The Role of Angular Momentum in the Critical Collapse of an Ideal Gas in Newtonian Gravity

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Outline

Motivation

Critical Gravitational Collapse (General Relativity/Newtonian Gravity)

$\Rightarrow$ Critical Phenomena

- Discovered by Choptuik (1993)–Scalar field
- Model dependent *e.g* Matter type, see Gundlach C. lrr-2007-5
- Far-reaching theoretical implications

Studied using Numerical methods

- Time consuming and high cost of computational resources
- Spherical symmetry
- Role of rotation largely unexplored

Rotation is a generic feature of gravitational collapse Investigate the role of angular momentum on critical fluid collapse

Critical Phenomena in Gravitational Collapse

- Model-independent description
- Initial data: one-parameter family p
  - $p < p^\star$: *subcritical*–weak field (explodes/disperses)
  - $p > p^\star$: *supercritical*–strong field (black hole (BH) formation)
- Tune $p = p^\star \Rightarrow$ critical solution (solution to the equations of Motion (EoM))

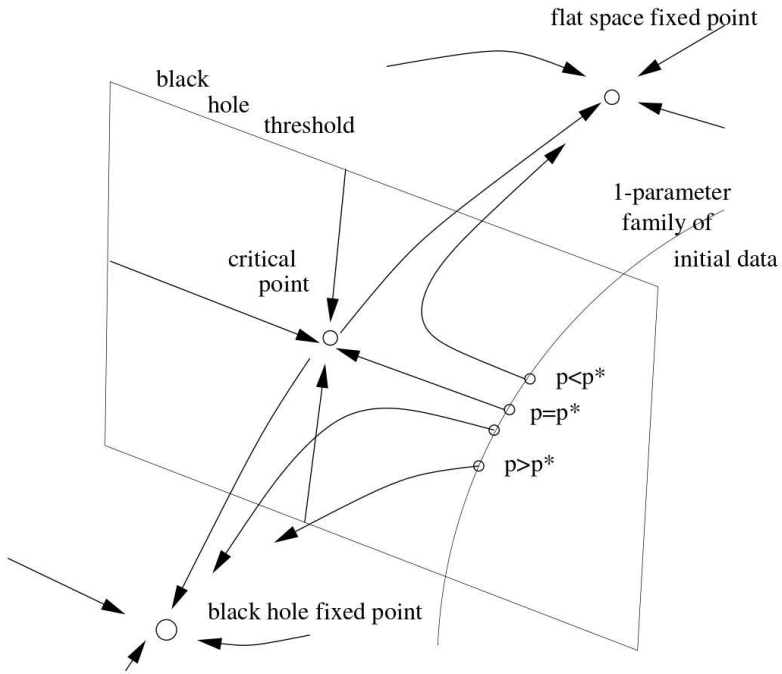


Figure: Credit: Gundlach C. lrr-2007-5

The Critical Solution

- Special mode structure in perturbation theory $\Rightarrow$ single unstable mode.
- Scaling laws.

Analogous to phase transitions in statistical mechanics. Find two types of threshold behavior:

- The order-parameter is the BH mass.
- Type I (with mass gap): static, meta-stable configurations
  - *e.g.* $Z(r,t) = Z^\star(r)$
  - $t \propto \sigma \ln |p - p^\star|$
- Type II (no mass gap): self-similar solutions
  - *e.g.* $Z(\eta r, \eta t) = Z(r,t) \equiv Z^\star(r/t)$ $(\eta \in \mathbb{R})$
  - $M_{\rm BH} \propto |p - p^\star|^\gamma$

σ and γ are the reciprocal of the Lyapunov exponent of the unstable mode

Perfect Fluid–Previous Work (Mostly with Equation of State, $P = k\rho$)

- GR Simulations for $k = 1/3 \Rightarrow$ Continuous self-similar (CSS) solution (type-II), $\gamma \approx 0.36$, Evans and Coleman (1994), Koike *et. al.* (1995)
- Critical solution and exponent γ showed dependence on k ($0 < k \leqslant 1$) Maison (1996), Neilsen and Choptuik (2000)
- Exploration of the $k \to 0$ (Newtonian limit (Ori and Piran (1990)) $\Rightarrow$ measurement of $\gamma \sim 0.11$–predicted type-II critical phenomena would emerge in Newtonian isothermal collapse, Harada and Maeda (2001)
- Evidence of type-II critical phenomena was found in critical collapse of an isothermal gas in Newtonian gravity Harada and Maeda (2001), (2003)

Are the parallels between critical fluid collapse in GR and Newtonian gravity preserved upon the addition of rotation?

Critical Collapse with Infinitesimal Initial Rotation

Perturbative analysis (GR perfect fluid with $P = k\rho$ EoS), Gundlach C. (2002)

- Non-spherical perturbations: unstable $\ell = 1$ axial mode, $\lambda_1(k)$
- Four-parameter (p, $\vec{q}$) families of initial data–($p = p^\star, \vec{q} = 0$)$\rightarrow$ Spherically symmetric CS
- Mass scaling: $M_{\rm BH} \propto |p - p^\star|^{1/\lambda_0}$
- Specific angular momentum scaling: $\vec{a}_{\rm BH} \propto \vec{q}|p - p^\star|^{(1-\lambda_1)/\lambda_0}$

Newtonian version (isentropic gas $P = K\rho^\Gamma$):

- Unstable axial mode (*Spin-up* mode) $\lambda_1 = 1/3$
- Collapsed mass scaling: $M \propto |p - p^\star|^{(4-3\Gamma)/\lambda_0}$
- Specific angular momentum scaling: $\vec{a} \propto \vec{q}|p - p^\star|^{(3-2\Gamma-\lambda_1)/\lambda_0}$

Hydrodynamics+Gravity

- Model: Euler's equations coupled to Newtonian gravity and subject to a polytropic equation-of-state (EoS)

$$
\begin{aligned}
\frac{\partial \rho}{\partial t} + \sum_i \frac{\partial}{\partial x_i}(\rho v_i) &= 0, \\
\frac{\partial(\rho v_j)}{\partial t} + \sum_i \frac{\partial}{\partial x_i}(\rho v_i v_j + P\delta_{ij}) &= -\rho \frac{\partial \Phi}{\partial x_j}, \\
\frac{\partial E}{\partial t} + \sum_i \frac{\partial}{\partial x_i}((E+P)v_i) &= -\rho \sum_j v_j \frac{\partial \Phi}{\partial x_j}, \\
\nabla^2 \Phi &= 4\pi G \rho, \\
P &= (\Gamma - 1)\rho\epsilon
\end{aligned}
$$

Where, $E = \rho\epsilon + \frac{1}{2}\sum_i \rho v_i^2$ and $\epsilon(\vec{x}, t)$ is the specific internal energy $\rho(\vec{x}, t),\ \vec{v}(\vec{x}, t),\ P(\vec{x}, t), \Phi(\vec{x}, t)$ and $\epsilon(\vec{x}, t)$.

