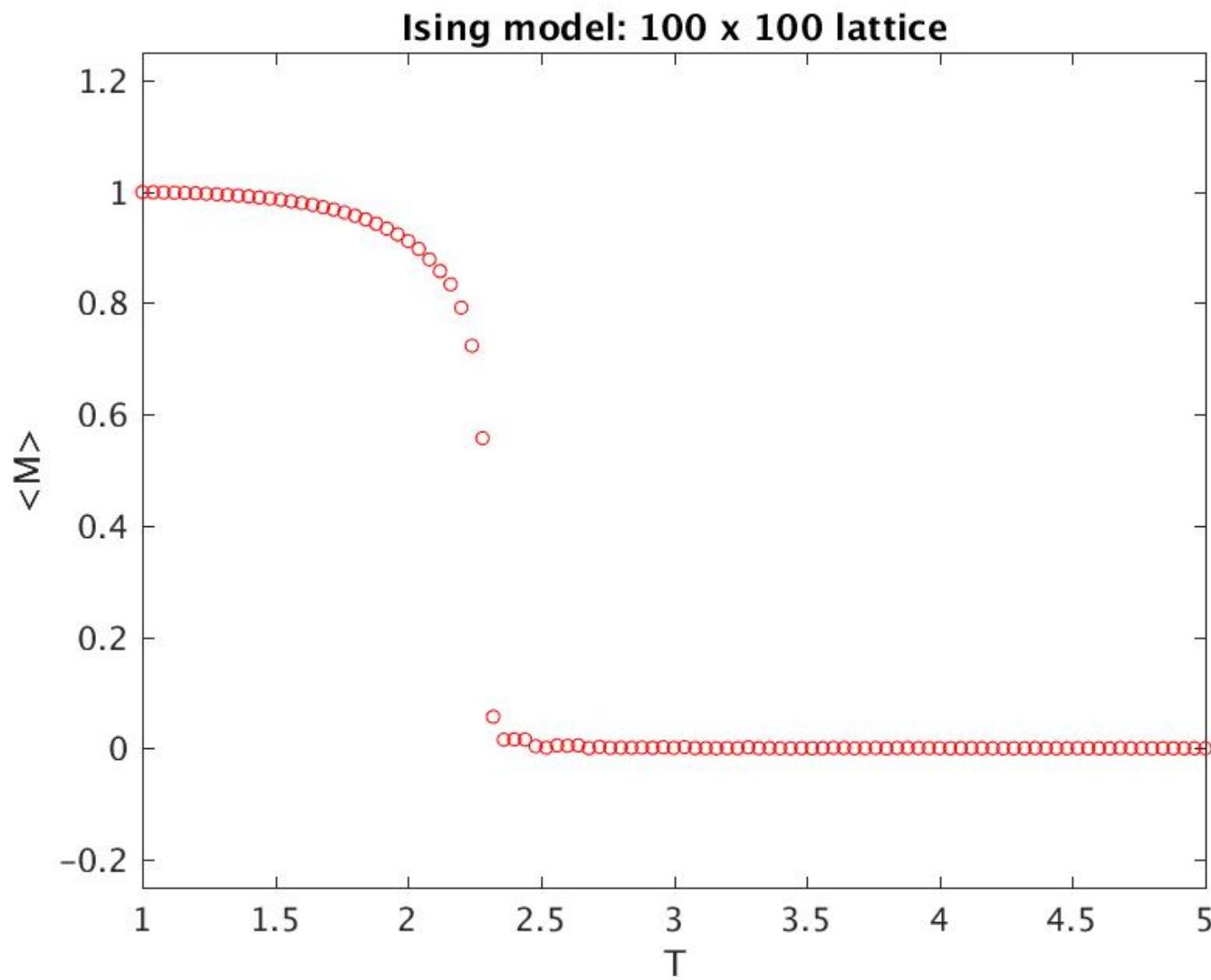
**Ising model: 100 x 100 lattice:  $T = 20$**



