Recent Developments in the 2-Body Problem in Numerical Relativity

Black Holes V
Theory and Mathematical Aspects
Banff, AB
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THANKS TO ...

1. THE ORGANIZERS

2. UofA / TPI

CIAR,CITA,PIMS,PITP

3. Frans Pretorius

[all simulations shown here]

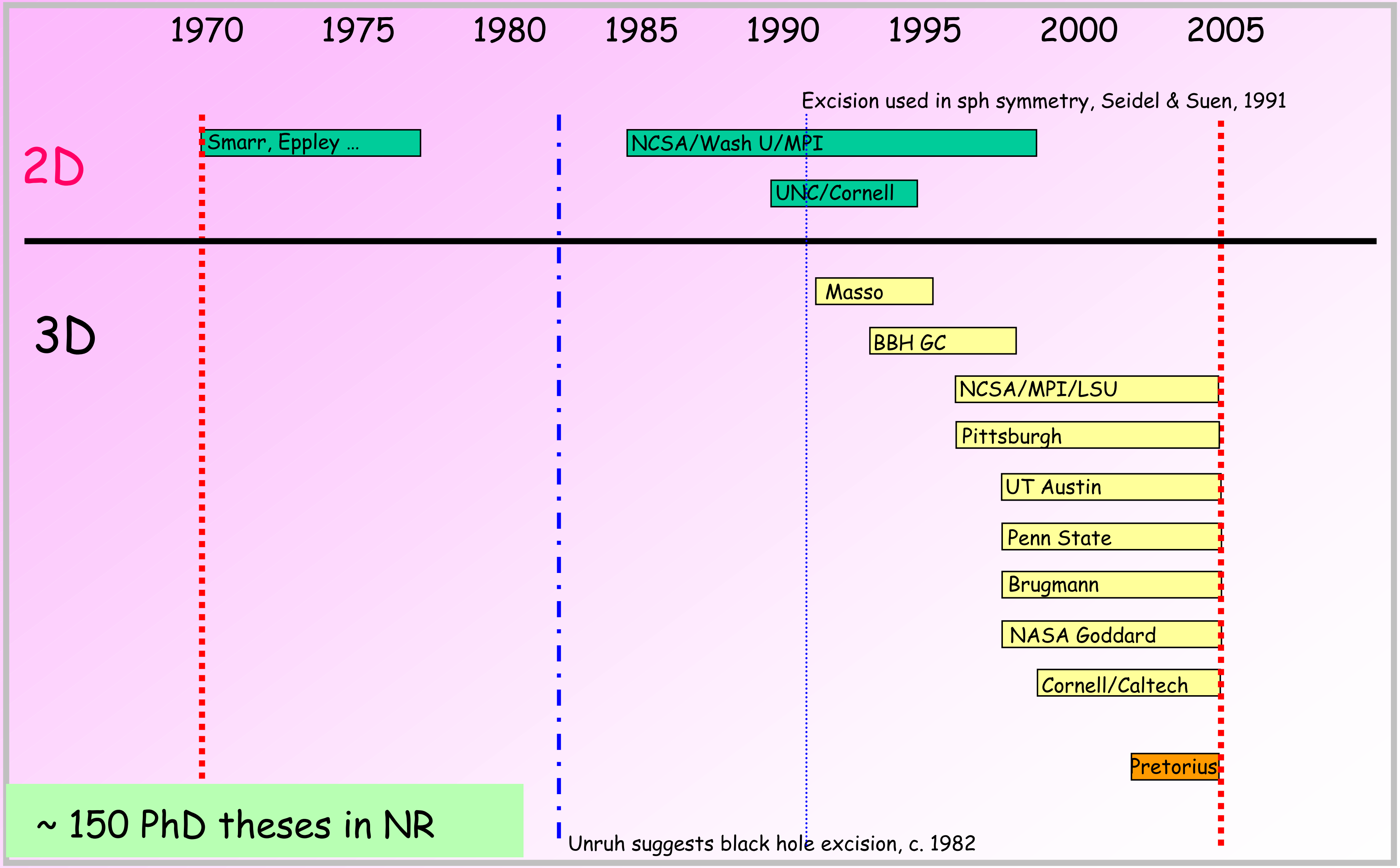
West of Banff on #1, 0600 May 14 2005

Outline

- Brief history of the dynamical binary black hole problem in numerical relativity
- Pretorius' new "generalized" harmonic code
  - axisymmetric black hole-boson star collisions
  - fully 3D collisions
- Prognosis

A Brief History of the 2 Black Hole Problem in NR

[DYNAMICS ONLY!; graphic preliminary & subject to correction/modifcation; apologies for omissions]



Pretorius's New Code
(in development for about 3 years)

- Key features
  - "ad hoc"; ignored much "conventional wisdom" (often when CW had no empirical basis)
  - Arguably only fundamentals retained from 30 years of cumulative experience in numerical relativity:

    1. Geometrodynamics is a useful concept (**Dirac**, **Wheeler** …)
    2. Pay attention to constraints (**Dewitt,** …)

Pretorius's New Code: Key Features

- GENERALIZED harmonic coordinates
- Second-order-in-time formulation and direct discretization thereof
- $O(h^2)$ finite differences with iterative, point-wise, Newton-Gauss-Seidel to solve implicit equations
- Kreiss-Oliger dissipation for damping high frequency solution components (stability)
- Spatial compactification
- Implements black hole excision
- Full Berger and Oliger adaptive mesh refinement
- Highly efficient parallel infrastructure (almost perfect scaling to hundreds of processors, no reason can't continue to thousands)
- Symbolic manipulation crucial for code generation

Pretorius' Generalized Harmonic Code

[Class. Quant. Grav. 22, 425, 2005,
following Garfinkle, PRD, 65:044029, 2002]

- Adds "source functions" to RHS of harmonic condition

$$\nabla^{\alpha}\nabla_{\alpha}x^{\mu} \equiv \frac{1}{\sqrt{-g}}\partial_{\alpha}\left(\sqrt{-g}g^{\alpha\mu}\right) = H^{\mu}$$

