Critical Collapse of a Scalar Field with Angular Momentum

(in spherical symmetry)

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http://laplace.physics.ubc.ca/People/inaki/emkgl/emkgl.html

Outline

- Introduction: Critical Phenomena,
  Angular Momentum and CP.

- Model: Procedure to get effective EOM,
  Example ($l = 1$),
  Equations.

- Our Code.

- Results: Low $l < 10$,
  Intermediate $l \approx 10$,
  High $l > 10$.

Introduction

Critical Phenomena

- Choose initial conditions (parametrized by one parameter p) for which there exists a value, p^*, of this parameter such that if $p > p^*$ it forms a black hole, and if $p < p^*$ the matter implodes and then completely disperses.

- Find the value of p^* and therefore the solution at the tresshold of BH formation choosing an initial bracket $[p_{\mathrm{lo}}, p_{\mathrm{hi}}]$ and narrowing it by dynamical evolutions.

- A decade playing this game and still finding interesting results.

Introduction (cont.)

Type II – Infinitesimal BH Mass at Threshold:

- CSS or DSS.
- Scaling Law: $\ln M_{BH} = \gamma \ln |p - p^*|$

Type I – Finite mass-gap:

- Static or periodic.
- Scaling Law: $\tau = \sigma \ln |p - p^*|$

Introduction (cont.)

Angular Momentum and critical phenomena

- Most of the studies have been done assuming spherical symmetry $\longrightarrow$ Very little is known about the role that angular momentum plays in critical phenomena.

- Perturbative studies of the spherical solutions show indications that the angular momentum is radiated away (Gardfinkle *et al.*).

- In our system we don't allow for this radiation since the angular momentum is *built in* in the EOM.

- Currently there is work underway to see what is the effect of the angular momentum in axisymmetry (GRAXI).

Equations

We consider the following $2l+1$ scalar fields:

$$\Psi_l^m(t,r,\theta,\phi) = \psi(t,r)\Theta_l^m(\theta,\phi), \qquad (1)$$

where $L^2\Theta_l^m(\theta,\phi) = l(l+1)\Theta_l^m(\theta,\phi)$.

For each scalar field with the same l, different m, we construct the SET:

$$T^{(lm)}{}_{ab} = \nabla_a\Psi_l^m\nabla_b\Psi_l^m - \frac{1}{2}g_{ab}(\nabla^c\Psi_l^m\nabla_c\Psi_l^m), \qquad (2)$$

Sum the SETs:

$$\mathcal{T}^{(l)}{}_{ab} = \sum_m T^{(lm)}{}_{ab}. \qquad (3)$$

EOM are calculated for $\psi(t,r)$ by:

$$G_{ab} = \mathcal{T}^{(l)}{}_{ab}, \qquad \nabla^a\mathcal{T}^{(l)}{}_{ab} = 0 \qquad (4)$$

Equations (cont.)

Example ($l = 1$):

$$\Psi_1^0 = \psi(t,r) Y_1^0(\theta,\phi), \tag{5}$$

$$\Psi_1^{-1} = \frac{i}{\sqrt{2}} \psi(t,r) \left[Y_1^1(\theta,\phi) + Y_1^{-1}(\theta,\phi) \right], \tag{6}$$

$$\Psi_1^1 = \frac{1}{\sqrt{2}} \psi(t,r) \left[Y_1^1(\theta,\phi) - Y_1^{-1}(\theta,\phi) \right]. \tag{7}$$

From these fields we calculate the SET:

$$\mathcal{T}_{ab}^{(l=1)} = T_{ab}^{(10)} + T_{ab}^{(1-1)} + T_{ab}^{(11)}. \tag{8}$$

This procedure amounts to do an statistical average over the angular variables. Incoherent sum.

Equations (cont.)

Metric:

$$ds^2 = -\alpha^2(t,r)dt^2 + a^2(t,r)dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2,$$

$$\frac{\partial \Phi}{\partial t} = \frac{\partial}{\partial r}(\frac{\alpha}{a}\Pi), \quad (9)$$

$$\frac{\partial \Pi}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\alpha}{a}\Phi) - l(l+1)a\alpha\frac{\psi}{r^2}, \quad (10)$$

$$\Phi(t,r) = \frac{\partial \psi}{\partial r}, \quad (11)$$

$$\Pi(t,r) = \frac{a}{\alpha}\frac{\partial \psi}{\partial t}. \quad (12)$$

Equations for the geometry:

$$\frac{1}{a}\frac{\partial a}{\partial r} = \frac{(2l+1)}{2}r(\Pi^2 + \Phi^2 + l(l+1)\frac{a^2}{r^2}\psi^2) - \frac{a^2-1}{2r},$$

$$\frac{1}{\alpha}\frac{\partial \alpha}{\partial r} = \frac{(2l+1)}{2}r(\Pi^2 + \Phi^2 - l(l+1)\frac{a^2}{r^2}\psi^2) + \frac{a^2-1}{2r},$$

$$\frac{\partial a}{\partial t} = (2l+1)r\alpha\Pi\Phi.$$

Mass aspect function:

$$M(t,r) = \frac{r}{2}\left(1 - \frac{1}{a^2}\right), \quad (13)$$

Code

- 2nd order finite difference implementation using RNPL.