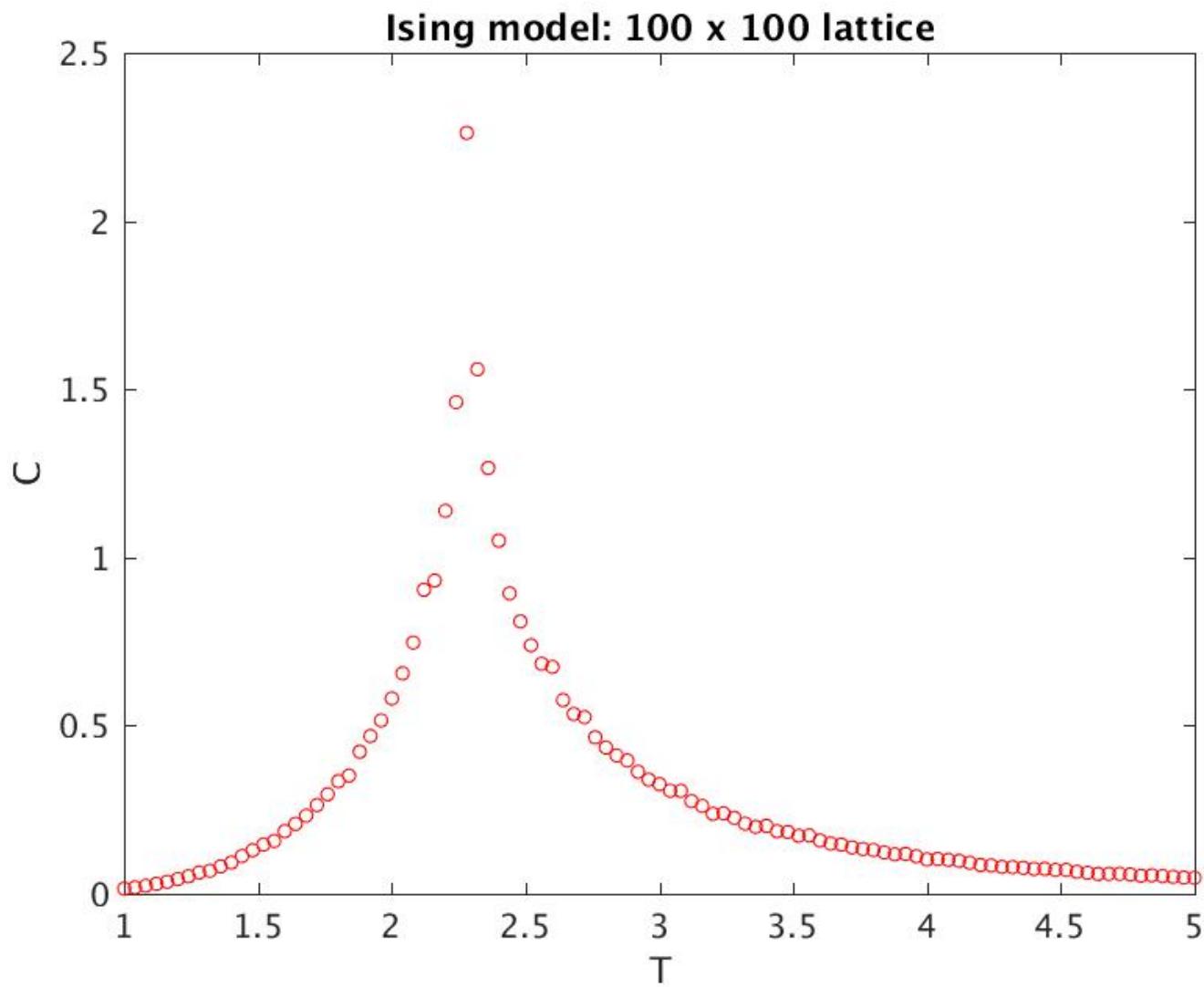
- Energy per spin vs temperature



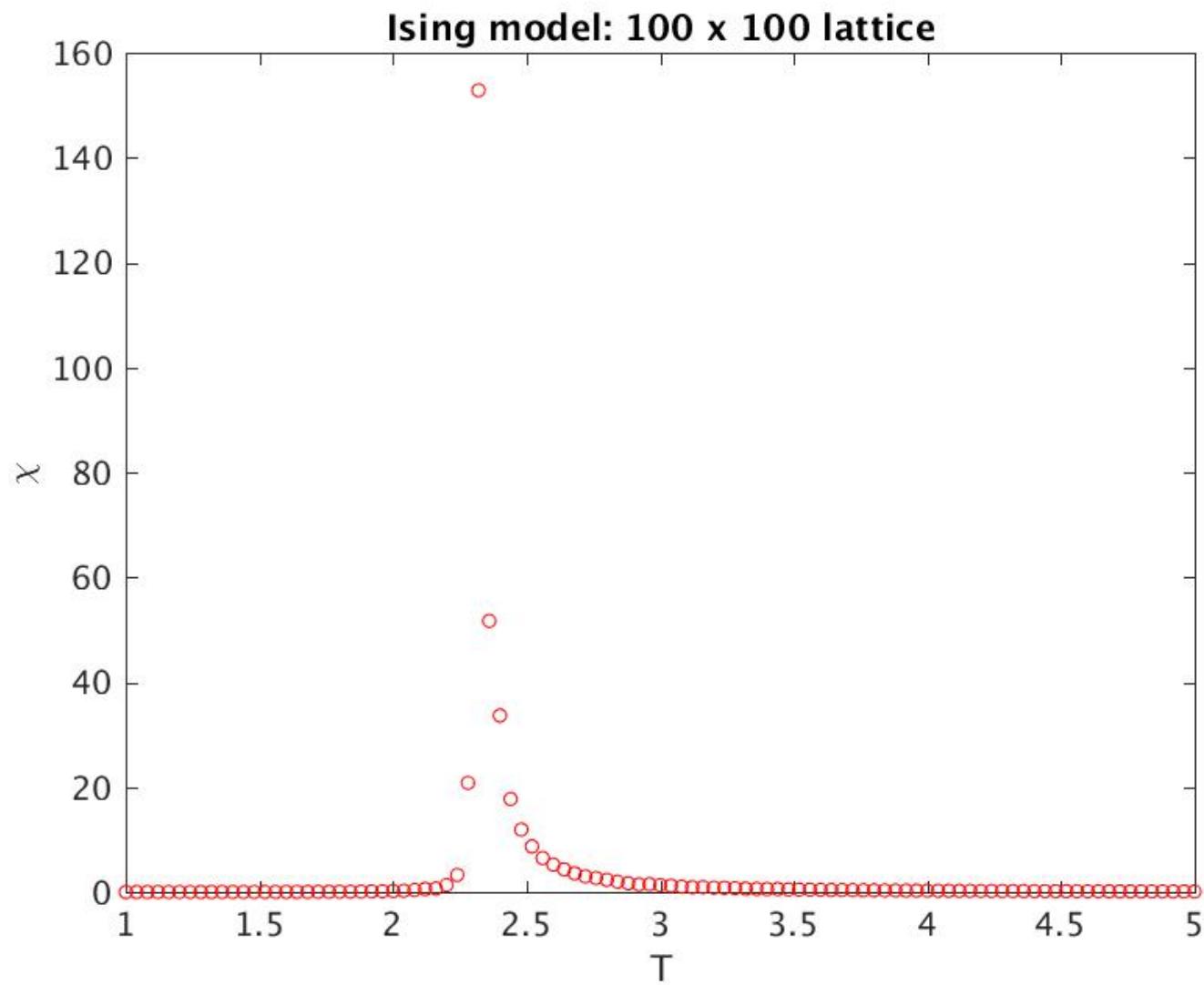
- Magnetization per spin vs temperature ( $T_C$  exact  $\approx 2.269$ )



