Quantum interest, negative energies and black holes

Motivation for studying matter that violates classical energy conditions, in particular negative energy:

- Such matter is required for many bizarre effects in GR, including worm holes, warp drives and time machines.
- Negative energies could be used to violate the 2^{nd} law of thermodynamics.
- Negative energy fluxes seem to play a crucial role in black hole evaporation.
- Classically, one needs to violate energy conditions to avoid singularities, so negative energy may play an important role in any singularity-free theory of quantum gravity.

Quantum Interest

The quantum inequalities of Ford and Roman, and in particular the "quantum interest" conjecture, place severe restrictions on the magnitude and duration of negative energy fluxes that may arise in free quantum fields in flat space.

L.H. Ford, *Phys. Rev.* **D 43**, 3972 (1991).

L. H. Ford and T. A. Roman, *Phys. Rev.* **D51**, 4277 (1995).

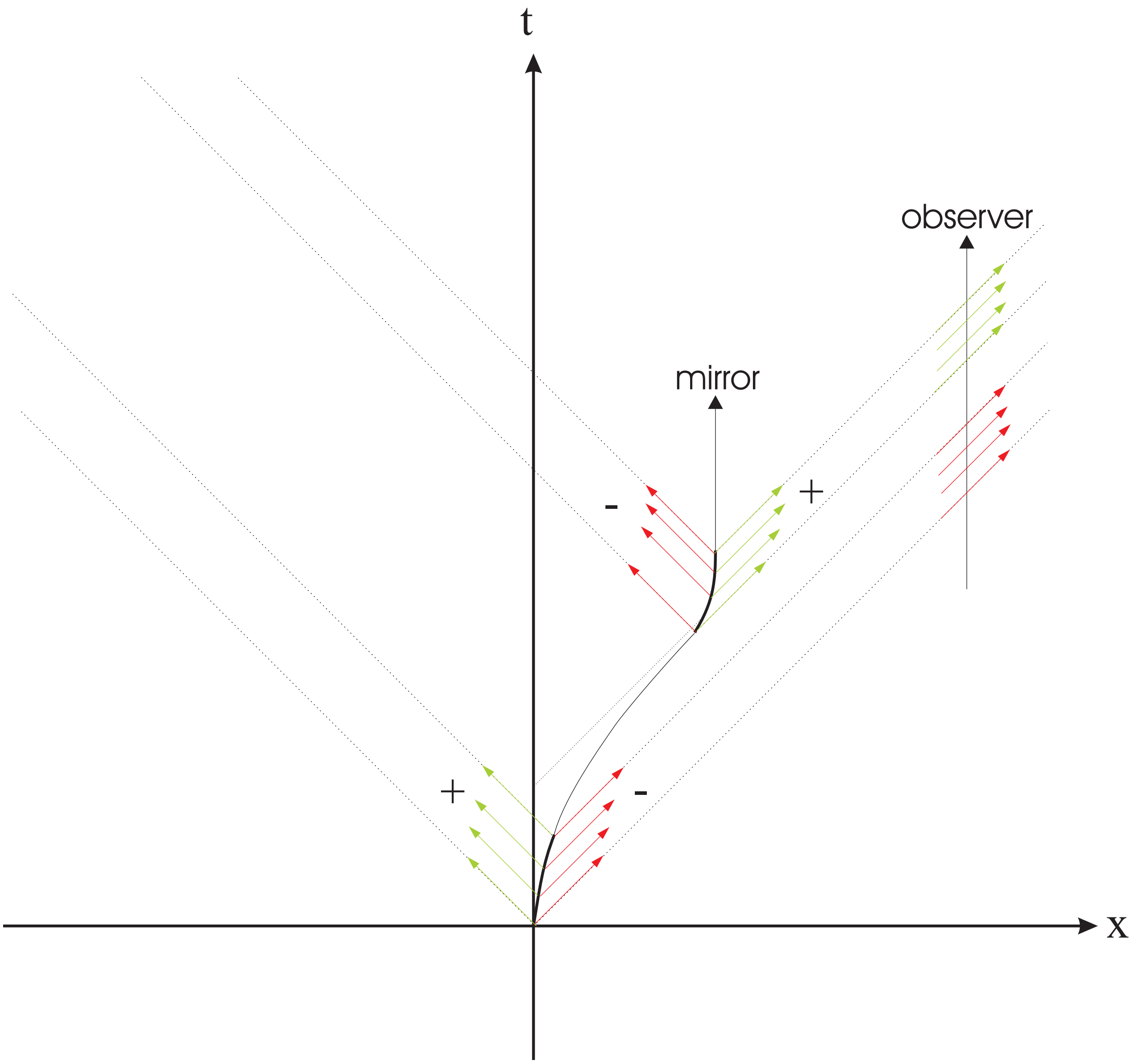
L.H. Ford and T.A. Roman, *The Quantum Interest Conjecture*, gr-qc/9901074