- It only requires Adaptive Mesh Refinement (AMR) for low l values ($l \leq 2$).

- Including numerical dissipation and enforcing the proper regularity condition has been crucial to get stable evolutions.

$$\psi(t, 0) = O(r^l) \qquad (14)$$

- Our implementation has passed all the usual convergence tests.

Results

Families of Initial Data:

Family	Form of initial data, $\psi(0,r)$	p
(a)	$A \exp\left(-(r-r_o)^2/\sigma^2\right)$	A
(b)	$-2A\,(r-r_o)/\sigma^2 \exp\left(-(r-r_o)^2/\sigma^2\right)$	A
(c)	$A\,r^2\,(\mathrm{atan}(r-r_o) - \mathrm{atan}(r-r_o-\sigma))$	A

Parameters:

Initial Data	Family	Parameters
1	(a)	$r_o = 70.0, \quad \sigma = 5.00$
2	(b)	$r_o = 70.0, \quad \sigma = 5.00$
3	(c)	$r_o = 70.0, \quad \sigma = 5.00$
4	(a)	$r_o = 40.0, \quad \sigma = 10.0$
5	(a)	$r_o = 40.0, \quad \sigma = 5.00$
6	(a)	$r_o = 70.0, \quad \sigma = 10.0$

Results (cont.)

Low values of l, $l < 10$

As in the original case $l = 0$ the solution at the treshold of black hole formation is Discrete Self Similar (DSS).

$$Z\left((t - t_\star), r\right) = Z\left(e^{n\Delta}(t - t_\star), e^{n\Delta} r\right) \qquad (15)$$

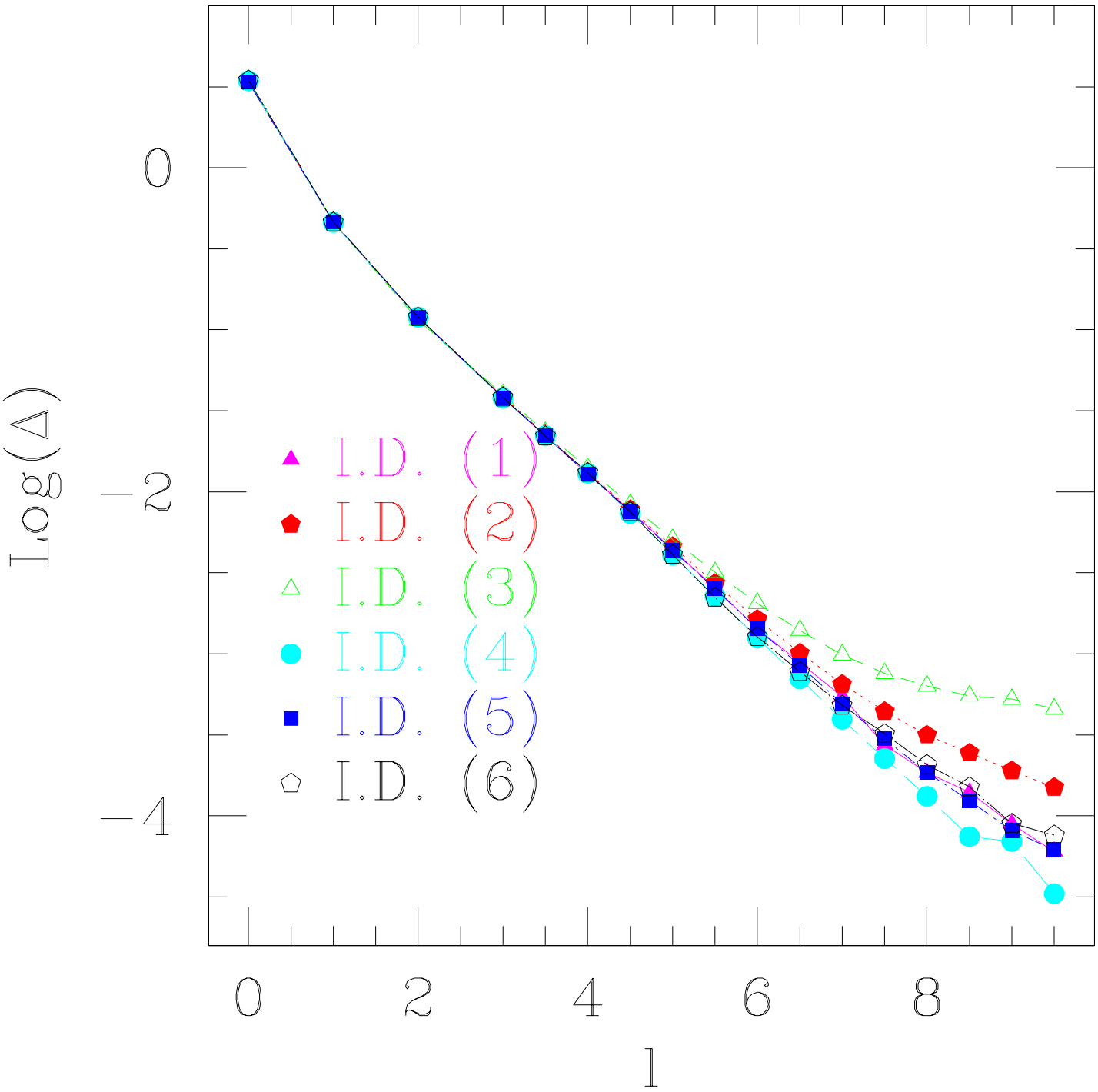
for some values of $t_\star$ and Δ.

