Metric

$$ds^2 = \left[-\alpha^2 + \psi^4\left(\beta^{r\,2} + \beta^{z\,2}\right)\right]dt^2 + 2\psi^4\beta^r dtdr + 2\psi^4\beta^z dtdz + \psi^4\left[dr^2 + dz^2 + b^2r^2d\Omega^2\right]$$

Evolution Equations

$$\begin{aligned}
\dot{\psi} &= -1/2\,\frac{k_{rr}\alpha}{(\psi)^3} + \beta^r\psi_{,r} + \beta^z\psi_{,z} + 1/2\,\psi\beta^r{}_{,r} \\
\dot{b} &= 2\,\frac{bk_{rr}\alpha}{(\psi)^4} - b\beta^r{}_{,r} + \beta^r b_{,r} + \frac{\beta^r b}{r} + \beta^z b_{,z} \\
\dot{k}_{rr} &= -2\,\frac{(k_{rr})^2\,\alpha}{(\psi)^4} + 2\,k_{rr}\beta^r{}_{,r} + 6\,\frac{\alpha\,(\psi_{,r})^2}{(\psi)^2} - \alpha_{,rr} \\
&\quad -8\,\alpha\pi\,\left(\frac{\partial}{\partial r}\phi\right)^2 - 4\,\frac{\alpha\psi_{,r}b_{,r}}{\psi b} - 4\,\frac{\alpha\psi_{,r}}{r\psi} + \beta^r(k_{rr})_{,r} \\
&\quad +\beta^z(k_{rr})_{,z} + 1/2\,\frac{(\psi)^4\,(\beta^z{}_{,r})^2}{\alpha} - 1/2\,\frac{(\psi)^4\,(\beta^r{}_{,z})^2}{\alpha} + 2\,\frac{\alpha_{,r}\psi_{,r}}{\psi} \\
&\quad -2\,\frac{\alpha_{,z}\psi_{,z}}{\psi} - 6\,\frac{\alpha\,(\psi_{,z})^2}{(\psi)^2} - 2\,\frac{\alpha\psi_{,zz}}{\psi} - 4\,\frac{\alpha b_{,r}}{br} - 6\,\frac{\alpha\psi_{,rr}}{\psi} \\
&\quad -4\,\frac{\alpha\psi_{,z}b_{,z}}{\psi b} - 2\,\frac{\alpha b_{,rr}}{b}
\end{aligned}$$

Constraint Equations

Hamiltonian Constraint:

$$
\begin{aligned}
&-1/6\,\frac{\psi}{(b)^2\,r^2}+1/6\,\frac{\psi}{r^2}+2\,\frac{\psi_{,z}b_{,z}}{b}+2\,\frac{\psi_{,r}b_{,r}}{b}+1/6\,\frac{\psi\,(b_{,z})^2}{(b)^2}+\frac{(\psi_{,z})^2}{\psi}+\frac{(\psi_{,r})^2}{\psi}\\
&+1/6\,\frac{\psi\,(b_{,r})^2}{(b)^2}+1/3\,\frac{\psi b_{,zz}}{b}+2\,\frac{\psi_{,r}}{r}+\psi_{,rr}+1/3\,\frac{\psi b_{,rr}}{b}+\frac{\psi b_{,r}}{br}+\psi_{,zz}+1/3\,\frac{(k_{rr})^2}{(\psi)^3}\\
&+1/24\,\frac{(\psi)^5\,(\beta^r{}_{,z})^2}{(\alpha)^2}+1/12\,\frac{(\psi)^5\,\beta^r{}_{,z}\beta^z{}_{,r}}{(\alpha)^2}+1/24\,\frac{(\psi)^5\,(\beta^z{}_{,r})^2}{(\alpha)^2}\\
&=-2/3\,\,(\psi)^5\,\pi\,\,(\Pi)^2-2/3\,\psi\pi\,\left(\frac{\partial}{\partial r}\phi\right)^2-2/3\,\psi\pi\,\left(\frac{\partial}{\partial z}\phi\right)^2
\end{aligned}
$$