F. Pretorius, *Quantum interest for scalar fields in Minkowski spacetime*, gr-qc/9903055

<u>The Quantum Interest Conjecture</u>: Any negative energy pulse observed (the "loan") must be accompanied ("repaid") by a positive pulse within a certain maximum time interval, and the longer the separation between the pulses the greater the excess of positive energy relative to the negative is needed to satisfy the quantum inequalities (i.e., " repaid with interest").

An accelerating mirror can produce a negative flux, and provides an intructive example of the quantum interest conjecture:

$$F_c = -\frac{1}{12\pi}\frac{da}{d\tau} \quad (\text{in 2D; } \hbar = c = 1)$$



The quantum interest conjecture can be proven as a direct consequence of the *quantum inequalities*, which are restrictions on the sampled energy density $\langle\rho\rangle$ that may be seen by an inertial observer:

$$\langle\rho\rangle \geq \langle\rho_{min}\rangle, \quad \text{where}$$

$$\langle\rho\rangle = \int_{-\infty}^{\infty} g(t)\rho(t)dt,$$

$$\int_{-\infty}^{\infty} g(t)dt = 1 \quad \text{and} \quad g(t) \geq 0 \ \ \forall t.$$

For the massless scalar field in 2D Minkowski spacetime

$$\langle\rho_{min}\rangle = -\frac{1}{24\pi}\int_{-\infty}^{\infty} \frac{g'(t)^2}{g(t)}dt,$$

(Flanagan's optimal bound –
É. É. Flanagan, *Phys. Rev.* **D 56**, 4922 (1997))

and in 4D

$$\langle\rho_{min}\rangle = -\frac{1}{16\pi^2}\int_{-\infty}^{\infty} \left(g^{1/2}(t)''\right)^2 dt, \quad \text{(4D)}.$$

(C.J. Fewster and S.P. Eveson, *Phys. Rev.* **D 58**, 084010 (1998))

To prove quantum interest, sample a distribution $\rho(t)$ *twice*, the 2^{nd} time with a scaled version $\bar{g}(t)$ of the original function $g(t)$:

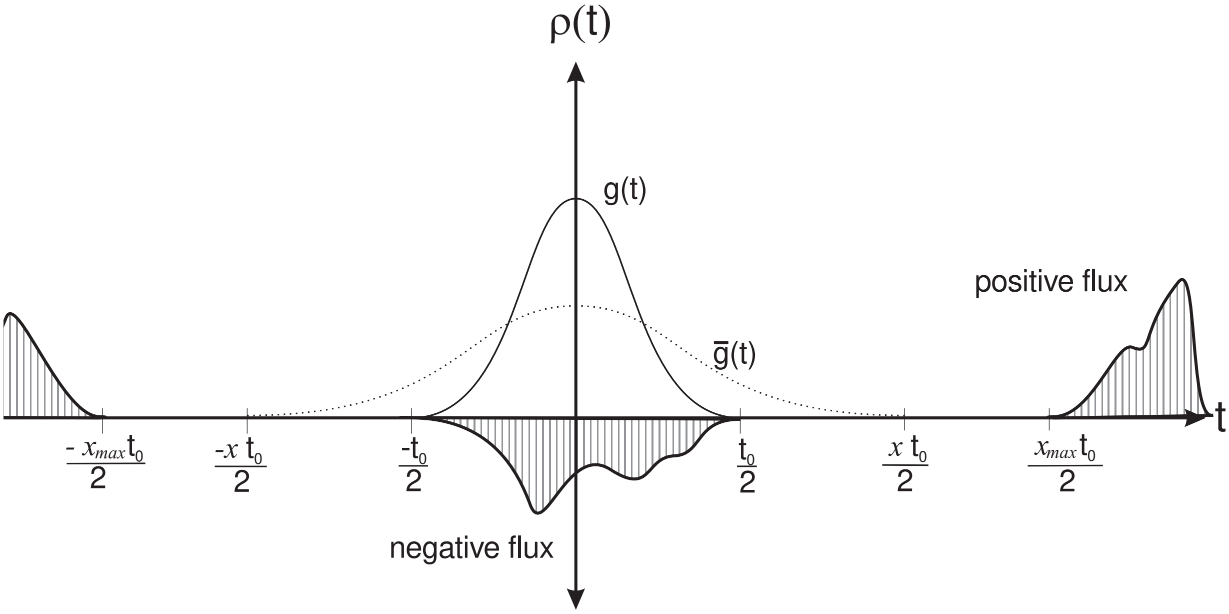
$$\bar{g}(t) = \frac{1}{x} g(t/x), \quad x \geq 1.$$

Then

$$\boxed{\langle \bar{\rho}_{min} \rangle = \frac{\langle \rho_{min} \rangle}{x^d}.}$$

In terms of this scaling relation, we can formulate the quantum interest conjecture as follows:

1. For any given distribution of negative energy sampled between $[-t_0/2, t_0/2]$, there is a maximum scale factor x_{max} corresponding to a sampling window $[-x_{max}t_0/2, x_{max}t_0/2]$ outside of which positive energy *must* be seen.



In particular, define y as $\langle\rho\rangle = y\langle\rho_{min}\rangle$. Then

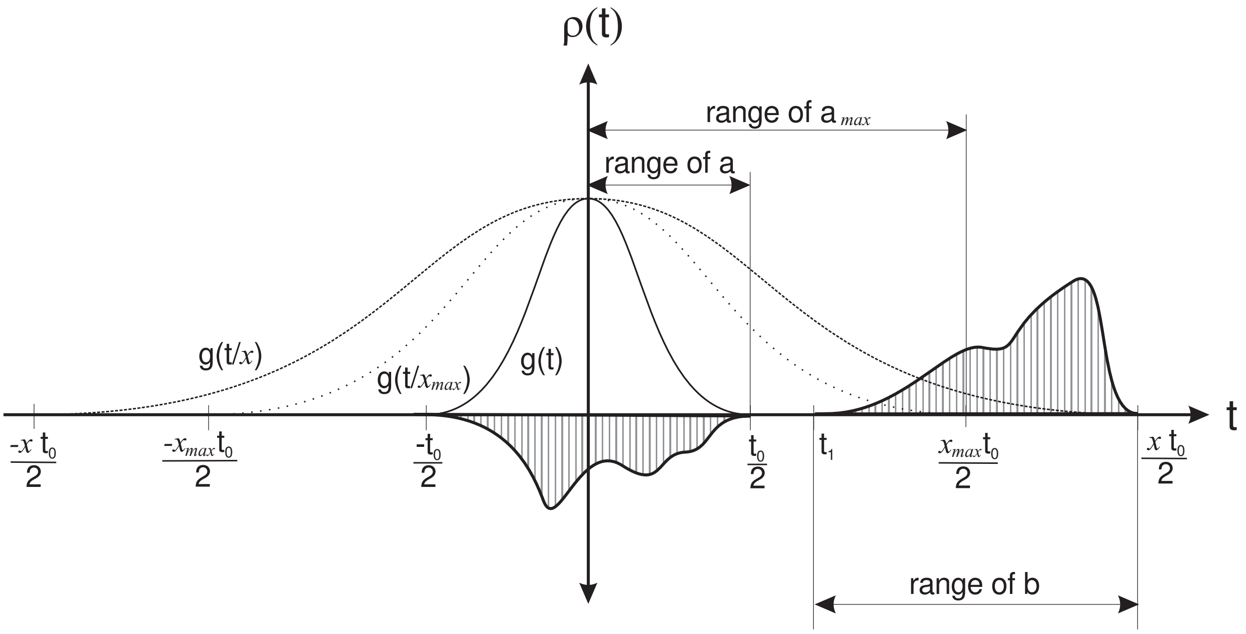
$$x_{max}^{d-1} = \frac{1}{y}\frac{\int_{-t_0/2}^{t_0/2}\rho(t)g(t)dt}{\int_{-t_0/2}^{t_0/2}\rho(t)g(t/x_{max})dt}.$$