- Substitute gradient of above into field equations, treat source functions as INDEPENDENT functions: retain key attractive feature (vis a vis solution as a Cauchy problem) of harmonic coordinates

$$g^{\gamma\delta}g_{\alpha\beta,\gamma\delta} + \ldots = 0$$

Principal part of continuum evolution equations for metric components is just a wave operator

Pretorius' Generalized Harmonic Code

- Einstein/harmonic equations (can be essentially arbitrary prescription for source functions)

$$g^{\gamma\delta} g_{\alpha\beta,\gamma\delta} + 2 g^{\gamma\delta}{}_{,(\alpha} g_{\beta)\delta,\gamma} + 2H_{(\alpha,\beta)} - 2H_{\delta}\Gamma^{\delta}_{\alpha\beta} + 2\Gamma^{\gamma}_{\delta\beta}\Gamma^{\delta}_{\gamma\alpha} + 8\pi\left(2T_{\alpha\beta} - g_{\alpha\beta}T\right) = 0$$

- Solution of above will satisfy Einstein equations if

$$C^{\mu}\Big|_{t=0} \equiv \left(H^{\mu} - \nabla^{\alpha}\nabla_{\alpha}x^{\mu}\right)\Big|_{t=0} = 0$$

$$C^{\mu}{}_{,t}\Big|_{t=0} \equiv \left(H^{\mu} - \nabla^{\alpha}\nabla_{\alpha}x^{\mu}\right)_{,t}\Big|_{t=0} = 0$$

Proof: $\nabla^{\alpha}\nabla_{\alpha}C^{\mu} = -R^{\mu}{}_{\nu}C^{\mu}$

Choosing source functions from consideration of behaviour of 3+1 kinematical variables

$$ds^2 = -\alpha^2 dt^2 + h_{ij}\left(dx^i + \beta^i dt\right)\left(dx^j + \beta^j dt\right)$$

$$H \cdot n \equiv H_\mu n^\mu = -n^\mu \partial_\mu \ln \alpha - K$$

$$\perp H^i \equiv H_\mu h^{i\mu} = \frac{1}{\alpha} n^\mu \partial_\mu \beta^i + h^{ij} \partial_j \ln \alpha - \bar{\Gamma}^i_{jk} h^{jk}$$

$$\partial_t \alpha = -\alpha^2 H \cdot n + \ldots$$

$$\partial_t \beta^i = \alpha^2 \perp H^i + \ldots$$

Choosing source functions from consideration of behaviour of 3+1 kinematical variables

- Can thus use source functions to drive 3+1 kinematical vbls to desired values

- Example: Pretorius has found that all of the following slicing conditions help counteract the "collapse of the lapse" that generically accompanies strong field evolution in "pure" harmonic coordinates

$$H_t = \xi \frac{\alpha - 1}{\alpha^n}$$

$$\partial_t H_t = \xi \partial_t \left(\frac{\alpha - 1}{\alpha^n} \right)$$

$$\nabla^\mu \nabla_\mu H_t = -\xi \frac{\alpha - 1}{\alpha^n} - \zeta \partial_t H_t$$

Constraint Damping

[Brodbeck et al, J Math Phys, 40, 909 (1999); Gundlach et al, gr-qc/0504114]

- Modify Einstein/harmonic equation via

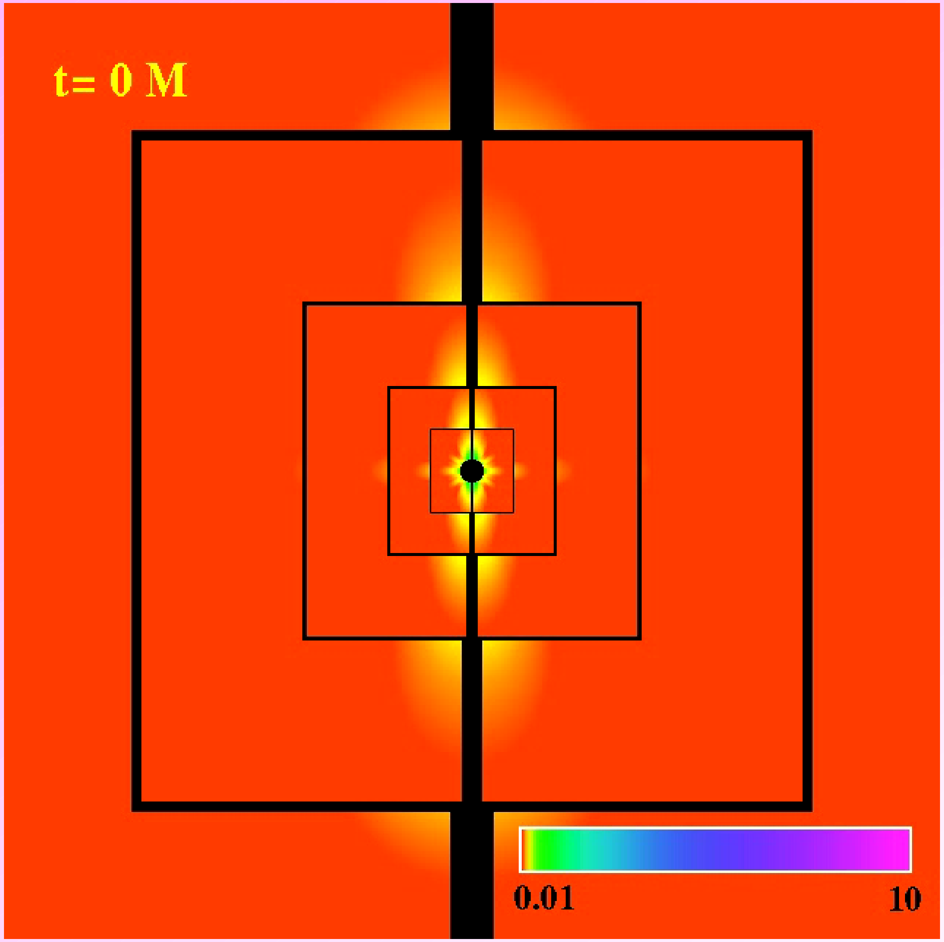
$$g^{\alpha\beta} g_{\mu\nu,\alpha\beta} + \ldots + \kappa \left(n_\mu C_\nu + n_\nu C_\mu - g_{\mu\nu} n^\alpha C_\alpha \right) = 0$$

where

$$C^\mu \equiv H^\mu - \nabla^\alpha \nabla_\alpha x^\mu$$
$$n_\mu \equiv -\alpha \nabla_\mu t$$

- Gundlach et al have shown that for all positive κ, (to be chosen empirically in general), all non-DC contraint-violations are damped for linear perturbations about Minkowski

Effect of constraint damping



- Axisymmetric simulation of single Schwarzschild hole
- Left/right calculations identical except that constraint damping is used in right case
- Note that without constraint damping, code blows up on a few dynamical times

Merger of eccentric binary system

[Pretorius, work in progress!]

