Lagrangian Hydrodynamics

(In Spherical Symmetry)

I) Mass Conservation (Continuity Equation)

$$V = 1/\rho = 4\pi r^2 \, \partial r/\partial m$$

II) Momentum Conservation

$$dv/dt = -4\pi r^2 \partial P/\partial m$$

III) Energy Conservation

$$d\epsilon/dt = -P dV/dt = P/\rho^2 \, d\rho/dt$$

With $d\rho/dt = -\rho \nabla \cdot \vec{v} = -\rho/r^2 \, \partial(r^2 v)/\partial r$

and $dm = 4\pi r^2 \rho dr$, the energy equation is:

$$d\epsilon/dt = -4\pi P \partial(r^2 v)/\partial m$$

IV) Equation of State

e.g. $P = (\gamma - 1)\rho\epsilon$