Numerical Methods: Finite-Volumes

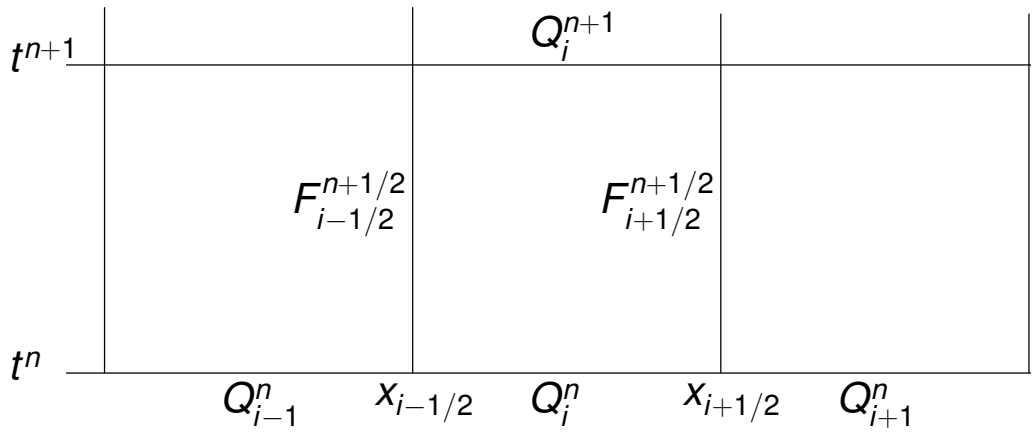
- The fluid model: Vector-conservation laws with sources

$$\frac{\partial \mathbf{q}}{\partial t} + \sum_j \frac{\partial \mathbf{F}^j(\mathbf{q})}{\partial x^j} = \mathbf{S}(\mathbf{q}, \vec{x})$$

- Integral formulations $\Rightarrow$ discrete-form for cell averages $\{Q_j^n\}$ (1-D example)

$$\frac{Q_j^{n+1} - Q_j^n}{\Delta t} + \frac{F_{j+1/2}^{n+1/2} - F_{j-1/2}^{n+1/2}}{\Delta x} = S_j^{n+1/2}$$

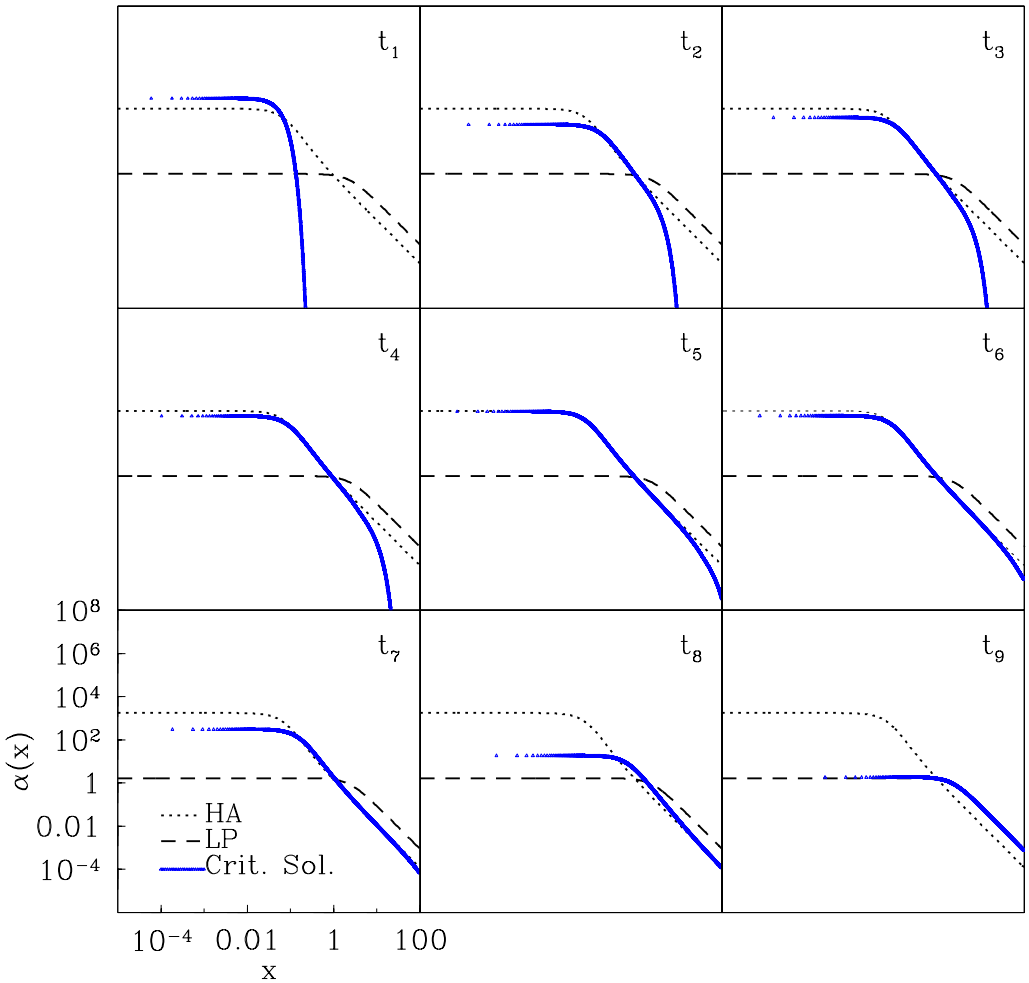
- Numerical Flux $F_{j+1/2}^{n+1/2}$
  - Apply Roe/HLLC Approximate Riemann solvers