- Initial data
  - Generated from prompt collapse of balls of massless scalar field, boosted towards each other
  - Spatial metric and time derivative conformally flat
  - Slice harmonic (gives initial lapse and time derivative of conformal factor)
  - Constraints solved for conformal factor, shift vector components
- Pros and cons to the approach, but point is that it serves to generate orbiting black holes

Merger of eccentric binary system

- Coordinate conditions

$$\nabla^{\mu}\nabla_{\mu}H_t = -\xi\frac{\alpha-1}{\alpha^n} - \zeta\partial_t H_t$$

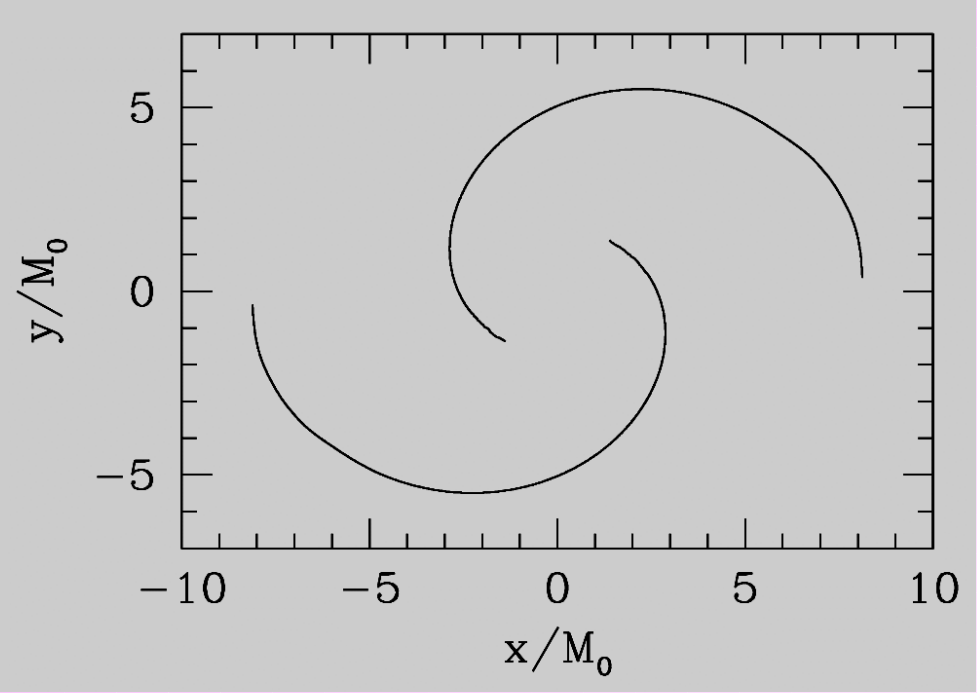
$$H_i = 0$$

$$\xi \sim 6/M, \quad \zeta \sim 1/M, \quad n = 5$$

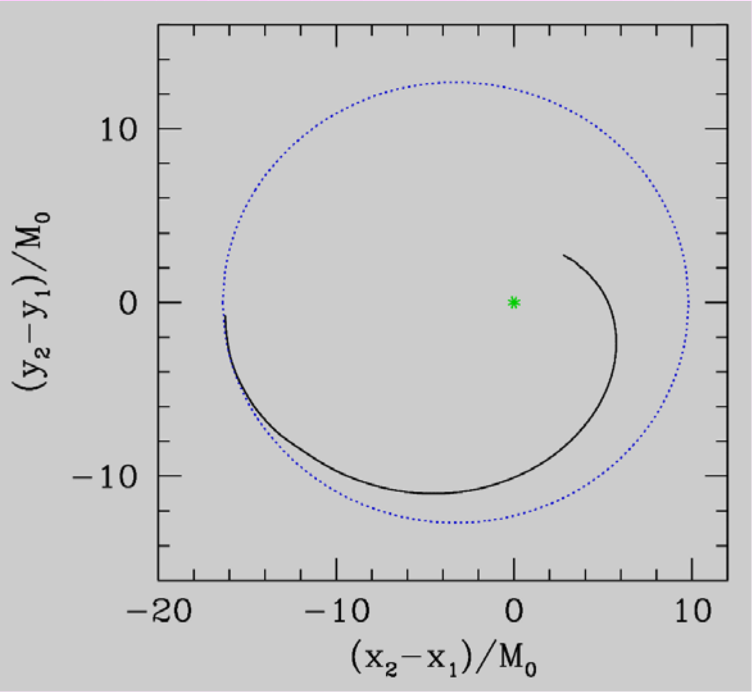
  - Strictly speaking, not spatially harmonic, which is defined in terms of "contravariant components" of source fcns

- Constraint damping coefficient: $\kappa \sim 1/M$

Orbit



Simulation (center of mass) coordinates



Reduced mass frame; solid black line is position of BH 1 relative to BH 2 (green star); dashed blue line is reference ellipse

