$$
\begin{aligned}
a(t,r,z) &= a_o(t,z) + a_1(t,z)r + \frac{1}{2}a_2(t,z)r^2 && (1)\\
\alpha(t,r,z) &= \alpha_o(t,z) + \alpha_1(t,z)r + \frac{1}{2}\alpha_2(t,z)r^2 && (2)\\
c(t,r,z) &= a_o(t,z) + c_1(t,z)r + \frac{1}{2}c_2(t,z)r^2 && (3)\\
\beta^r(t,r,z) &= \beta^r_o(t,z) + \beta^r_1(t,z)r + \frac{1}{2}\beta^r_2(t,z)r^2 && (4)\\
\beta^z(t,r,z) &= \beta^z_o(t,z) + \beta^z_1(t,z)r + \frac{1}{2}\beta^z_2(t,z)r^2 && (5)\\
k_{rr}(t,r,z) &= k_{rro}(t,z) + \beta^r_1(t,z)r + \frac{1}{2}k_{rr2}(t,z)r^2 && (6)\\
\psi(t,r,z) &= \psi_o(t,z) + \psi_1(t,z)r + \frac{1}{2}\psi_2(t,z)r^2 && (7)\\
\Pi(t,r,z) &= \Pi_o(t,z) + \Pi_1(t,z)r + \frac{1}{2}\Pi_2(t,z)r^2 && (8)\\
& && (9)
\end{aligned}
$$

Slicing Condition

$O(0)$:

$$\alpha_o\left(a_o^2 - 1\right) = 0 \qquad (10)$$

from this equation we get:

$$\boxed{a_o = 1}$$

$O(1)$:

$$-2a_o\alpha_o c_1 + 4a_1\alpha_o c_o - 2a_o\alpha_1 c_o + a_o^3\alpha_1 c_o = 0 \qquad (11)$$

substitution of $a_o = 1$ results:

$$-2\alpha_o c_1 + 4a_1\alpha_o c_o - 2\alpha_1 c_o = 0 \qquad (12)$$

and taking into account Frans conditions (i.e. $\alpha_1 = 0$ and $a_1 = 0$) we get:

$$\boxed{c_1 = 0,\ a_1 = 0,\ \alpha_1 = 0}$$

r^2 Hamiltonian Constraint

$O(0)$:

$$3a_o^2\beta_o^{r2} - a_o^2\alpha_o^2 + \alpha_o^2 = 0 \tag{13}$$

Using that $a_o = 1$ then we get:

$$\boxed{\beta_o^r = 0}$$

$O(1)$:

$$-3a_o^3c_o\alpha_1\beta_o^{r2} + a_o\alpha_o^3c_1 - 2c_oa_1\alpha_o^3 + 3\alpha_oa_o^3c_o\beta_o^r\beta_1^r + a_o\alpha_o^2c_o\beta_o^rk_{rro} = 0 \tag{14}$$

Using that $\beta_o^r = 0$ and $a_o = 1$ then:

$$c_1 = 2c_oa_1\alpha_o \tag{15}$$

R- Momentum Constraint

$O(0)$:

$$a_o^2\alpha_oc_1\beta_o^r + a_o^2\alpha_1c_o\beta_o^r - a_o^2\alpha_oc_o\beta_1^r + \alpha_o^2c_ok_{rro} = 0 \tag{16}$$

Using $a_o = 1$ and $\beta_o^r = 0$:

$$\boxed{\beta_1^r = \alpha_ok_{rro}}$$

$O(1)$: Already using the fact that the scalar field goes quadratic, that $a_o = 1$ and that $\beta_o^r = 0$:

$$\begin{aligned}&4\alpha_o^2c_1k_{rro} + \alpha_o\frac{\partial c_o}{\partial z}\beta_1^z - 2\alpha_oc_o\beta_2^r + 4c_o\alpha_1\beta_1^r \\ &+6\alpha_o^2c_ok_{rr1} - \frac{\partial\alpha_o}{\partial z}c_o\beta_1^z + 4\alpha_oc_1\beta_1^r - 12c_oa_1\alpha_o^2k_{rro} + \alpha_oc_o\frac{\partial\beta_1^z}{\partial z} = 0\end{aligned}$$

Using now Frans' results ($a_1 = 0$, $\alpha_1 = 0$ and $c_1 = 0$) and $\beta_1^z = 0$:

$$\beta_2^r = 3\alpha_ok_{rr1} \tag{17}$$

Z- Momentum Constraint

$O(0)$:

$$-a_o^2\alpha_o\frac{\partial\beta_o^r}{\partial z}+\alpha_o c_o^2\beta_1^z+2a_o^2\frac{\partial\alpha_o}{\partial z}\beta_o^r-2\frac{a_o}{\partial z}a_o\alpha_o\beta_o^r=0 \tag{18}$$

Using $a_o=1$ and $\beta_o^r=0$:

$$\boxed{\beta_1^z=0}$$

$O(1)$: Already using the fact that $a_o=1$, $\beta^r=0$

$$\begin{aligned}&5a_1\alpha_o c_o^2\beta_1^z+3c_o^2\alpha_1\beta_1^z+\alpha_o\frac{\partial\beta_1^r}{\partial z}\\&+2\alpha_o^2\frac{k_{rro}}{\partial z}-7\alpha_o c_o c_1\beta_1^z-3\beta_2^z\alpha_o c_o^2\\&-4\frac{\partial\alpha_o}{\partial z}\beta_1^r-16\frac{\partial\psi_o}{\partial z}\Pi_o\pi\alpha_o^2=0\end{aligned}$$

Evolution equation for a

$O(0)$:

$$-\alpha_o k_{rro}+a_o^2\beta_1^r+a_o a_1\beta_o^r a_o+\frac{\partial a_o}{\partial z}\beta_o^z=0 \tag{19}$$

Using $a_o=1$ and $\beta_o^r=0$:

$$\beta_1^r=\alpha_o k_{rro} \tag{20}$$

Evolution equation for c

$O(1)$: Already with $a_o=1$ and $\beta_o^r=0$ and Frans' results:

$$c_o\frac{\partial\beta_1^z}{\partial z}+c_o\beta_2^r+\alpha_o c_o k_{rr1}+\frac{\partial c_o}{\partial z}\beta_1^z=0 \tag{21}$$

Using (14) we get:

$$\beta_2^r=-\alpha_o k_{rr1} \tag{22}$$

with (12) then:

$$\boxed{k_{rr1}=0\longrightarrow\beta_2^r=0}$$