A particular *feature* (the maximum in space) will reach the maximum at times t_n:

$$t_n - t_\star = (t_o - t_\star)\, e^{n\Delta} \qquad (16)$$

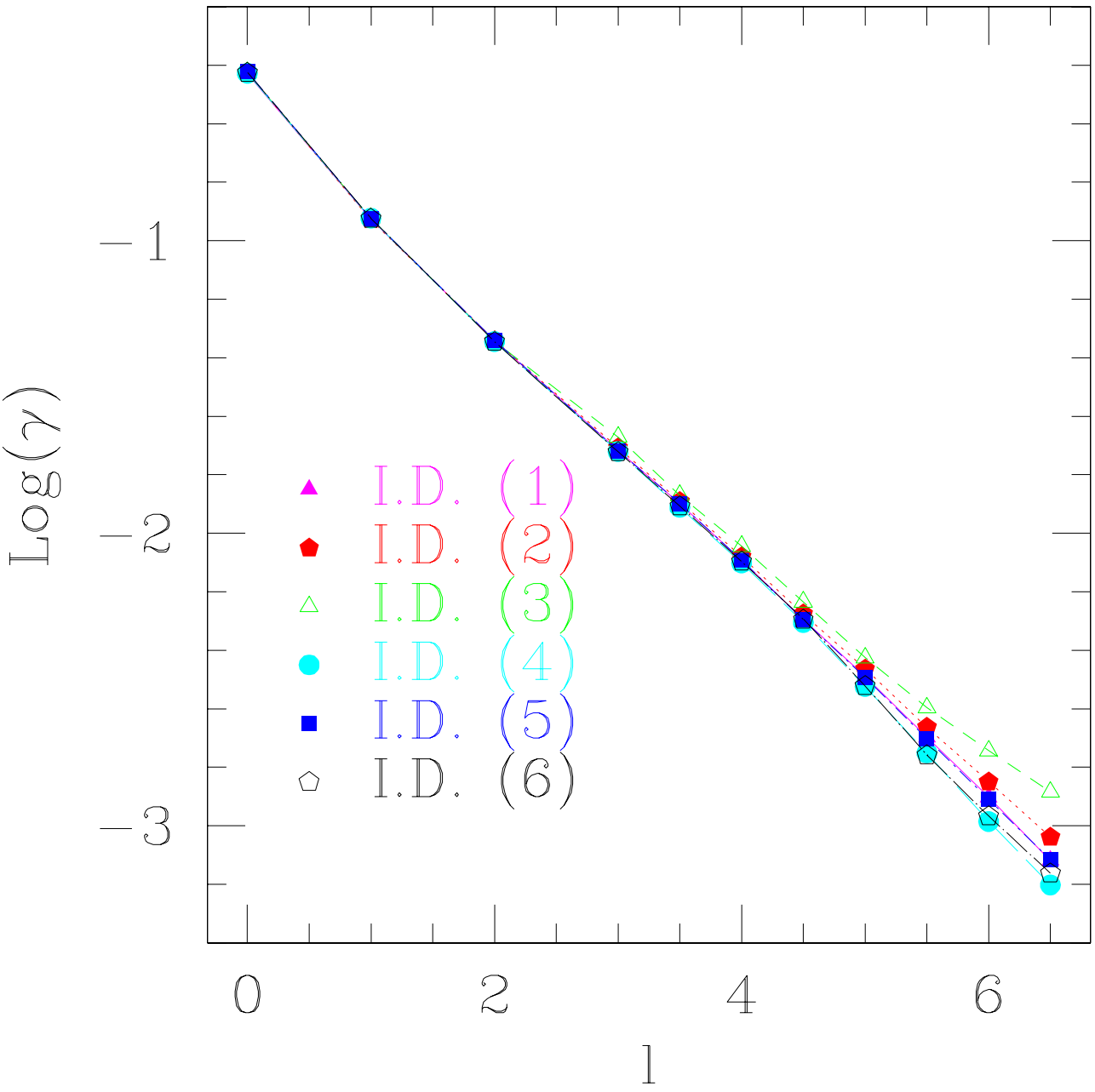
Results (cont.)

Low values of l, $l < 10$



Results (cont.)