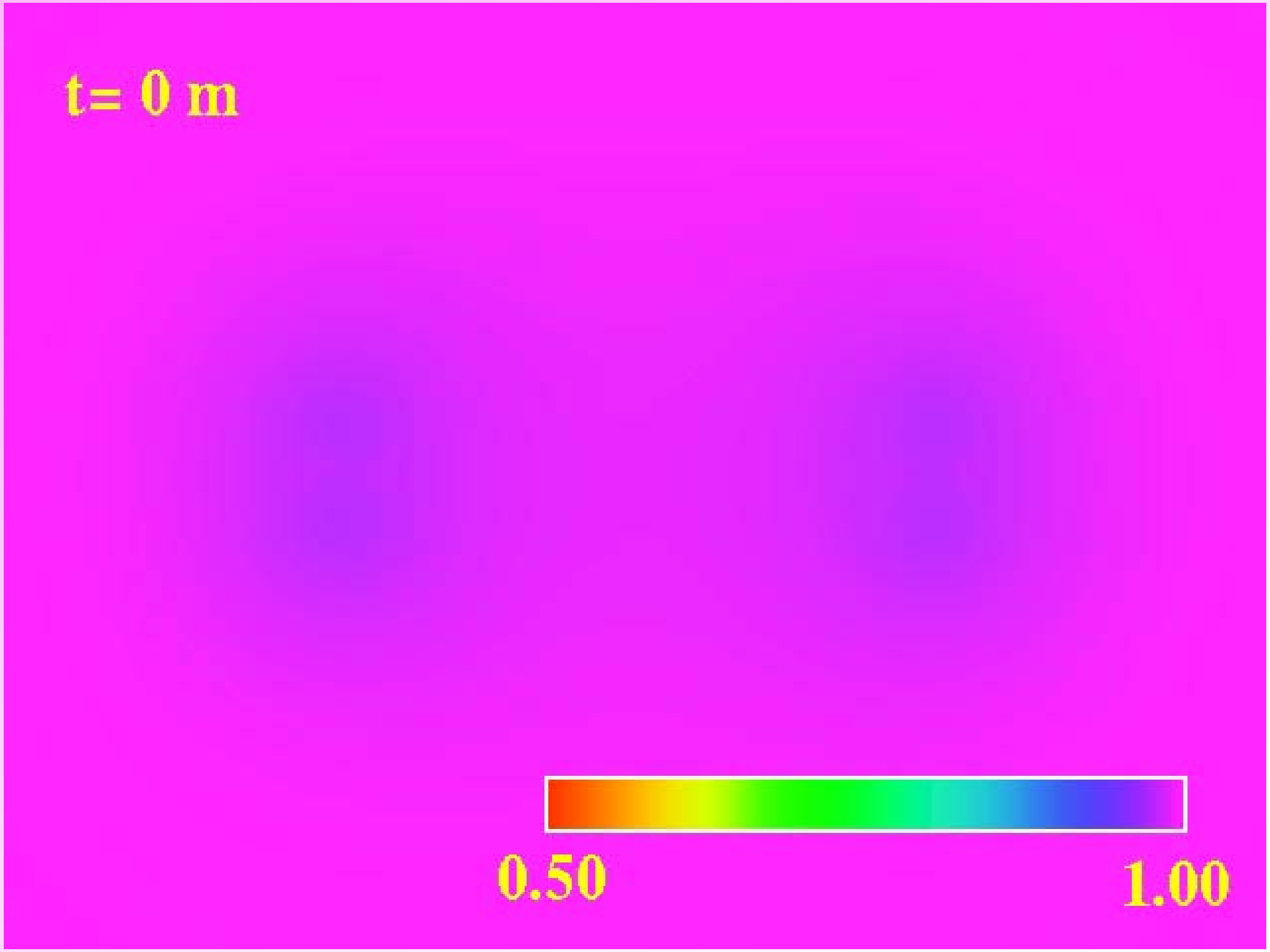
t=0

- Equal mass components
- Eccentricity ~ 0.25
- Coord. Separation ~ 16M
- Proper Separation ~ 20M
- Velocity of each hole ~ 0.12
- Spin ang mom of each hole = 0

t ~ 200

- Final BH mass ~ 1.85M
- Kerr parameter a ~ 0.7
- Estimated error ~ 10%

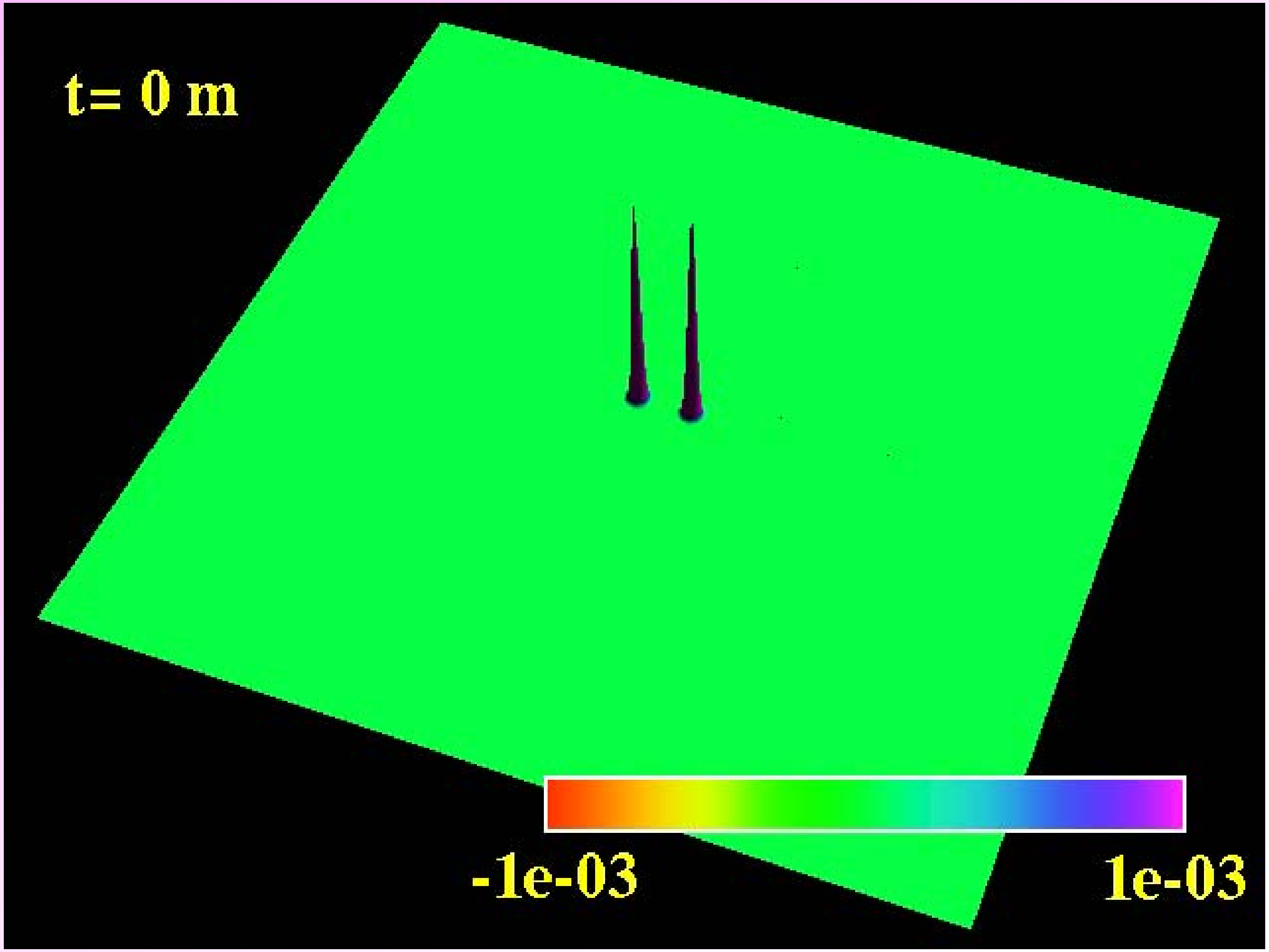
Lapse function
Uncompactified coordinates