R-Momentum Constraint:

$$
\begin{aligned}
&\frac{(k_{rr})_{,r}}{(\psi)^4}+4\,\frac{\psi_{,z}\beta^r{}_{,z}}{\alpha\psi}+4\,\frac{\psi_{,z}\beta^z{}_{,r}}{\alpha\psi}+1/2\,\frac{\beta^r{}_{,zz}}{\alpha}+1/2\,\frac{\beta^z{}_{,rz}}{\alpha}-1/2\,\frac{\alpha_{,z}\beta^r{}_{,z}}{(\alpha)^2}\\
&-1/2\,\frac{\alpha_{,z}\beta^z{}_{,r}}{(\alpha)^2}+4\,\frac{k_{rr}\psi_{,r}}{(\psi)^5}+\frac{b_{,z}\beta^r{}_{,z}}{b\alpha}+\frac{b_{,z}\beta^z{}_{,r}}{b\alpha}+4\,\frac{k_{rr}b_{,r}}{b\,(\psi)^4}+4\,\frac{k_{rr}}{r\,(\psi)^4}=-8\,\left(\frac{\partial}{\partial r}\phi\right)\Pi\pi
\end{aligned}
$$

Z-Momentum Constraint:

$$
\begin{aligned}
&4\,\frac{\psi_{,r}\beta^r{}_{,z}}{\alpha\psi}+4\,\frac{\psi_{,r}\beta^z{}_{,r}}{\alpha\psi}+1/2\,\frac{\beta^r{}_{,rz}}{\alpha}+1/2\,\frac{\beta^z{}_{,rr}}{\alpha}-1/2\,\frac{\alpha_{,r}\beta^r{}_{,z}}{(\alpha)^2}-1/2\,\frac{\alpha_{,r}\beta^z{}_{,r}}{(\alpha)^2}\\
&+\frac{(k_{rr})_{,z}}{(\psi)^4}+4\,\frac{k_{rr}\psi_{,z}}{(\psi)^5}+\frac{b_{,r}\beta^r{}_{,z}}{b\alpha}+\frac{b_{,r}\beta^z{}_{,r}}{b\alpha}+\frac{\beta^r{}_{,z}}{r\alpha}+\frac{\beta^z{}_{,r}}{r\alpha}+4\,\frac{k_{rr}b_{,z}}{b\,(\psi)^4}=-8\,\left(\frac{\partial}{\partial z}\phi\right)\Pi\pi
\end{aligned}
$$

Slicing Condition:

$$-8\ (\psi)^4\,\alpha\pi\ (\Pi)^2 - 4\,\frac{(k_{rr})^2\,\alpha}{(\psi)^4} - 1/2\,\frac{(\psi)^4\,(\beta^r{}_{,z})^2}{\alpha} - 1/2\,\frac{(\psi)^4\,(\beta^z{}_{,r})^2}{\alpha}$$
$$-\frac{(\psi)^4\,\beta^r{}_{,z}\beta^z{}_{,r}}{\alpha} + 2\,\frac{\alpha_{,z}b_{,z}}{b} + 2\,\frac{\alpha_{,r}}{r} + \alpha_{,rr} + \alpha_{,zz} + 2\,\frac{\alpha_{,r}b_{,r}}{b} + 4\,\frac{\alpha_{,r}\psi_{,r}}{\psi} + 4\,\frac{\alpha_{,z}\psi_{,z}}{\psi}$$

Scalar Field

$$\begin{aligned}
\dot{\Pi} &= \beta^r\frac{\partial}{\partial r}\Pi - \beta^r{}_{,r}\Pi + \beta^z{}_{,z}\Pi + \beta^z\frac{\partial}{\partial z}\Pi + \frac{\alpha_{,r}\frac{\partial}{\partial r}\phi}{(\psi)^4} + \frac{\alpha\frac{\partial^2}{\partial r^2}\phi}{(\psi)^4} + \frac{\alpha_{,z}\frac{\partial}{\partial z}\phi}{(\psi)^4} + \frac{\alpha\frac{\partial^2}{\partial z^2}\phi}{(\psi)^4} \\
&\quad + 4\,\frac{\psi_{,r}\left(\frac{\partial}{\partial r}\phi\right)\alpha}{(\psi)^5} + 2\,\frac{b_{,r}\left(\frac{\partial}{\partial r}\phi\right)\alpha}{b\,(\psi)^4} + 4\,\frac{\psi_{,z}\left(\frac{\partial}{\partial z}\phi\right)\alpha}{(\psi)^5} + 2\,\frac{\left(\frac{\partial}{\partial r}\phi\right)\alpha}{(\psi)^4\,r} + 2\,\frac{b_{,z}\left(\frac{\partial}{\partial z}\phi\right)\alpha}{b\,(\psi)^4} \\
\dot{\phi} &= \alpha\Pi + \beta^z\frac{\partial}{\partial z}\phi + \beta^r\frac{\partial}{\partial r}\phi
\end{aligned}$$