

# Building the `series_op` Procedure Interactively

First define the series as in Abramowitz & Stegun

```
[ > s[1] := 1 + a[1]*x + a[2]*x^2 + a[3]*x^3;  
       $s_1 := 1 + a_1 x + a_2 x^2 + a_3 x^3$ 
```

```
[ > s[2] := 1 + b[1]*x + b[2]*x^2 + b[3]*x^3;  
       $s_2 := 1 + b_1 x + b_2 x^2 + b_3 x^3$ 
```

```
[ > s[3] := 1 + c[1]*x + c[2]*x^2 + c[3]*x^3;  
       $s_3 := 1 + c_1 x + c_2 x^2 + c_3 x^3$ 
```

Define a set of unknowns (the coeffs. of `s3`)

```
[ > unknowns := {c[1],c[2],c[3]};  
       $unknowns := \{c_1, c_2, c_3\}$ 
```

Define a 'shorthand' procedure for converting an expression to a polynomial

```
[ > P := proc(x) convert(x,polynom) end;  
       $P := \mathbf{proc}(x) \text{ convert}(x, \text{polynom}) \mathbf{end}$ 
```

## Define a specific series to re-express

```
> series_in := 1 / s[1];
```

$$\text{series\_in} := \frac{1}{1 + a_1 x + a_2 x^2 + a_3 x^3}$$

## Perform a series expansion to high enough order

```
> series(% , x=0 , 4 );
```

$$1 - a_1 x + (-a_2 + a_1^2) x^2 + (-a_3 + a_1 a_2 + (a_2 - a_1^2) a_1) x^3 + O(x^4)$$

## Convert the power series to a polynomial

```
> p1 := P(%);
```

$$p1 := 1 - a_1 x + (-a_2 + a_1^2) x^2 + (-a_3 + a_1 a_2 + (a_2 - a_1^2) a_1) x^3$$

## Subtract the converted series and s[3] (equivalent to equating the series)

```
> s[3] - p1;
```

$$c_1 x + c_2 x^2 + c_3 x^3 + a_1 x - (-a_2 + a_1^2) x^2 - (-a_3 + a_1 a_2 + (a_2 - a_1^2) a_1) x^3$$

## Extract the coefficients with respect to $x$

```
[ > coeffs(% , x) ;  
       $c_1 + a_1, c_2 + a_2 - a_1^2, c_3 + a_3 - a_1 a_2 - (a_2 - a_1^2) a_1$ 
```

**Covert the coefficient sequence to a set. Order by order, the coefficients must vanish, and Maple assumes "= 0" is there is no "=" in an equation**

```
[ > {%} ;  
       $\{c_2 + a_2 - a_1^2, c_3 + a_3 - a_1 a_2 - (a_2 - a_1^2) a_1, c_1 + a_1\}$ 
```

## Solve the set of equations for $c[1]$ , $c[2]$ , $c[3]$

```
[ > solve(% , unknowns) ;  
       $\{c_2 = -a_2 + a_1^2, c_1 = -a_1, c_3 = -a_3 + 2 a_1 a_2 - a_1^3\}$ 
```

Now read the `series_op` procedure from a plain-text file, and display the procedure definition.

```
[ > read series4;

> op(series_op) ;
proc(series_in::anything)
    solve( { coeffs(P(s[3]) - P(series(series_in, x = 0, 5)), x) },
    unknowns)
end
```

`series_op` returns a **SET** of equations which define the coefficients `c[1]`, `c[2]`, ... etc. in terms of the `a[i]` and `b[i]`. To extract the value of a specific coefficient, use the `subs` command.

Here's an example showing how to extract the coefficient `c[4]` for the case `s[3] := 1 / s[1]`

```
[ > s[1] := 1 + a[1]*x + a[2]*x^2 +
>          a[3]*x^3 + a[4]*x^4 ;
          s1 := 1 + a1 x + a2 x2 + a3 x3 + a4 x4

> subs( series_op( 1 / s[1]), c[4] );
          -a4 + 2 a1 a3 + a22 - 3 a2 a12 + a14
```