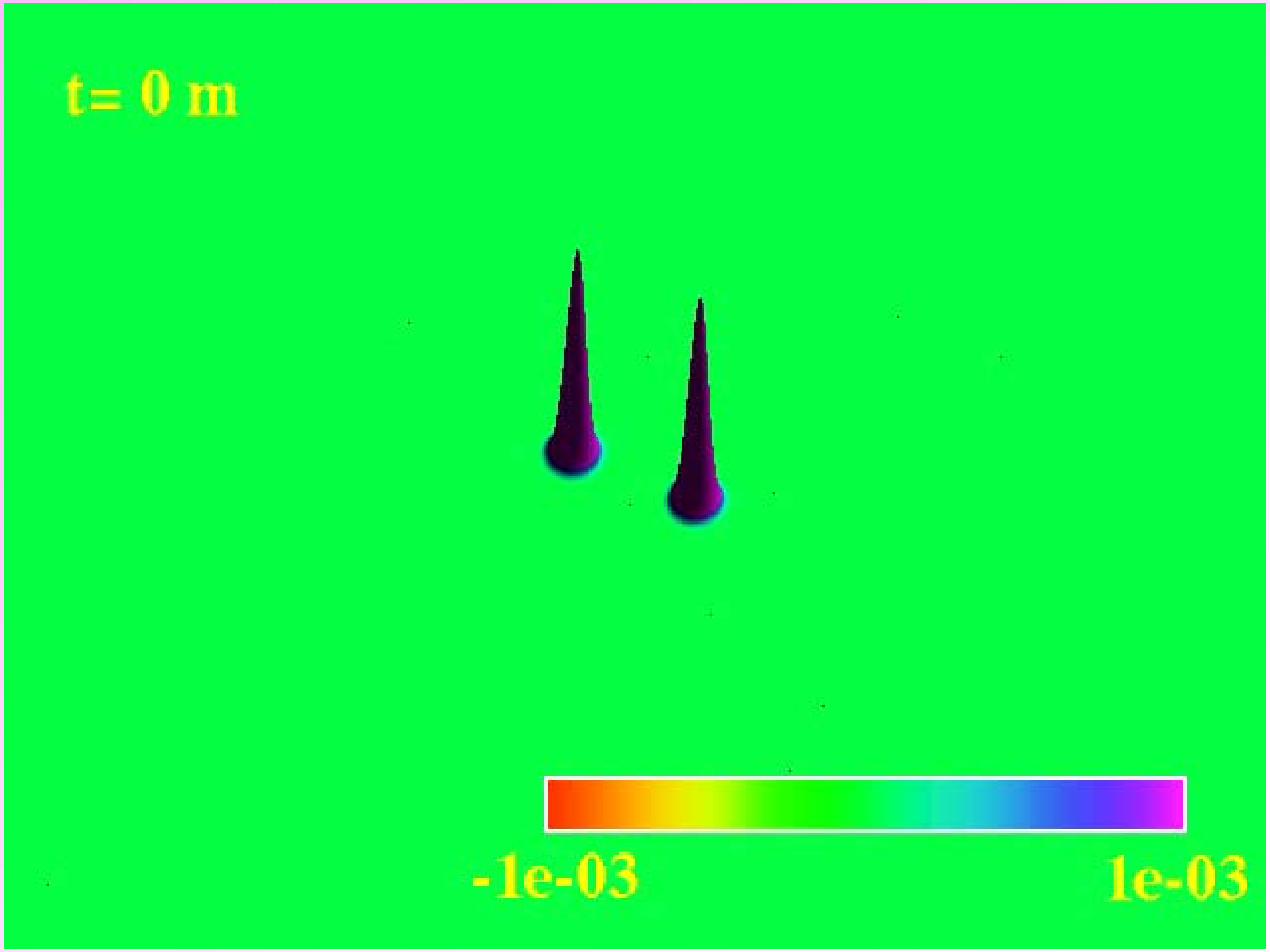
- All animations show quantities on the z=0 plane
- Time measured in units of M