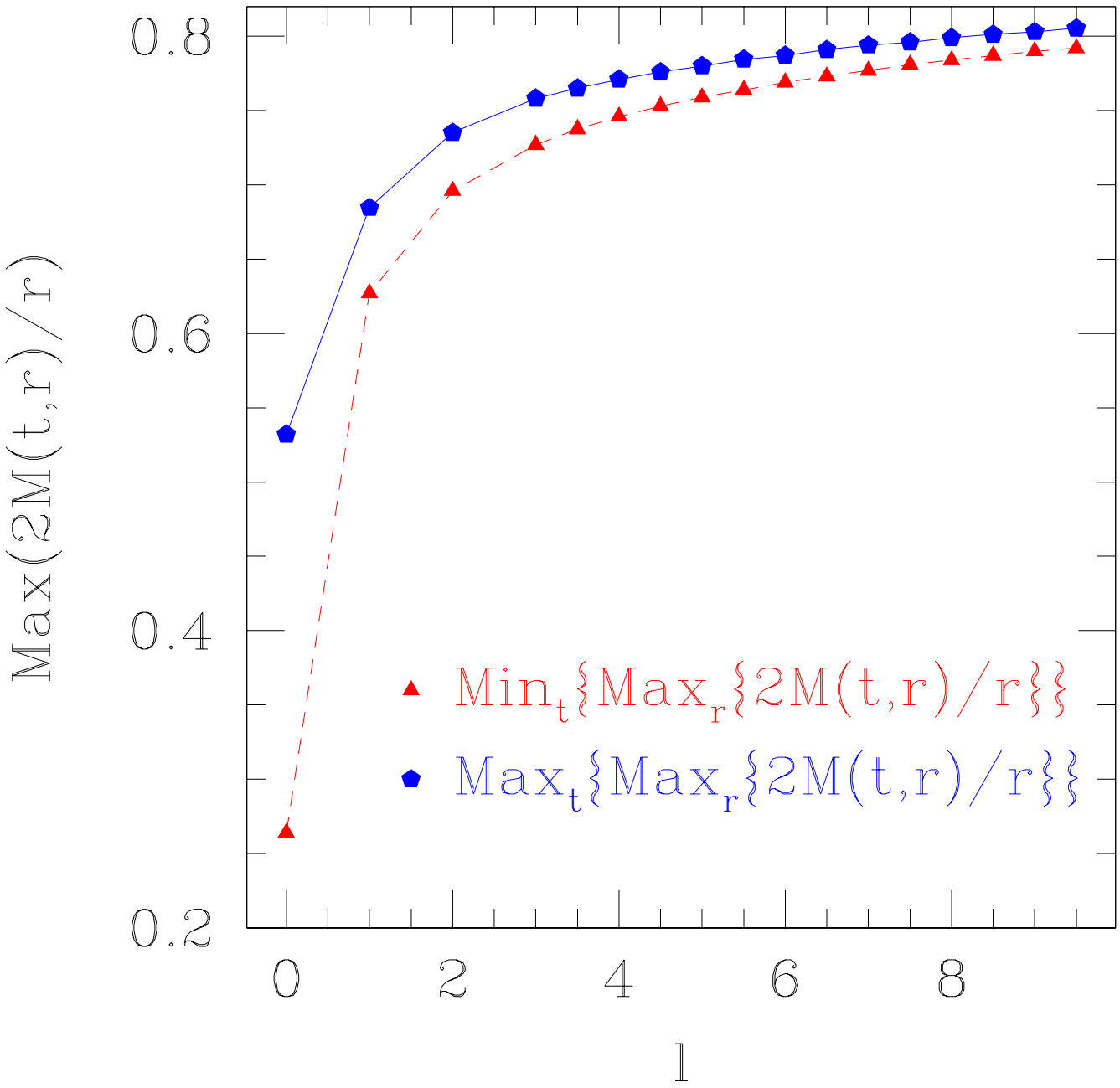
Low values of l, $l < 10$



$$\ln\left(Max\{\mathcal{T}^a{}_a\}\right) = 2\gamma \ln|p - p^*| \qquad (17)$$

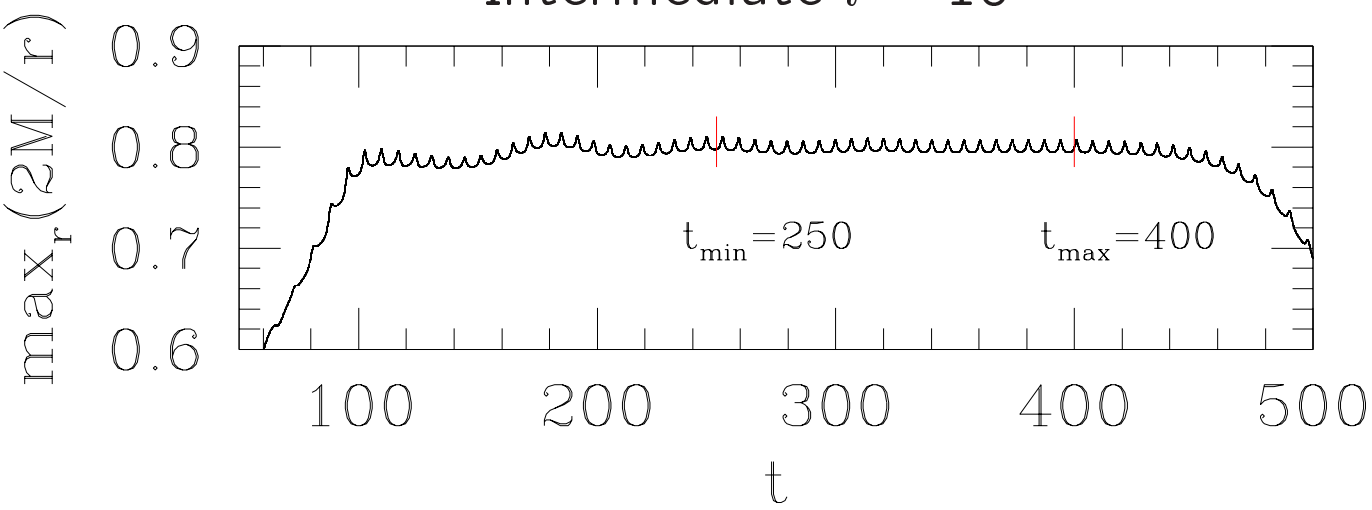
Results (cont.)