- Heat capacity per spin vs temperature

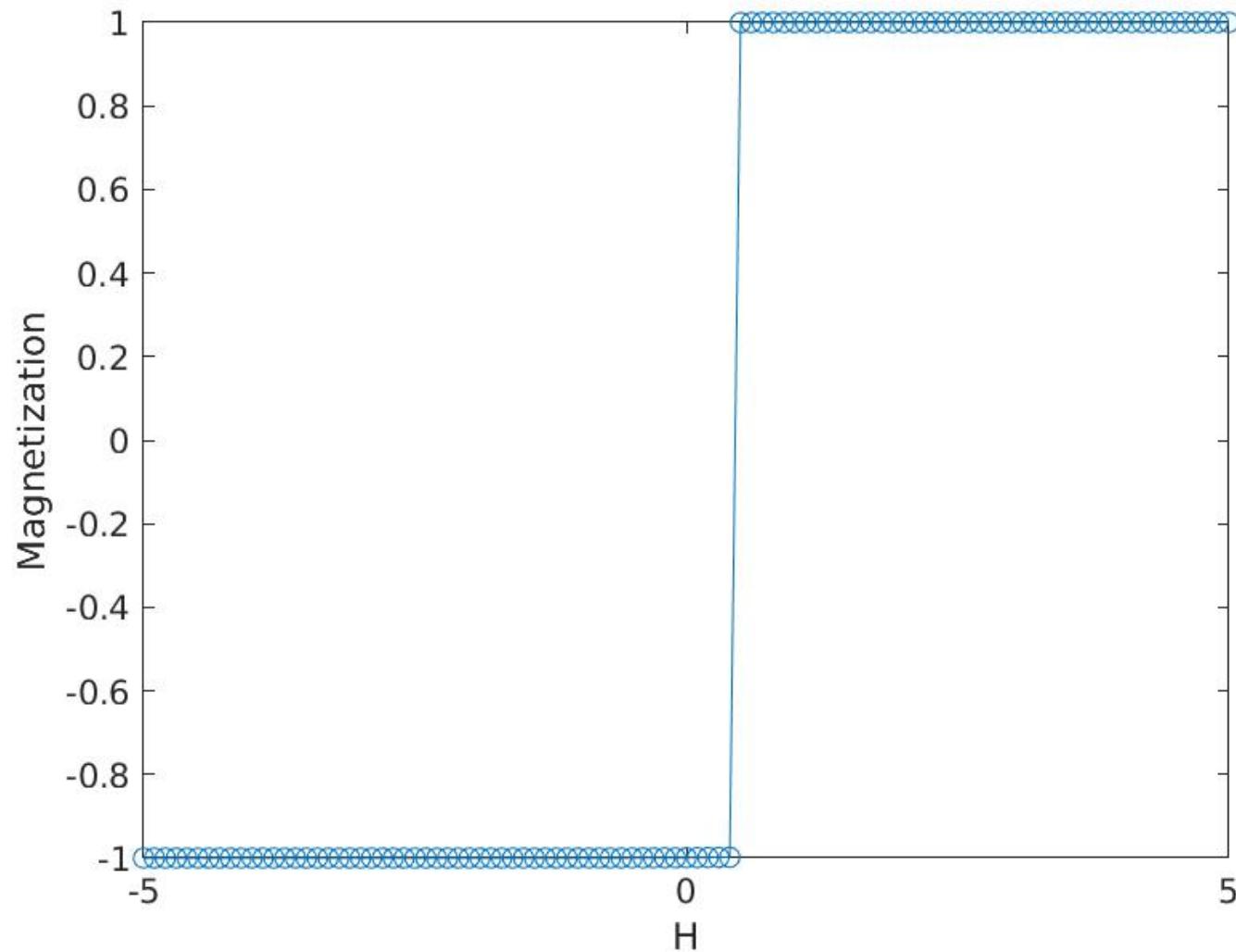


- Susceptibility per