Scalar field modulus
Compactified (code) coordinates

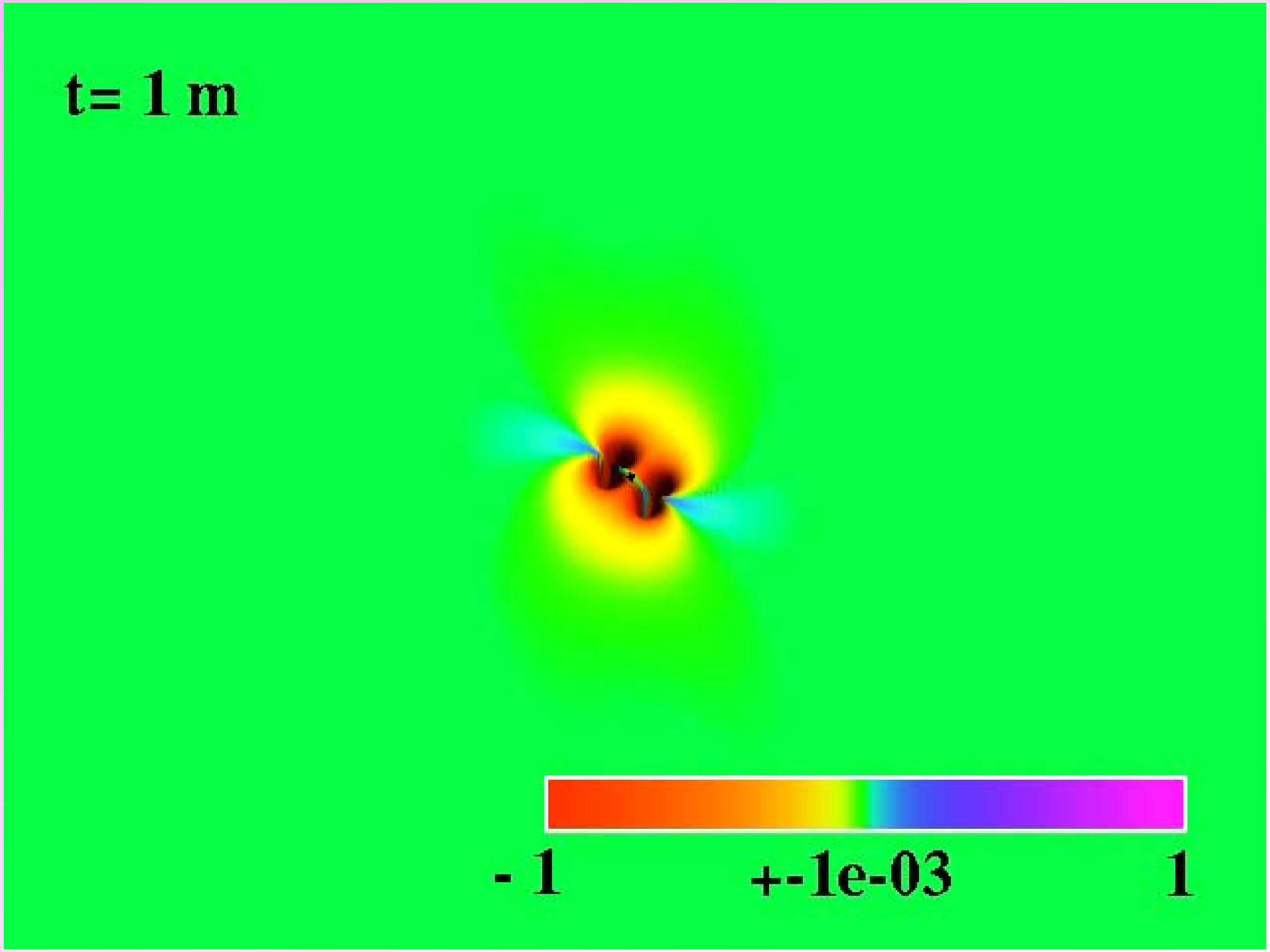
$$\bar{x} = \tan(x\pi / 2),\ \bar{y} = \tan(y\pi / 2),\ \bar{z} = \tan(z\pi / 2)$$



Scalar field modulus
Uncompactified coordinates



Gravitational Radiation
Uncompactified coordinates



Real component of the Newman-Penrose scalar: $r\Psi_4$

Computation vital statistics

- Base grid resolution: 48 x 48 x 48
  - 9 levels of 2:1 mesh refinement
    - Effective finest grid 12288 x 12288 x 12288
- Data shown (calculation still running)
  - ~ 60,000 time steps on finest level
  - CPU time: about 70,000 CPU hours (8 CPU years)
    - Started on 48 processors of our local P4/Myrinet cluster
    - Continues of 128 nodes of WestGrid P4/gig cluster
  - Memory usage: ~ 20 GB total max
  - Disk usage: ~ 0.5 TB with infrequent output!

Hardware
[CFI/ASRA/BCKDF funded HPC infrastructure]

November 1999

vn.physics.ubc.ca
128 x 0.85 GHz PIII, 100 Mbit
Up continuously since 10/98
MTBF of node: 1.9 yrs

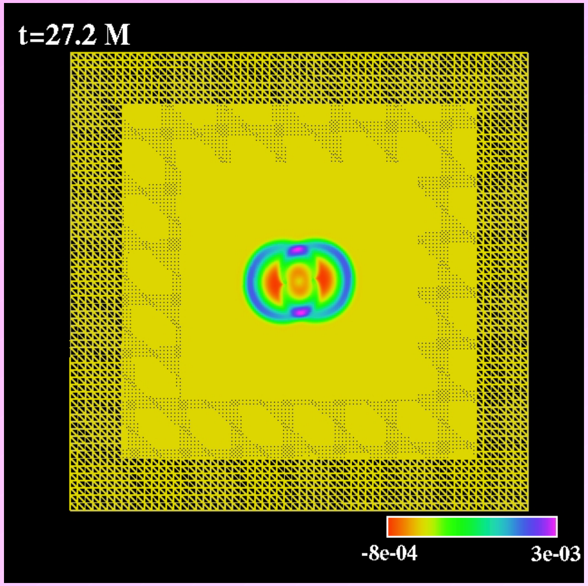
March 2005

vnp4.physics.ubc.ca
110 x 2.4 GHz P4/Xeon, Myrinet
Up continuously since 06/03
MTBF of node: 1.9 yrs