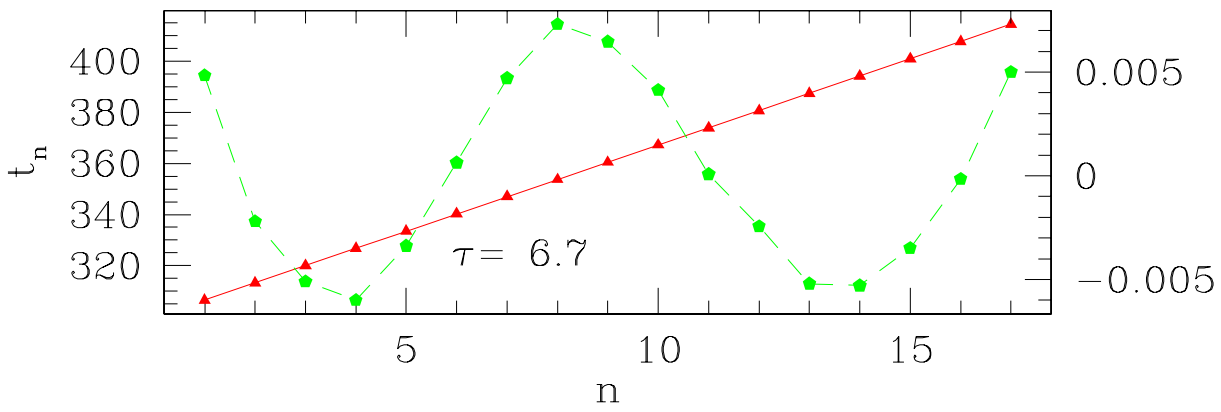
Low values of l, $l < 10$

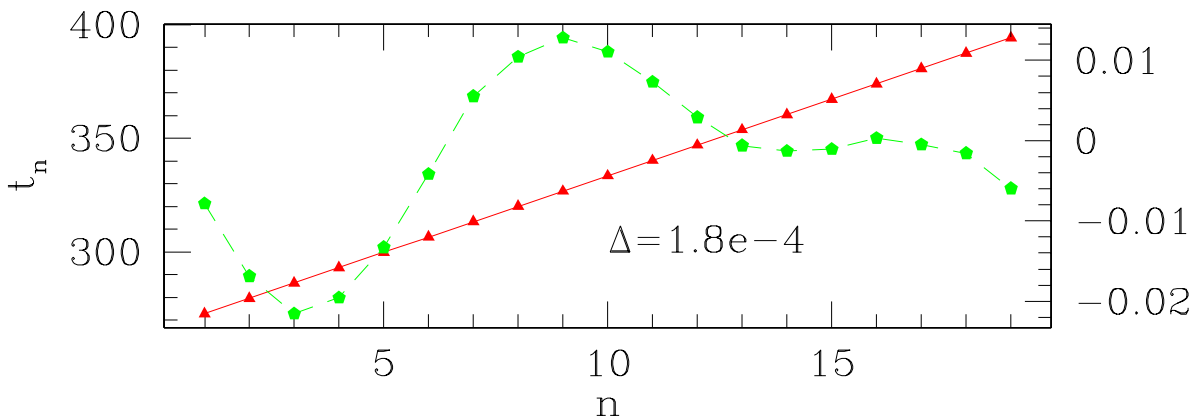


Results (cont.)

