# Metric

$$ds^2 = \left(-\alpha^2 + a^2\beta^{r\,2} + c^2\beta^{z\,2}\right) dt^2 + 2a^2\beta^r dt dr + 2c^2\beta^z dt dz + a^2 dr^2 + c^2 dz^2 + r^2 d\Omega^2$$

# Evolution Equations

$$\begin{aligned}
\dot{a} &= -\frac{\alpha\Pi_a}{a} + 1/2\, a\beta^r{}_{,r} + 1/2\,\frac{(c)^2\,\beta^z{}_{,z}}{a} + \beta^r a_{,r} + \beta^z a_{,z} \\
\dot{c} &= -\frac{\alpha\Pi_c}{c} + 1/2\,\frac{(a)^2\,\beta^r{}_{,r}}{c} + 1/2\, c\beta^z{}_{,z} + \beta^z c_{,z} + \beta^r c_{,r} \\
\dot{\Pi}_a &= \Pi_a\beta^r{}_{,r} - \frac{\alpha c_{,rr}}{c} + 1/2\,\frac{(c)^2\,(\beta^z{}_{,r})^2}{\alpha} - 1/2\,\frac{(a)^2\,\frac{d}{dt}\beta^r{}_{,r}}{\alpha} + 1/2\,\frac{(a)^2\,\beta^r\beta^r{}_{,rr}}{\alpha} \\
&\quad + \frac{a_{,r}\alpha_{,r}}{a} - \frac{a\alpha a_{,zz}}{(c)^2} + 1/2\,\frac{(a)^2\,\beta^z\beta^r{}_{,rz}}{\alpha} - 1/2\,\frac{(c)^2\,\beta^z\beta^z{}_{,zz}}{\alpha} - 1/2\,\frac{(c)^2\,\beta^r\beta^z{}_{,rz}}{\alpha} \\
&\quad + \frac{(c)^2\,(\beta^z{}_{,z})^2}{\alpha} - 1/2\,\frac{(a)^2\,\beta^r\beta^r{}_{,r}\alpha_{,r}}{(\alpha)^2} - \alpha_{,rr} + 2\,\frac{\alpha a_{,r}}{ar} + 1/2\,\frac{(a)^2\,\beta^r{}_{,r}\frac{\partial}{\partial t}\alpha}{(\alpha)^2} \\
&\quad + \frac{\alpha c_{,r}a_{,r}}{ac} - 1/2\,\frac{(a)^2\,\beta^z\beta^r{}_{,r}\alpha_{,z}}{(\alpha)^2} + \beta^r(\Pi_a)_{,r} - \frac{aa_{,z}\alpha_{,z}}{(c)^2} + 1/2\,\frac{(c)^2\,\beta^z\beta^z{}_{,z}\alpha_{,z}}{(\alpha)^2} \\
&\quad + 1/2\,\frac{(c)^2\,\beta^r\beta^z{}_{,z}\alpha_{,r}}{(\alpha)^2} - 1/2\,\frac{(a)^4\,(\beta^r{}_{,z})^2}{\alpha\,(c)^2} - 1/2\,\frac{(c)^2\,\beta^z{}_{,z}\frac{\partial}{\partial t}\alpha}{(\alpha)^2} + 2\,\frac{(c)^2\,\beta^r\beta^z{}_{,z}}{\alpha r} \\
&\quad - \frac{(c)^4\,(\beta^z{}_{,z})^2}{(a)^2\,\alpha} + 3\,\frac{(c)^2\,\Pi_a\beta^z{}_{,z}}{(a)^2} + \frac{a\alpha a_{,z}c_{,z}}{(c)^3} + \beta^z(\Pi_a)_{,z} - 2\,\frac{\alpha\,(\Pi_a)^2}{(a)^2} \\
&\quad + 1/2\,\frac{(c)^2\,\frac{\partial^2}{\partial t\partial z}\beta^z}{\alpha} - 8\,\alpha\pi\,\left(\frac{\partial}{\partial r}\psi\right)^2
\end{aligned}$$

# Constraint Equations

Hamiltonian Constraint:

$$
\begin{aligned}
&-2\,\frac{a_{,zz}}{a\,(c)^2}-2\,\frac{c_{,rr}}{c\,(a)^2}+4\,\frac{a_{,r}}{(a)^3\,r}-2\,\frac{(a)^2\,\beta^r\beta^r{}_{,r}}{r\,(c)^2\,(\alpha)^2}+2\,\frac{\beta^r\beta^r{}_{,r}}{r\,(\alpha)^2}-1/2\,\frac{(a)^2\,(\beta^r{}_{,r})^2}{(c)^2\,(\alpha)^2}\\
&+\frac{\beta^r{}_{,r}\Pi_c}{(c)^2\,\alpha}-\frac{\Pi_a\beta^r{}_{,r}}{(c)^2\,\alpha}-1/2\,\frac{(c)^2\,(\beta^z{}_{,r})^2}{(a)^2\,(\alpha)^2}+2\,\frac{(\beta^r)^2}{r^2\,(\alpha)^2}+4\,\frac{\beta^r\Pi_a}{r\,(a)^2\,\alpha}+2\,\frac{c_{,r}a_{,r}}{(a)^3\,c}\\
&-4\,\frac{c_{,r}}{(a)^2\,rc}+4\,\frac{\beta^r\Pi_c}{r\,(c)^2\,\alpha}+2\,r^{-2}+\frac{\beta^z{}_{,z}\beta^r{}_{,r}}{(\alpha)^2}-\frac{\beta^r{}_{,z}\beta^z{}_{,r}}{(\alpha)^2}+2\,\frac{\beta^r\beta^z{}_{,z}}{r\,(\alpha)^2}\\
&-2\,\frac{(c)^2\,\beta^r\beta^z{}_{,z}}{(a)^2\,r\,(\alpha)^2}-\frac{\beta^z{}_{,z}\Pi_c}{(a)^2\,\alpha}-1/2\,\frac{(c)^2\,(\beta^z{}_{,z})^2}{(a)^2\,(\alpha)^2}-1/2\,\frac{(a)^2\,(\beta^r{}_{,z})^2}{(c)^2\,(\alpha)^2}+\frac{\Pi_a\beta^z{}_{,z}}{(a)^2\,\alpha}\\
&+2\,\frac{a_{,z}c_{,z}}{(c)^3\,a}+2\,\frac{\Pi_a\Pi_c}{(a)^2\,(c)^2}-2\,\frac{1}{r^2\,(a)^2}=8\,\pi\,(\Pi_\psi)^2+8\,\frac{\pi\,\left(\frac{\partial}{\partial r}\psi\right)^2}{(a)^2}+8\,\frac{\pi\,\left(\frac{\partial}{\partial z}\psi\right)^2}{(c)^2}
\end{aligned}
$$

R-Momentum Constraint:

$$
\begin{aligned}
&-\frac{\beta^r{}_{,r}\alpha_{,r}}{\alpha}+2\,\frac{\beta^r{}_{,r}a_{,r}}{a}-\frac{cc_{,r}\beta^z{}_{,z}}{(a)^2}+\frac{(c)^2\,\beta^z{}_{,z}\alpha_{,r}}{\alpha\,(a)^2}-2\,\frac{(c)^2\,\beta^r{}_{,r}}{(a)^2\,r}\\
&+4\,\frac{(c)^2\,\beta^r\alpha_{,r}}{r\,(a)^2\,\alpha}+2\,\frac{c\alpha c_{,r}\Pi_a}{(a)^4}+\frac{cc_{,r}\beta^r{}_{,r}}{(a)^2}-\frac{(c)^3\,c_{,r}\beta^z{}_{,z}}{(a)^4}-2\,\frac{(c)^4\,\beta^z{}_{,z}}{(a)^4\,r}\\
&+4\,\frac{(c)^2\,\alpha\Pi_a}{(a)^4\,r}+3\,\frac{a_{,z}\beta^r{}_{,z}}{a}+\frac{(c)^2\,a_{,z}\beta^z{}_{,r}}{(a)^3}-\frac{c_{,z}\beta^r{}_{,z}}{c}+\frac{cc_{,z}\beta^z{}_{,r}}{(a)^2}+\beta^r{}_{,zz}\\
&-\frac{\alpha_{,z}\beta^r{}_{,z}}{\alpha}-\frac{(c)^2\,\alpha_{,z}\beta^z{}_{,r}}{\alpha\,(a)^2}+2\,\frac{\alpha c_{,r}\Pi_c}{c\,(a)^2}-2\,\frac{\alpha(\Pi_c)_{,r}}{(a)^2}-\frac{c_{,r}\beta^r{}_{,r}}{c}+\beta^r{}_{,rr}\\
&=-16\,\frac{\left(\frac{\partial}{\partial r}\psi\right)(c)^2\,\alpha\Pi_\psi\pi}{(a)^2}
\end{aligned}
$$

Z-Momentum Constraint:

$$\frac{aa_{,r}\beta^{r}{}_{,z}}{(c)^{2}}-\frac{a_{,r}\beta^{z}{}_{,r}}{a}+3\,\frac{\beta^{z}{}_{,r}c_{,r}}{c}+\beta^{z}{}_{,rr}-\frac{(a)^{2}\alpha_{,r}\beta^{r}{}_{,z}}{\alpha\,(c)^{2}}-\frac{\alpha_{,r}\beta^{z}{}_{,r}}{\alpha}+2\,\frac{a\alpha a_{,z}\Pi_{c}}{(c)^{4}}$$
$$-\frac{(a)^{3}a_{,z}\beta^{r}{}_{,r}}{(c)^{4}}+\frac{aa_{,z}\beta^{z}{}_{,z}}{(c)^{2}}+2\,\frac{\alpha a_{,z}\Pi_{a}}{a\,(c)^{2}}-\frac{aa_{,z}\beta^{r}{}_{,r}}{(c)^{2}}-\frac{a_{,z}\beta^{z}{}_{,z}}{a}+\frac{(a)^{2}c_{,r}\beta^{r}{}_{,z}}{(c)^{3}}$$
$$-2\,\frac{(a)^{2}\beta^{r}{}_{,z}}{(c)^{2}r}+2\,\frac{\beta^{z}{}_{,r}}{r}-2\,\frac{\alpha(\Pi_{a})_{,z}}{(c)^{2}}+\frac{(a)^{2}\beta^{r}{}_{,r}\alpha_{,z}}{\alpha\,(c)^{2}}+2\,\frac{\beta^{z}{}_{,z}c_{,z}}{c}-\frac{\beta^{z}{}_{,z}\alpha_{,z}}{\alpha}+\beta^{z}{}_{,zz}$$
$$+4\,\frac{(a)^{2}\beta^{r}\alpha_{,z}}{\alpha\,(c)^{2}r}=-16\,\frac{\left(\frac{\partial}{\partial z}\psi\right)(a)^{2}\alpha\Pi_{\psi}\pi}{(c)^{2}}$$