- Gravity (Poisson's equation): finite differencing
- Adaptive Mesh Refinement (AMR)
- Parallel computations: PAMR–infrastructure (Pretorius)

Spherically symmetric critical collapse

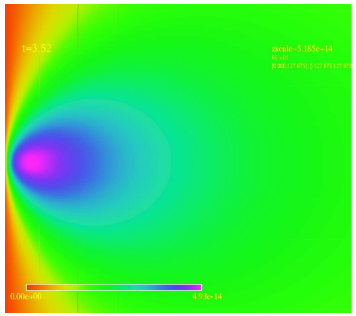
- Evidence of type-II critical phenomena $\Gamma \in [1,\, 6/5)$
  - Scale-invariant solution: $Z^{\star}_{\Gamma}(x \propto r/t^{2-\Gamma})$ ('Hunter-A')
    - Example: dimensionless density $\alpha(x) \propto t^2 \rho(r,t)$ ($\Gamma = 1.00001$)
- Type-I behavior 'turns on' at $\Gamma \geq 6/5$



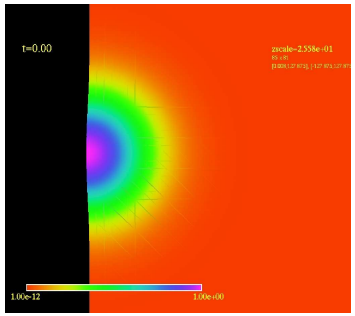
Axi-symmetric Simulations: $\Gamma = 1.00001$ and $q = 10^{-14}$

- Solved Euler's equations in Axial symmetry
- Study critical collapse
  - Initial conditions [generic]: three types, A, B, and C

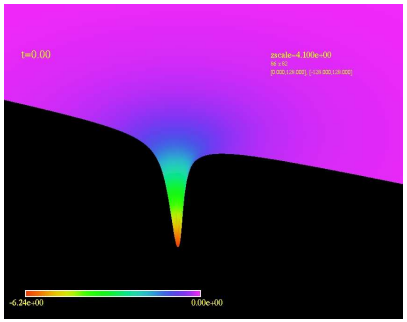
Example: Model A



v_ϕ



ρ



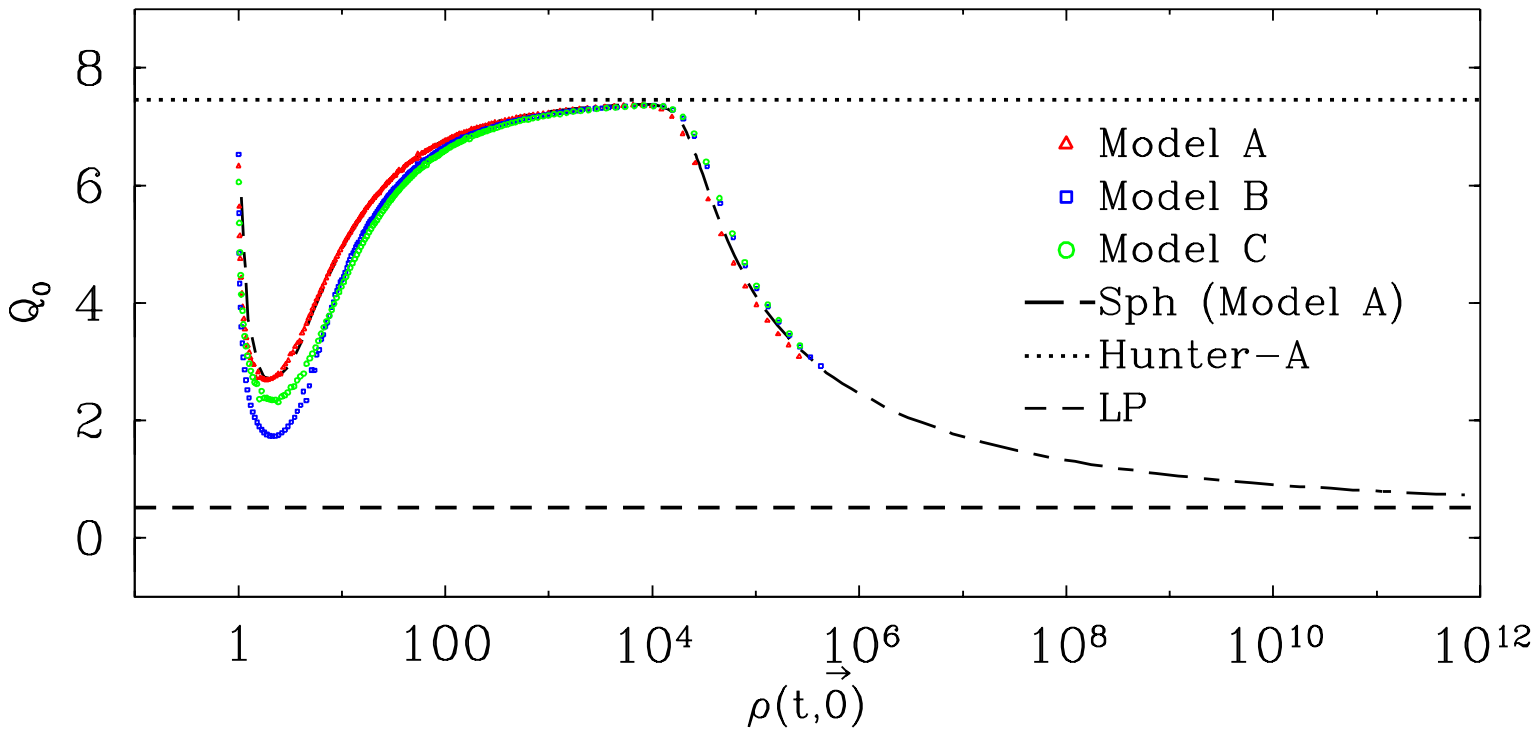
Φ

- Observed growth of axial perturbation: angular dependence $\propto \dfrac{\partial}{\partial\theta} Y_1^0(\theta,\phi) \sim \sin\theta$ (spin-up)
- Scalar quantities such as density ρ and Φ remain spherically symmetric

