Derivation of the Valencia Formulation

Start from the Divergence of the mixed components of the SET:

$$
\begin{aligned}
&\nabla_\mu \left(T^\mu{}_\nu\right) = \\
&\nabla_\mu \left(g_{\delta\nu} T^{\mu\delta}\right) = \\
&T^{\mu\delta} \nabla_\mu g_{\delta\nu} + g_{\delta\nu} \nabla_\mu T^{\mu\delta} = \\
&g_{\delta\nu} \left[\frac{1}{\sqrt{-g}}\, \partial_\mu \left(\sqrt{-g} T^{\mu\delta}\right) + \Gamma^\delta_{\mu\lambda} T^{\mu\lambda}\right] = \\
&g_{\delta\nu} \left[\frac{1}{\sqrt{-g}}\, \partial_\mu \left(\sqrt{-g}\, g^{\xi\delta} T^\mu{}_\xi\right) + \Gamma^\delta_{\mu\lambda} T^{\mu\lambda}\right] = \\
&g_{\delta\nu} \left[\partial_\mu \left(g^{\xi\delta}\right) T^\mu{}_\xi + g^{\xi\delta} \frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g} T^\mu{}_\xi\right) + \Gamma^\delta_{\mu\lambda} T^{\mu\lambda}\right] = 0
\end{aligned}
$$

Therefore:

$$
\begin{aligned}
g_{\delta\nu} g^{\xi\delta} \frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g} T^\mu{}_\xi\right) &= -g_{\delta\nu} \partial_\mu \left(g^{\xi\delta}\right) T^\mu{}_\xi - \Gamma^\delta_{\mu\lambda} T^{\mu\lambda} g_{\delta\nu} \\
g_{\delta\nu} g^{\xi\delta} \frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g} T^\mu{}_\xi\right) &= \left[-\delta_\mu \left(g_{\delta\nu} g^{\xi\delta}\right) + g^{\xi\delta} \partial_\mu \left(g_{\delta\nu}\right)\right] T^\mu{}_\xi - \Gamma^\delta_{\mu\lambda} T^{\mu\lambda} g_{\delta\nu} \\
\delta^\xi{}_\nu \frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g} T^\mu{}_\xi\right) &= g^{\xi\delta} \partial_\mu \left(g_{\delta\nu}\right) T^\mu{}_\xi - \Gamma^\delta_{\mu\lambda} T^{\mu\lambda} g_{\delta\nu} \\
\frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g} T^\mu{}_\nu\right) &= \partial_\mu \left(g_{\delta\nu}\right) T^{\mu\delta} - \Gamma^\delta_{\mu\lambda} T^{\mu\lambda} g_{\delta\nu}
\end{aligned}
$$

and renaming the dummy indexes:

$$
\frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g} T^\mu{}_j\right) = \partial_\mu \left(g_{\nu j}\right) T^{\mu\nu} - \Gamma^\delta_{\mu\nu} T^{\mu\nu} g_{\delta j} \tag{1}
$$

References

[1]