spin vs temperature



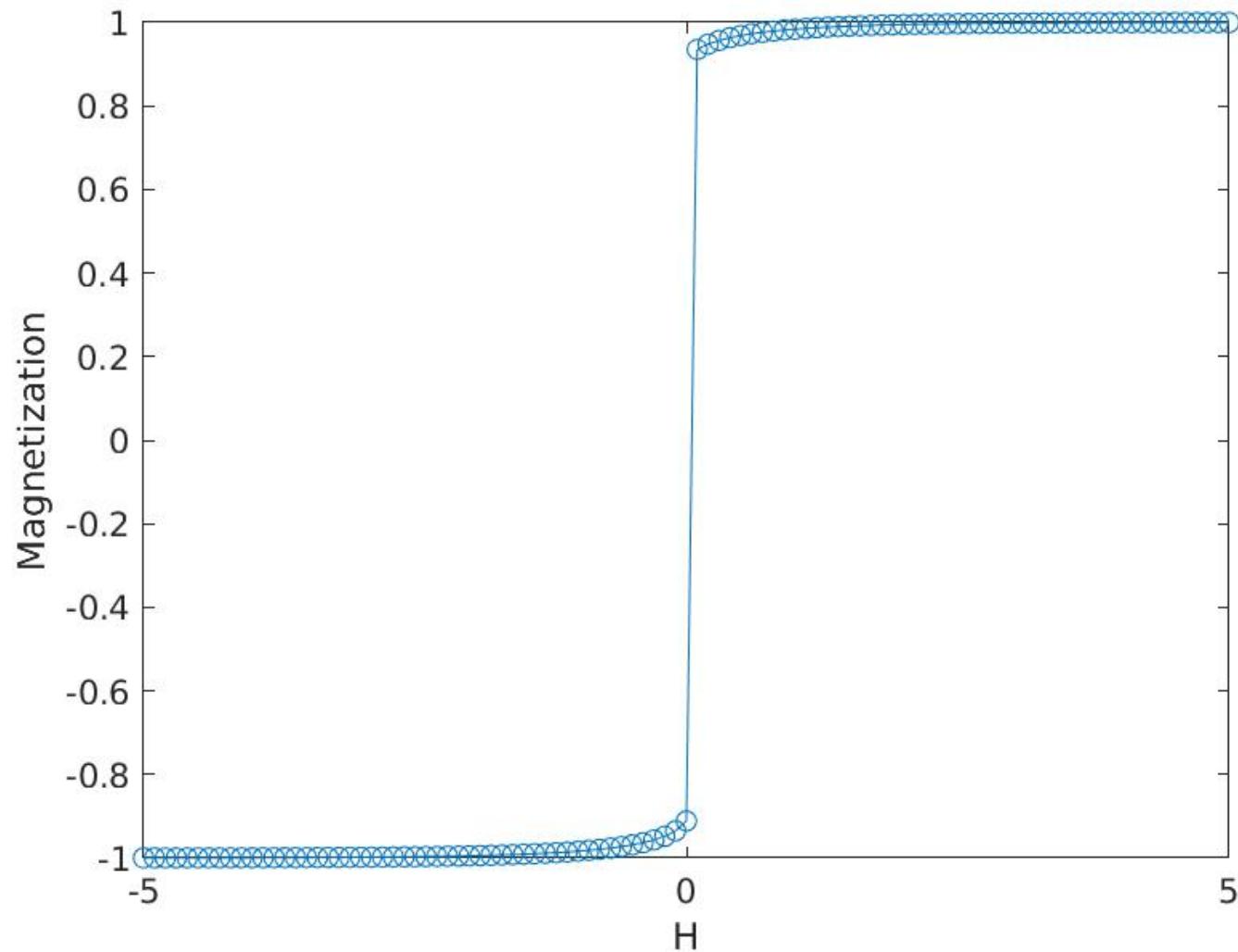
- Magnetization per spin vs magnetic field