Slicing Condition (still $Tr\dot{K}=0$):

$$8\,\frac{\alpha\pi\,\left(\frac{\partial}{\partial r}\psi\right)^{2}}{(a)^{2}}+2\,\frac{\alpha_{,r}}{r\,(a)^{2}}-2\,\frac{\alpha}{r^{2}}+2\,\frac{\alpha}{r^{2}\,(a)^{2}}+\frac{\alpha_{,rr}}{(a)^{2}}-\frac{a_{,r}\alpha_{,r}}{(a)^{3}}-4\,\frac{\alpha a_{,r}}{(a)^{3}r}$$
$$+\frac{a_{,z}\alpha_{,z}}{a\,(c)^{2}}+2\,\frac{\alpha a_{,zz}}{a\,(c)^{2}}+2\,\frac{\alpha c_{,rr}}{c\,(a)^{2}}-2\,\frac{\alpha c_{,r}a_{,r}}{(a)^{3}c}+4\,\frac{\alpha c_{,r}}{r\,(a)^{2}c}+8\,\frac{\alpha\pi\,\left(\frac{\partial}{\partial z}\psi\right)^{2}}{(c)^{2}}$$
$$+\frac{c_{,r}\alpha_{,r}}{c\,(a)^{2}}-2\,\frac{\alpha a_{,z}c_{,z}}{(c)^{3}a}-\frac{c_{,z}\alpha_{,z}}{(c)^{3}}+\frac{\alpha_{,zz}}{(c)^{2}}$$

Equation for $\Pi_c$ from $TrK=0$:

$$\begin{aligned}\Pi_{c} &= -\frac{(c)^{2}\Pi_{a}}{(a)^{2}}-1/2\,\frac{(c)^{2}\beta^{r}{}_{,r}}{\alpha}+1/2\,\frac{(c)^{4}\beta^{z}{}_{,z}}{(a)^{2}\alpha}\\ &\quad +1/2\,\frac{(a)^{2}\beta^{r}{}_{,r}}{\alpha}-1/2\,\frac{(c)^{2}\beta^{z}{}_{,z}}{\alpha}-2\,\frac{\beta^{r}\,(c)^{2}}{r\alpha}\end{aligned}$$

## Scalar Field

$$
\begin{aligned}
\Pi_\psi &= \frac{c_{,r}\alpha\frac{\partial}{\partial r}\psi}{c\left(a\right)^2}+\frac{\alpha\Pi_\psi\Pi_c}{\left(c\right)^2}-1/2\,\frac{\left(a\right)^2\Pi_\psi\beta^r{}_{,r}}{\left(c\right)^2}+1/2\,\beta^z{}_{,z}\Pi_\psi+\beta^z\frac{\partial}{\partial z}\Pi_\psi \\
&\quad+\frac{a_{,z}\alpha\frac{\partial}{\partial z}\psi}{a\left(c\right)^2}+\frac{\alpha_{,z}\frac{\partial}{\partial z}\psi}{\left(c\right)^2}+\frac{\alpha\Pi_\psi\Pi_a}{\left(a\right)^2}+1/2\,\beta^r{}_{,r}\Pi_\psi-1/2\,\frac{\left(c\right)^2\Pi_\psi\beta^z{}_{,z}}{\left(a\right)^2} \\
&\quad+\beta^r\frac{\partial}{\partial r}\Pi_\psi+\frac{\alpha_{,r}\frac{\partial}{\partial r}\psi}{\left(a\right)^2}+\frac{\alpha\frac{\partial^2}{\partial r^2}\psi}{\left(a\right)^2}-\frac{a_{,r}\alpha\frac{\partial}{\partial r}\psi}{\left(a\right)^3}+\frac{\alpha\frac{\partial^2}{\partial z^2}\psi}{\left(c\right)^2}-\frac{c_{,z}\alpha\frac{\partial}{\partial z}\psi}{\left(c\right)^3} \\
&\quad+2\,\frac{\beta^r\Pi_\psi}{r}+2\,\frac{\left(\frac{\partial}{\partial r}\psi\right)\alpha}{r\left(a\right)^2} \\
\dot{\psi} &= \alpha\Pi_\psi+\beta^z\frac{\partial}{\partial z}\psi+\beta^r\frac{\partial}{\partial r}\psi
\end{aligned}
$$