Einstein's equations can be divided into two classes -constraint equations and evolution equations. The constraint equations form a system of four elliptic partial differential equations, whereas there are six hyperbolic evolution equations. In contrast to the case of many other systems of time-dependent PDE's, the initial data for evolution in general relativity cannot be specified freely, but must obey the constraint equations. Thus, even to obtain consistent initial data is a highly non-trivial task. I am looking for asymptotically flat initial data sets that contain apparent horizons. I assume axisymmetry, and use cylindrical coordinates (rho,z,phi). My problem will amount to solving the Laplace equation for a particular geometric function known as the conformal factor. To solve the equation I use a finite element method, as well as the software package PLTMG. I impose regularity conditions on the axis of symmetry and Robin boundary conditions on the outer edge of the computational domain. On the apparent horizon itself, I impose the condition that the divergence of any out going null geodesic has to be zero. I can choose the shape of the apparent horizon to be whatever I want and this is where our interest lies. By choosing the shape we can explore certain extreme limiting cases which may shed some light on outstanding conjectures in relativity, such as Thorne's hoop conjecture, or Penrose's cosmic censorship conjecture. The adaptive algorithm employed will be very important. The algorithm which will yield the most positive results with this particular type of elliptic equation is that which is discussed by Bank and Holst (2003). This approach will require the use of parallel computation and serves to reduce the processor load balancing problem to the numerical solution of a small elliptic problem on a single processor, using a sequential adaptive solver. The small elliptic problem is then used to produce a posteriori error estimate to predict future element sizes and densities in the mesh, which are then used in a weighted recursive spectral bisection of the the initial mesh.