**Ising model: 100 x 100 lattice: T = 1**



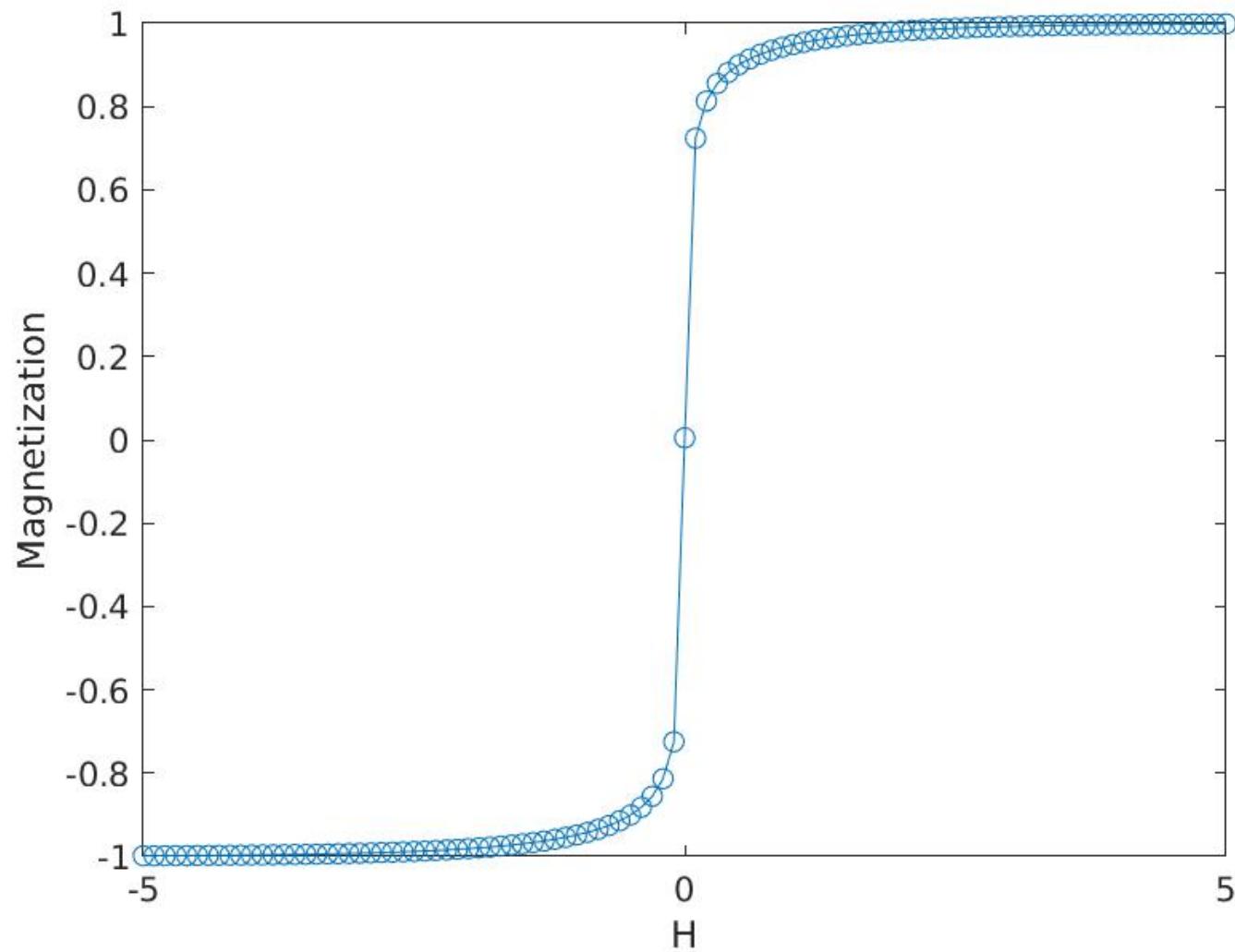
- Magnetization per spin vs magnetic field