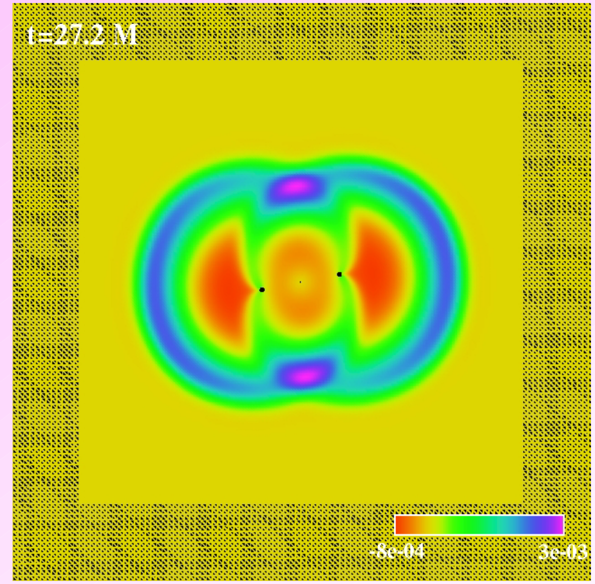
glacier.westgrid.ca
1600 x 3.06 GHz P4, Gigiabit
Ranked #54 in Top 500 11/04 (Top in Canada)

Sample Mesh Structure

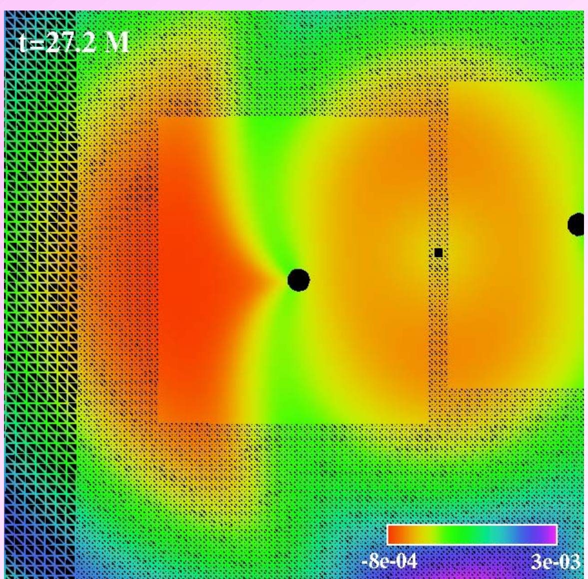
1



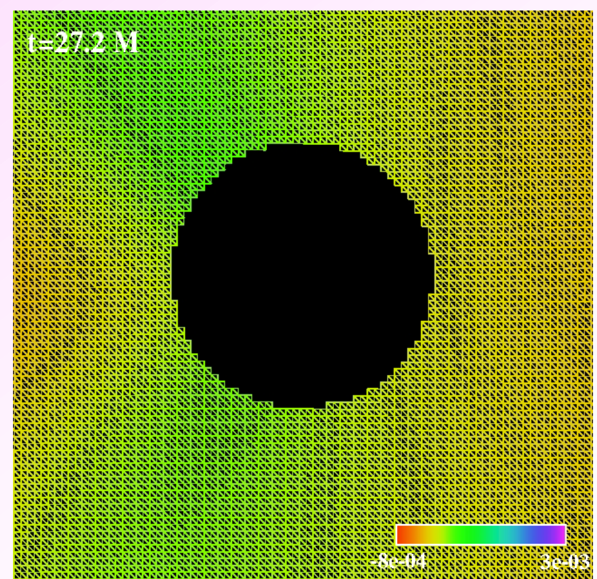
2



3



4



Boson star – Black hole collisions

[Pretorius, in progress]

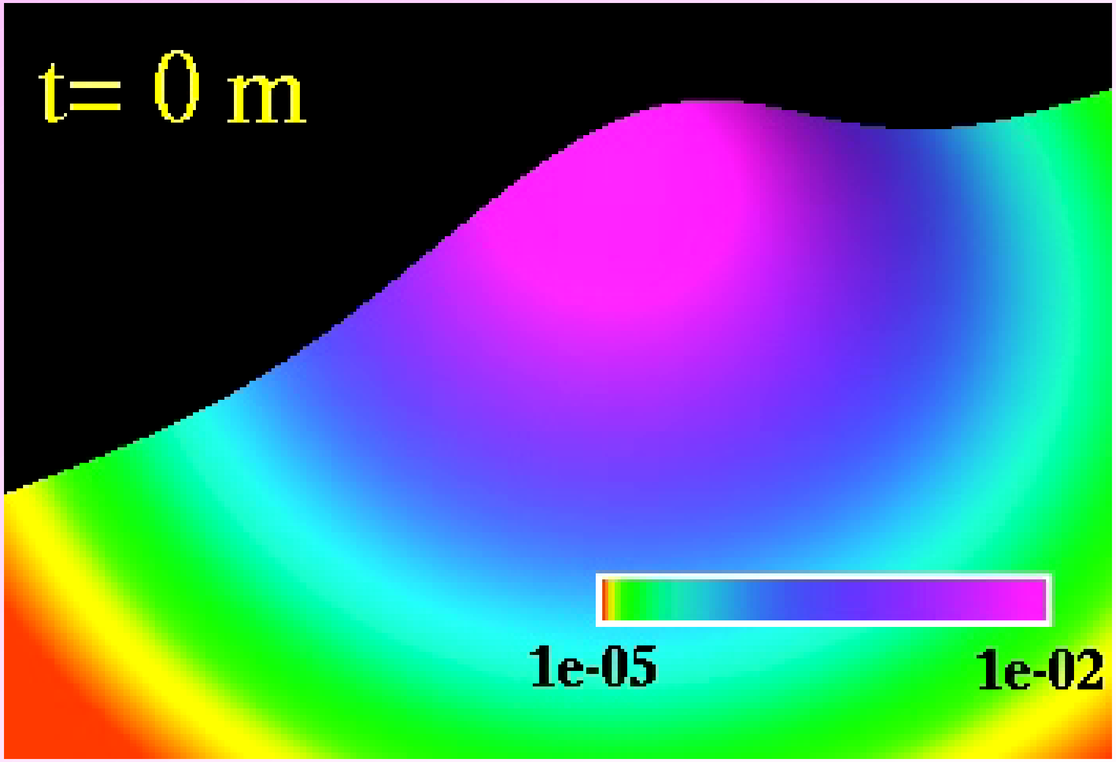
- Axisymmetric calculations; uses modified "Cartoon" method originally proposed by J. Thornburg in his UBC PhD thesis
- Work in Cartesian coordinates (rather than polar-spherical or cylindrical); restrict to z=0 plane; reexpress z-derivatives in terms of x and y (in plane) derivatives using symmetry