Convergence to the Hunter-A solution: evolution of Q_0

$Q_0(t) = \ln(4\pi G(t_0 - t)^2 \rho(t, 0, 0))$

- Q_0 characterizes the similarity solutions *i.e.* Hunter-A, Larson-Penston, ...
  - Hunter-A: $Q_0 \approx 7.46$ at $\Gamma = 1.00001$
  - Larson-Penston: $Q_0 \approx 0.51$ at $\Gamma = 1.00001$

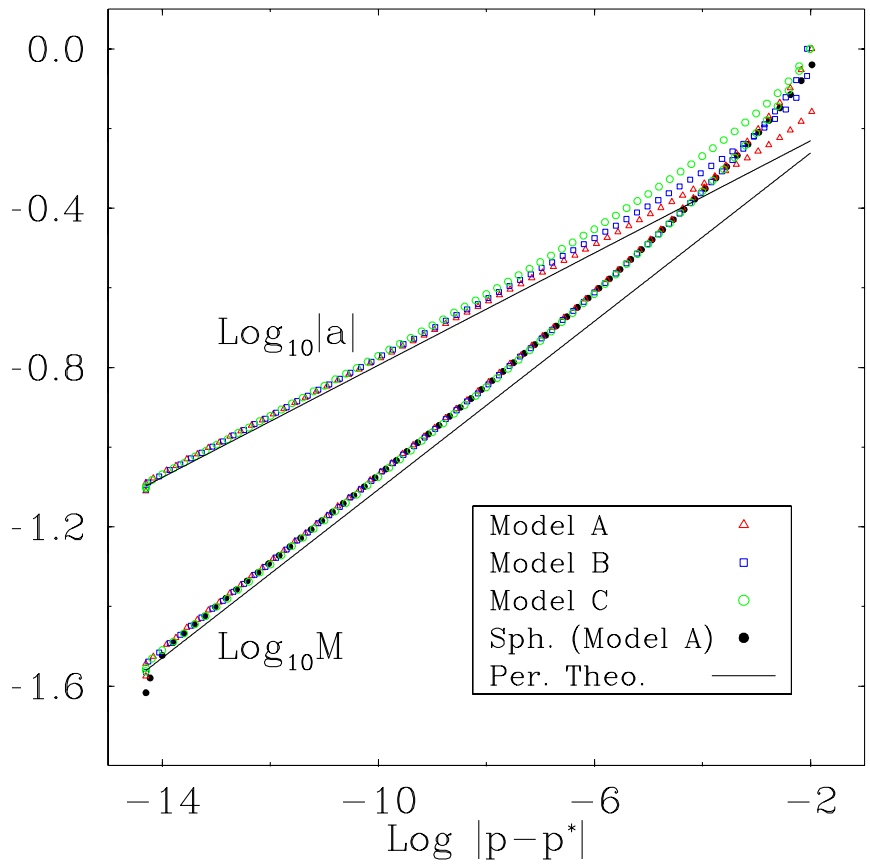


- Evidence of Convergence and Universality

Scaling Laws ($\Gamma = 1.00001$, $q = 10^{-14}$)

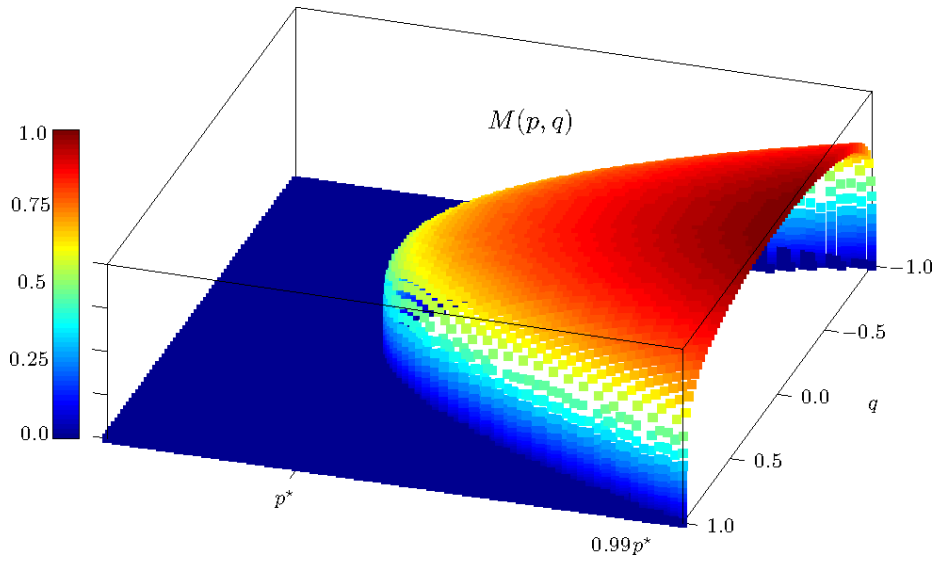
The critical evolutions are described by linear regime (Hunter-A plus 2-modes)

- Order-parameters: $M(\text{collapsed})$ and $\vec{a}$
  - $M \propto |p - p^\star|^{(4-3\Gamma)/\lambda_0}$
  - $\vec{a} \propto \vec{q}|p - p^\star|^{(3-2\Gamma-\lambda_1)/\lambda_0}$
- Agreement with results from perturbation analysis ($\lambda_0 = 9.46430101$, $\lambda_1 = 1/3$)
- Found evidence of Universality



Parameter Survey: M

- Found critical curve on $p - q$ plane (64×64 grid)
  - Fine tune parameters along the two independent directions
  - For small q–behavior of the collapsed mass is universal near the critical curve (no mass gap)