Intermediate $l = 10$

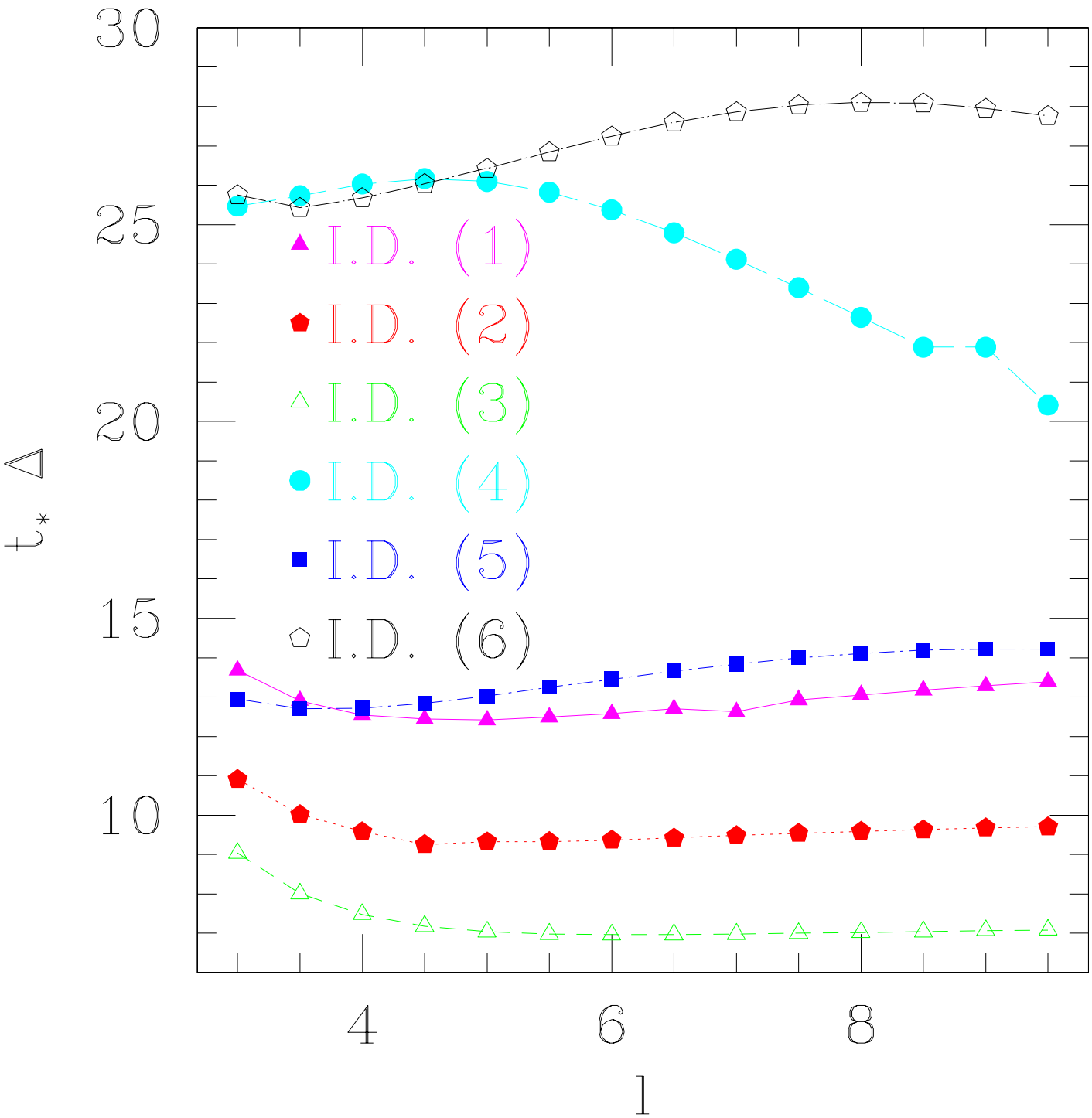






Results (cont.)

Intermediate $l = 9.5$



Results (cont.)

Intermediate $l = 9.5$

As $\Delta \longrightarrow 0$ and $t_\star \longrightarrow \infty$ the solution approaches to be periodic with period $(\Delta t_\star)$

$$t_n - t_\star = (t_o - t_\star)\, e^{n\Delta} \approx (t_o - t_\star)\,(1 + n\Delta)$$