- Initial data
  - (Mini) boson-star on the stable branch
  - Again form black hole via prompt collapse of initial massless scalar field configuration, and further boost this configuration towards the black hole

Boson Star – Black Hole Collision: Case 1

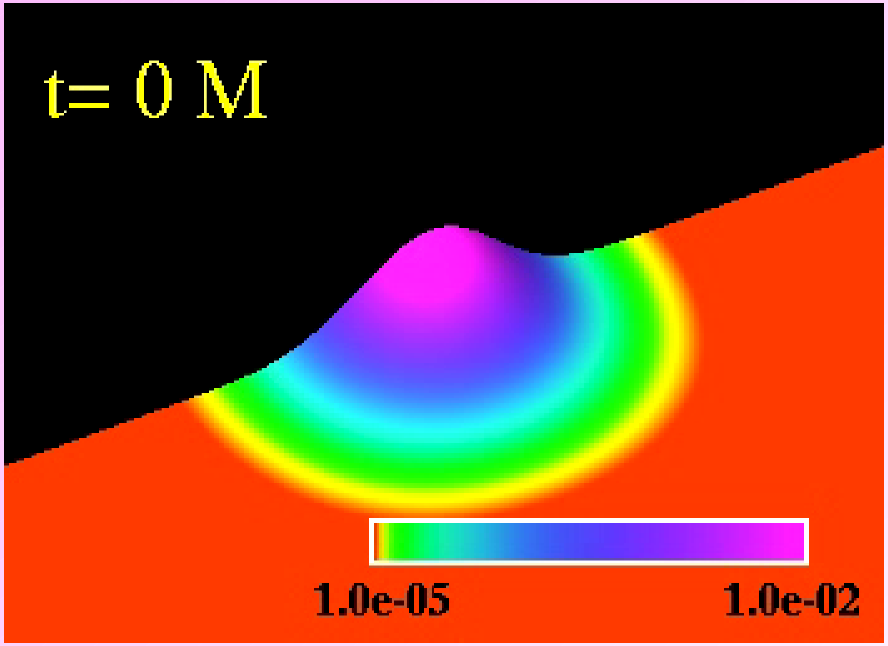
- MBS/MBH ~ 0.75
- RBS/RBH ~ 12.5
- BH initially just outside BH and moving towards it with v ~ 0.1 c

$$|\phi(t,\rho,z)|$$

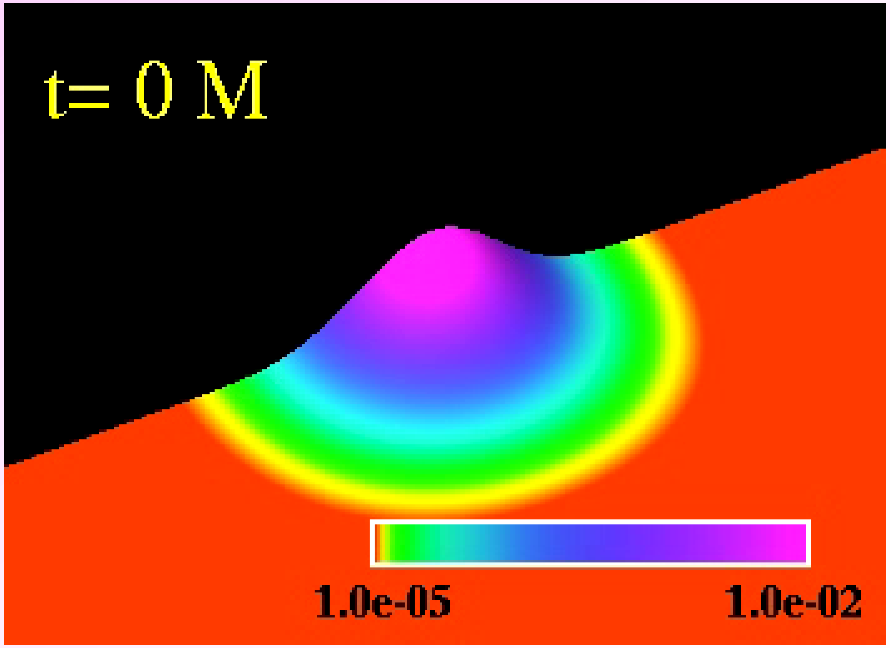


Boson Star – Black Hole Collision: Case 2

- MBS/MBH ~ 3.00
- RBS/RBH ~ 50.0
- BH initially just outside BS, and at rest



mesh spacing 2h



mesh spacing h

PROGNOSIS

- The golden age of numerical relativity is nigh, and we can expect continued exciting developments in near term
- Have scaling issues to deal with, particularly with low-order difference approximations in 3 (or more!) spatial dimensions; but there are obvious things to be tried

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- Have scaling issues to deal with, particularly with low-order difference approximations in 3 (or more!) spatial dimensions; but there are obvious things to be tried
- Can expect swift incorporation of fluids into code, will vastly extend astrophysical range of code
- STILL LOTS TO DO AND LEARN IN AXISYMMETRY AND EVEN SPHERICAL SYMMETRY!!

APS Metropolis Award Winners
(for best dissertation in computational physics)

1999	LUIS LEHNER
2000	Michael Falk
2001	John Pask
2002	Nadia Lapusta
2003	FRANS PRETORIUS
2004	Joerg Rottler
2005	HARALD PFEIFFER