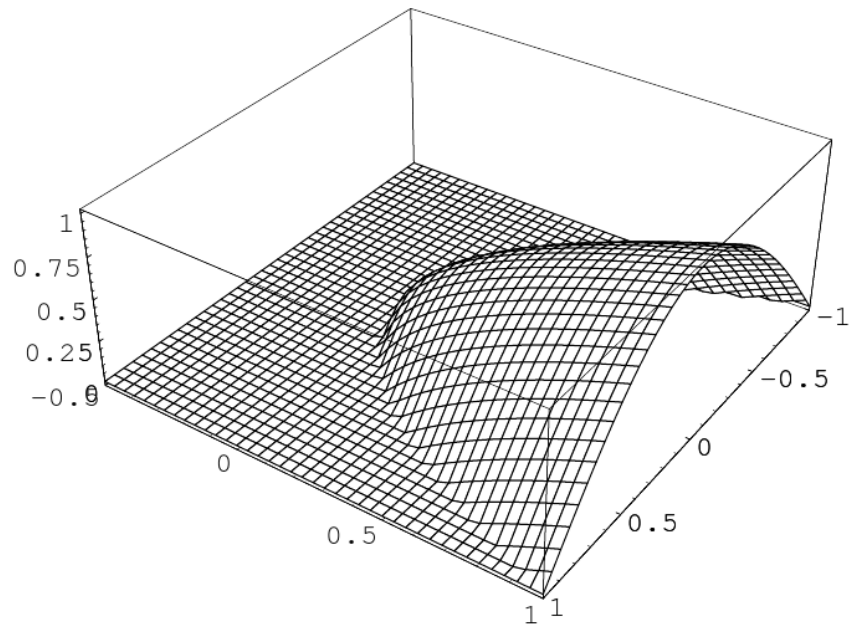
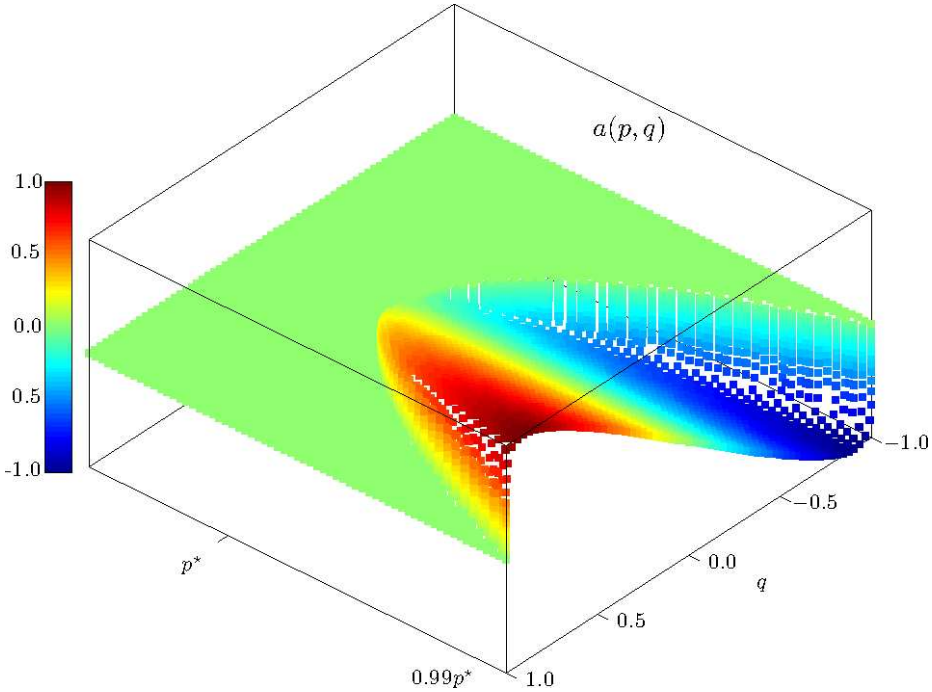


Figure: Credit: Gundlach C. Phys. Rev. D 65 (2002)

- Calculations are similar to Gundlanch's predictions (right plot)

Parameter Survey: $\vec{a}$

- Angular momentum of compact object ($\vec{a}$) aligns with $\vec{q}$ from the initial state
- Scaling of $\vec{a}$ is universal across the critical curve



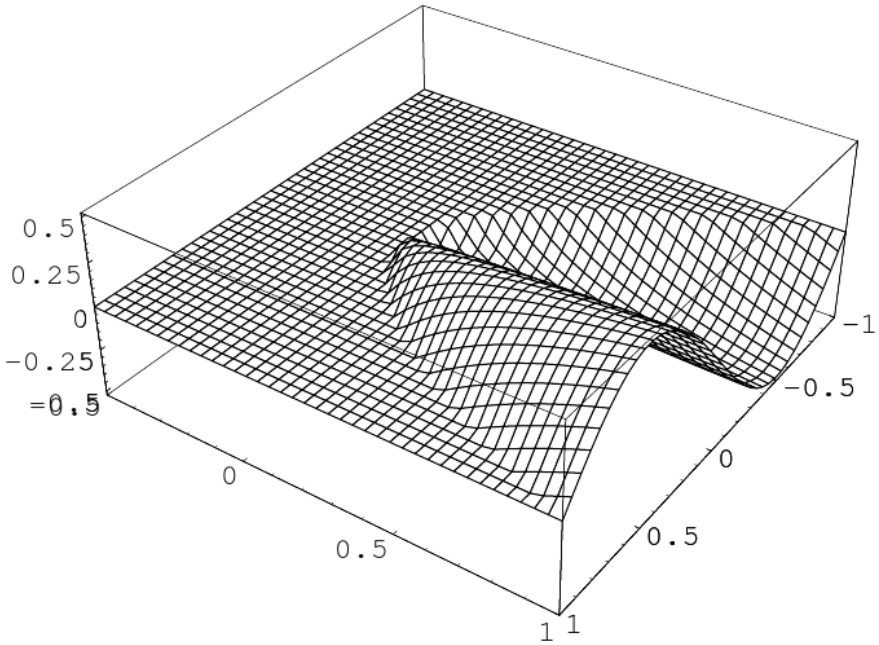


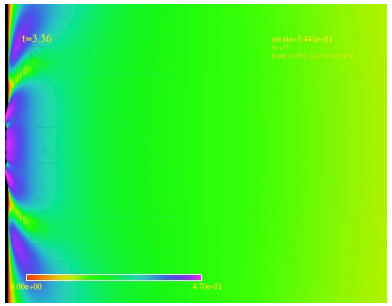
Figure: Credit: Gundlach C. Phys. Rev. D 65 (2002)

- Results are consistent with Gundlach's predictions

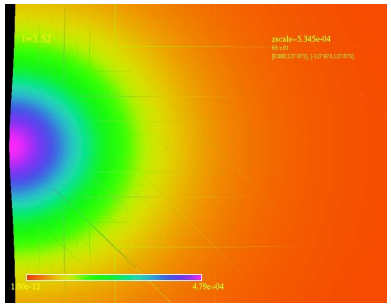
The large q regime ($\Gamma = 1.00001$)