$$t_n = t_o - n\,(\Delta\ t_\star) \qquad (18)$$

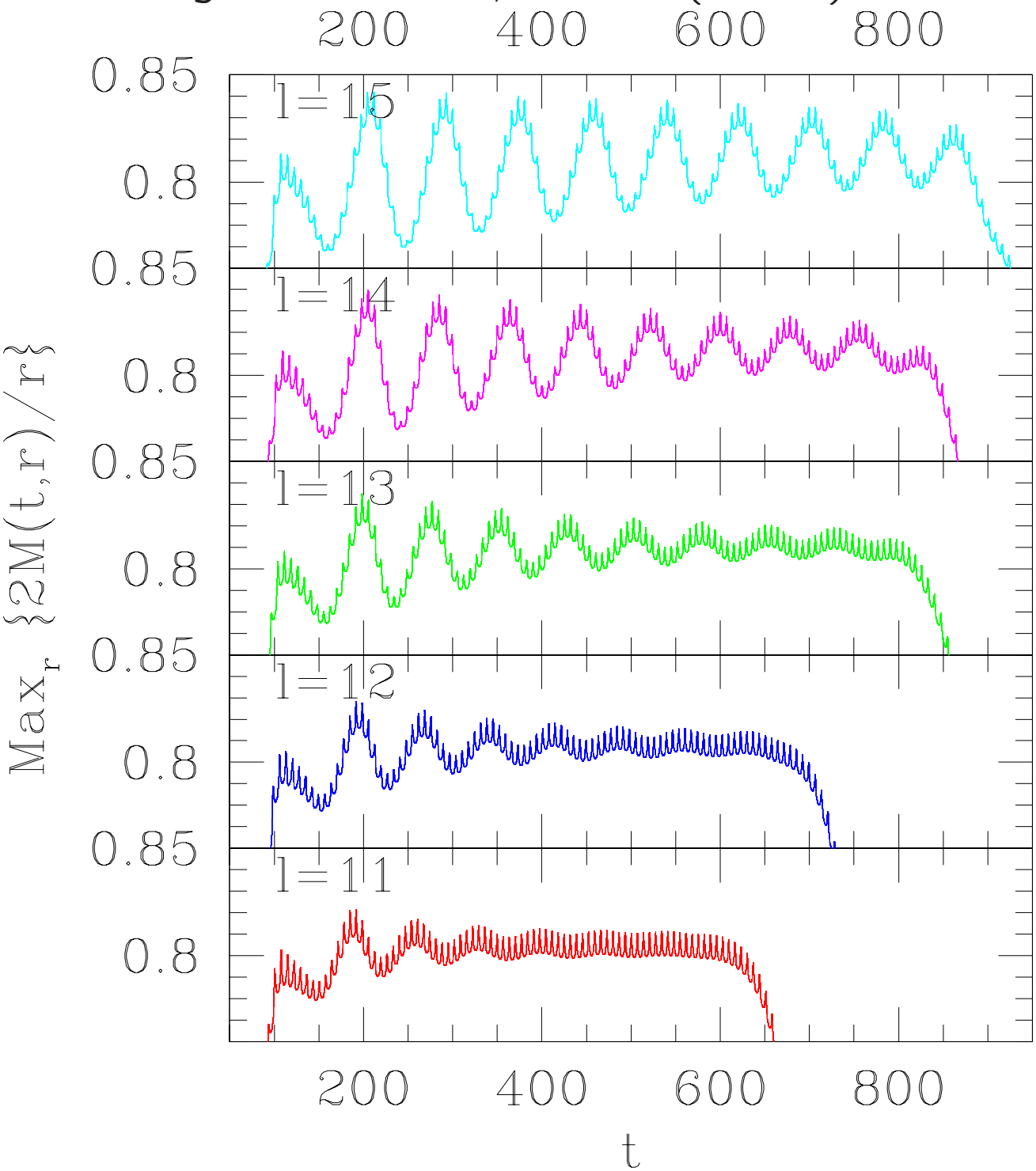
Initial data set a scale after rescaling to match the position r_o at which the maximum of $2M(t,r)/r$ oscillates we get:

$$f_o = r_o/(\Delta\ t_\star) = 4.35 \pm 0.01 \qquad (19)$$

Independently of the Initial Data.

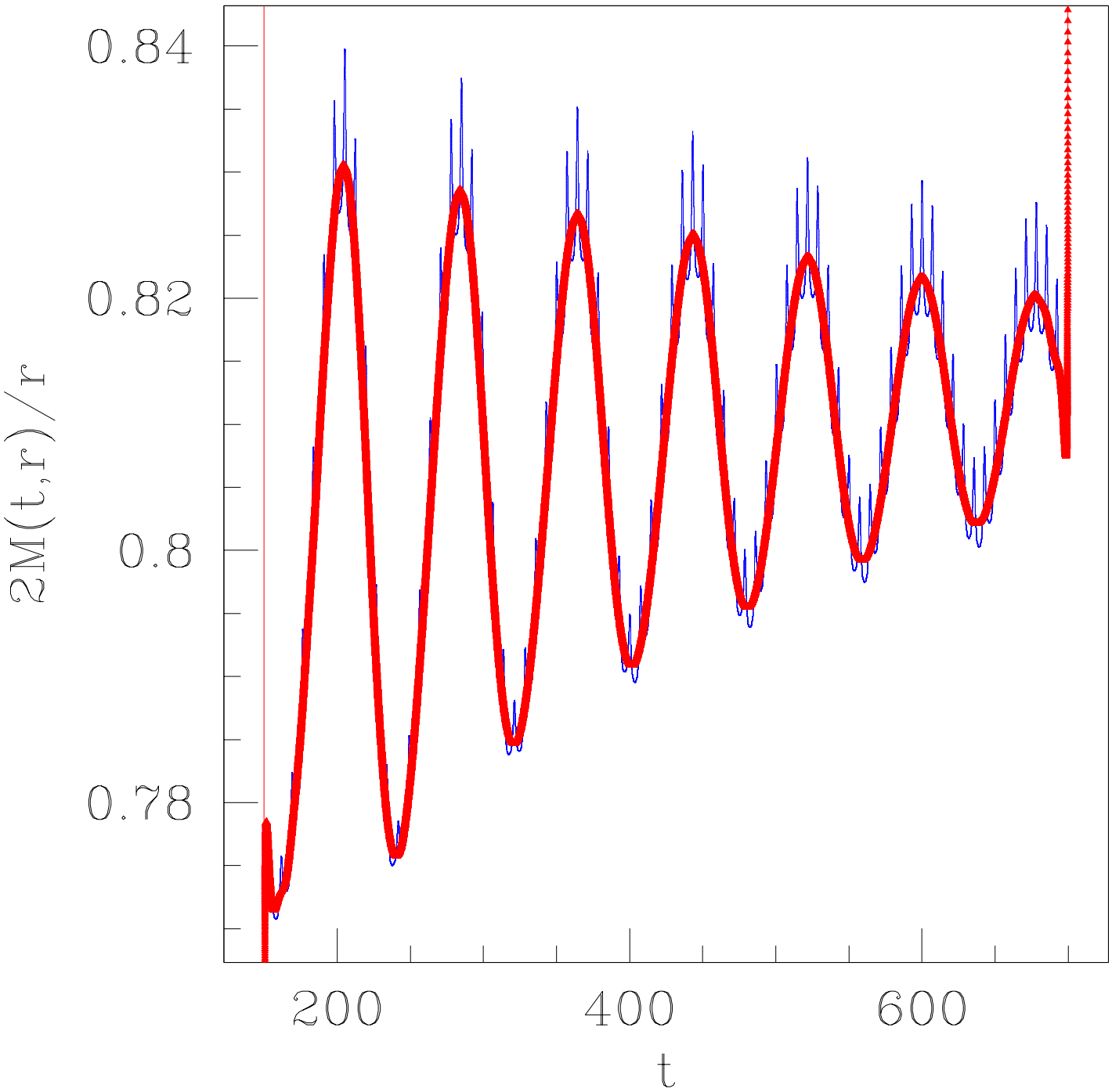
Results (cont.)