**Ising model: 100 x 100 lattice: T = 2**



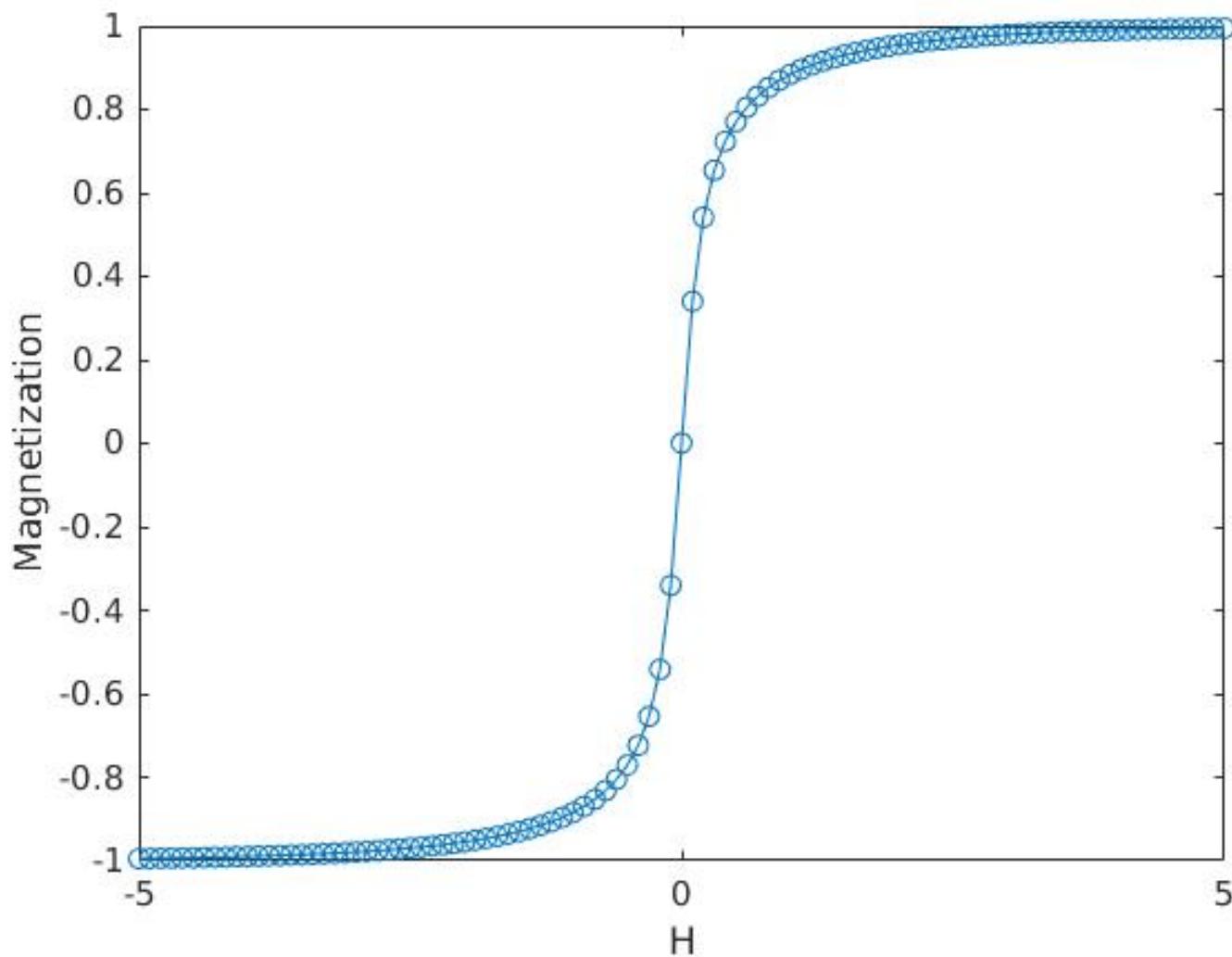
- Magnetization per spin vs magnetic field

**Ising model: 100 x 100 lattice: T = 2.5**



- Magnetization per spin vs magnetic field

**Ising model: 100 x 100 lattice: T = 3**



- Magnetization per spin vs magnetic field

**Ising model: 100 x 100 lattice: T = 5**