Critical collapse at $0.1 < |\vec{q}| < 0.5$

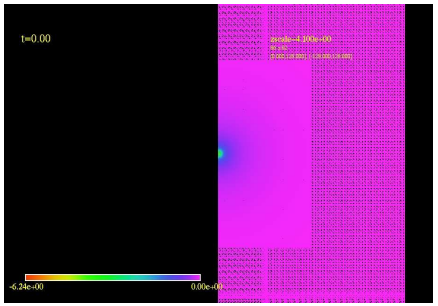
- Change in qualitative behavior of the field v_ϕ
  - Angular dependence of v_ϕ is no longer given by $\dfrac{\partial}{\partial\theta} Y_1^0(\theta, \phi)$



v_ϕ



ρ

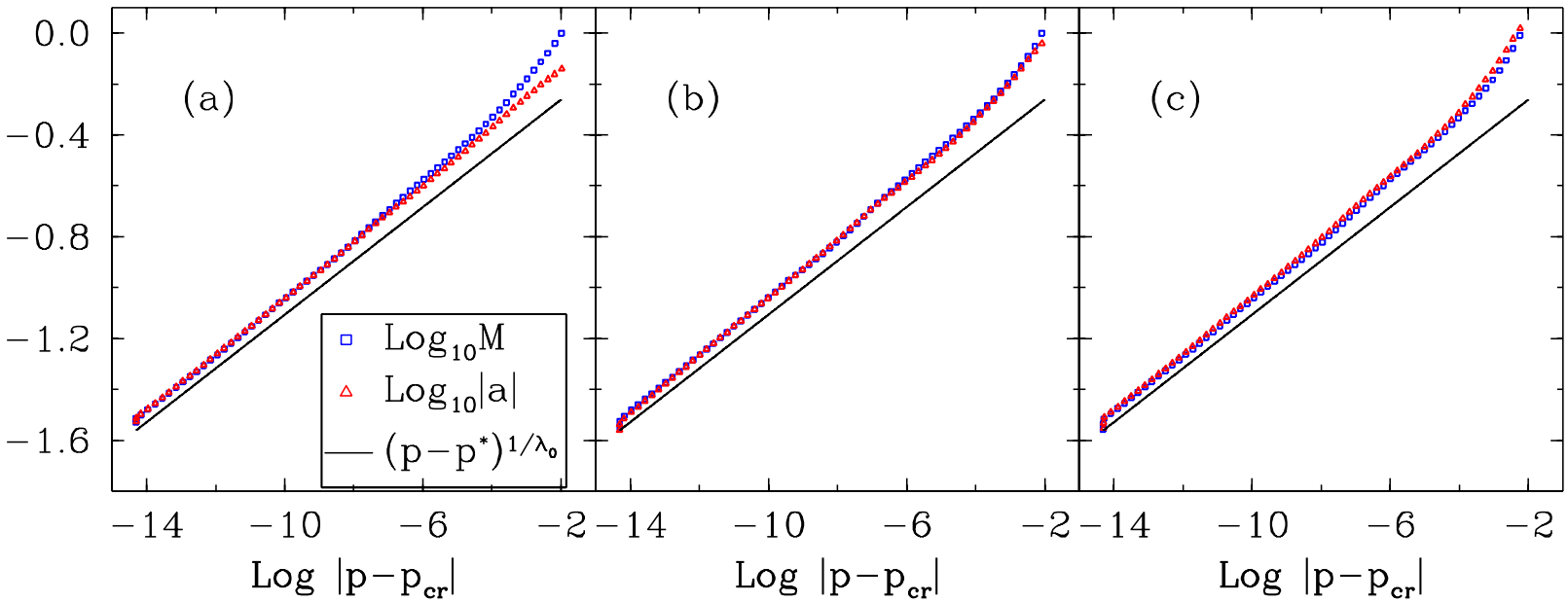


Φ

- Critical solution seems to be a self-similar solution
  - Universal solution
  - Non-spherically symmetric solution

Scaling Behavior

- Generic initial data: families A, B, and C
- Mass scaling remains unchanged
- Measured a change in the scaling of $\vec{a}$
  - Its scaling is indistinguishable from the scaling of M (up to an overall scale)
  - Calculations are consistent with convergence to single-unstable-mode non-spherical $Z^{\star}(x, \theta, \phi)$
  - Speculative: Growth rate of unstable mode $\sim \lambda_0$



Conclusion

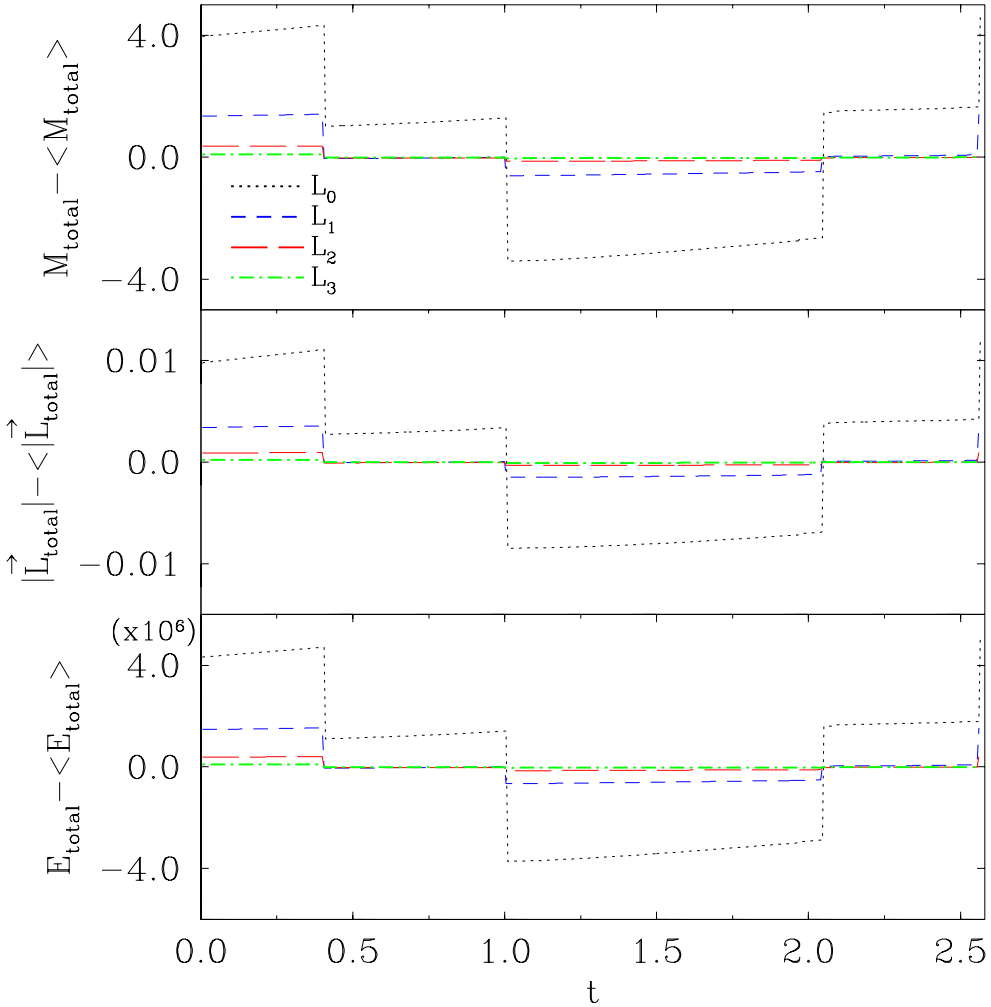
First dynamical investigation of the role of angular momentum during critical fluid collapse