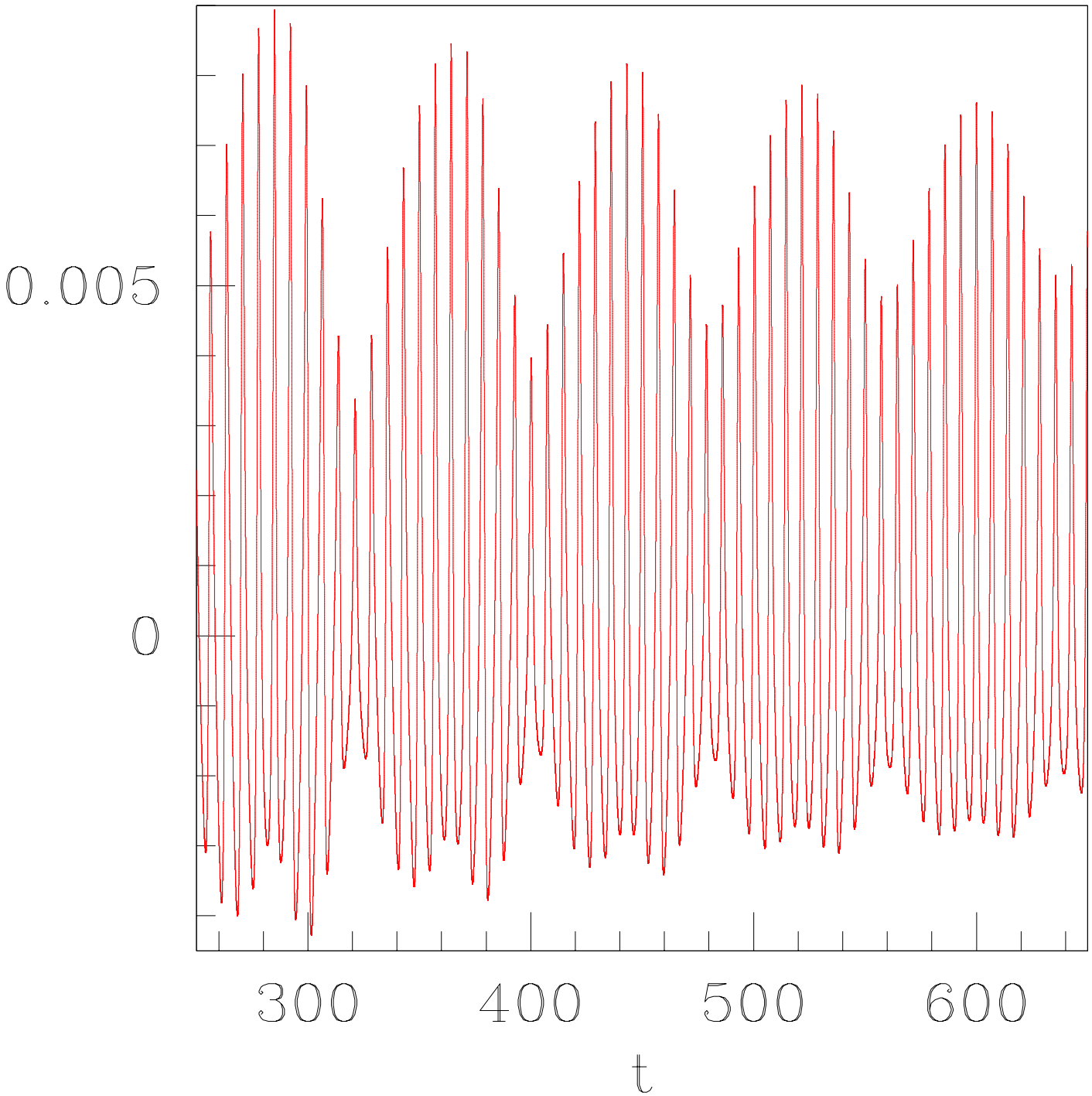
High Values of l, $l = 9.5$ (I.D. 1)



Results (cont.)

High Values of l, $l = 9.5$ (I.D. 1)

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