Answered questions raised Gundlach in Phys. Rev. D **65** (2002) in the context of a Newtonian-model fluid collapse

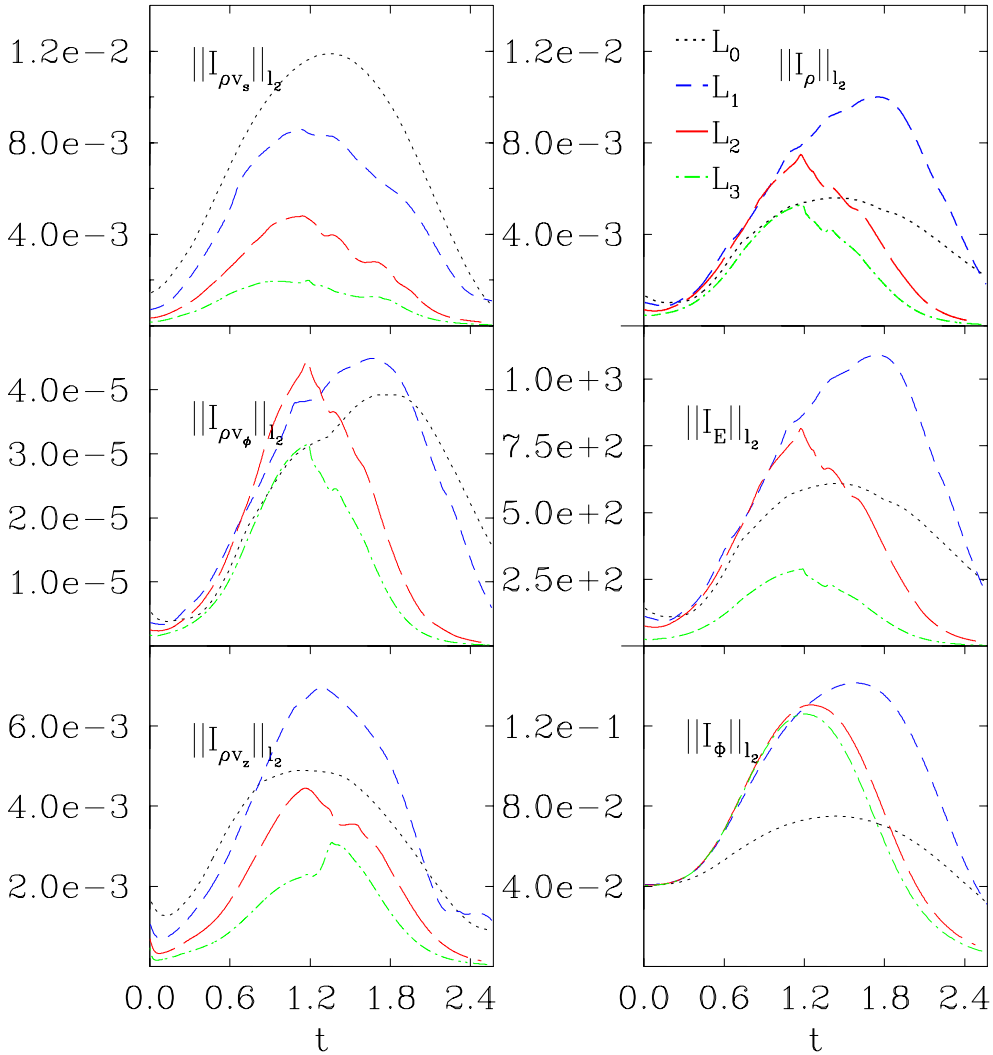
- The critical solution's departure from spherical symmetry is carried by and axial growing mode, the spin-up mode in the Newtonian context
  - Two growing modes $\Rightarrow$ Scaling laws
  - Dynamical calculations agree with scaling laws
- Behavior of solution near the critical curve
  - Universal
- Extension of the analogy with statistical physics suggested in Phys. Rev. D **65** and Irr-2007-5: ferromagnetic material at high temperature
  - $\vec{q} \rightarrow \vec{B}_{\rm ext}$ (applied magnetic field) and $\vec{a} \rightarrow \vec{m}$ (magnetization)

Code diagnostics: Axial Symmetry

Conserved quantities

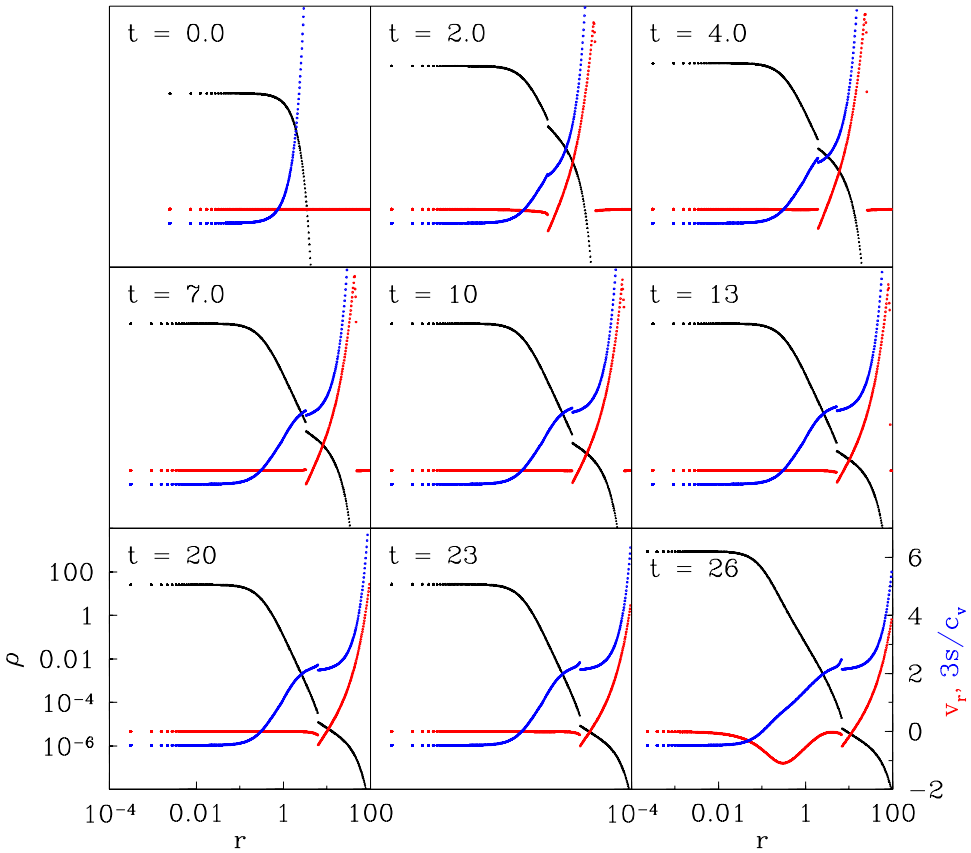


Independent residual



Spherically symmetry: Type-I critical behavior

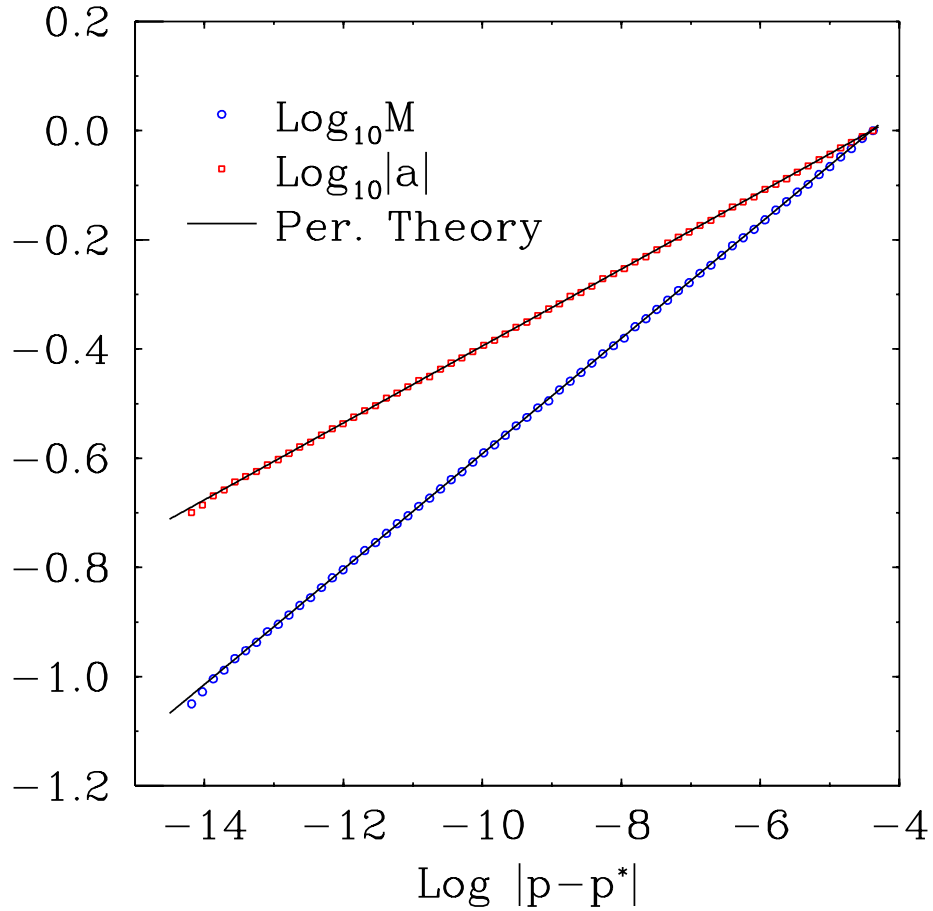
- Type-I behavior 'turns on' at $\Gamma \geq 6/5$
  - Meta-stable solution: $Z^{\star}_{\Gamma}(r)$ ($\Gamma = 1.28$)



SCALING LAWS ($\Gamma = 1.00001$, $q \to 0$)

Order-parameters: $M(\text{collapsed})$ and $|\vec{a}|$

- Initial data chosen near $Z^\star(x)$ plus a a small but generic perturbation controlled by $\tilde{p}$ and $\tilde{\vec{q}}$
- Fine tune to threshold of collapse: Note this is not $\tilde{p}_{\text{cr}} \neq 0$ and $\tilde{q}_{\text{max}} \neq 0$
- Eliminate from the outset decaying perturbations which obscure $Z^\star$
  - Strong attraction to $Z^\star(x)$ before the spherical mode has a chance to grow
  - Evident in the scaling laws



Other values of Γ ($\Gamma = 1.12$, $q \to 0$)

Order-parameters: $M(\text{collapsed})$ and $|\vec{a}|$

- Initial data near $Z^{\star}_{\Gamma}(x)$ with $q = 10^{-14}$
- Measurements of M and $\vec{a}$ are in agreement with scaling laws