2. Define the quantum interest ϵ as

$$\frac{E_p}{|E_m|} = (1+\epsilon).$$

Then one can always find a scale factor x beyond which $\epsilon \geq 0$:

$$(1+\epsilon) \geq \frac{1}{g(b/x)}\left(g(a/x) - g(a_{max}/x_{max})\left(\frac{x_{max}}{x}\right)^{d-1}\right).$$



For large x:

$$\epsilon \geq \frac{|g''(0)|}{2g(0)} \frac{b^2 - a^2}{x^2} - \frac{g(a_{max}/x_{max})}{g(0)} \left(\frac{x_{max}}{x}\right)^{d-1} + O(1/x^3).$$

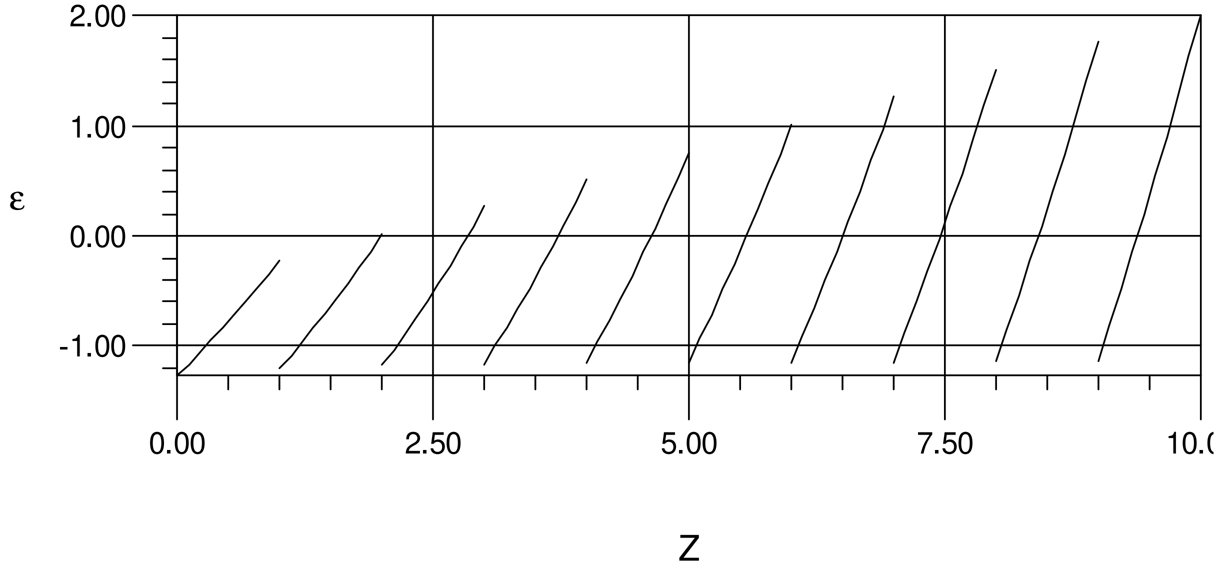
To better illustrate quantum interest, choose the following sampling function $g(t)$:

$$g(t) \propto \left(t^2 - \frac{t_0^2}{4}\right)^2, \quad -\frac{t_0}{2} < t < \frac{t_0}{2}, 0 \quad elsewhere.$$

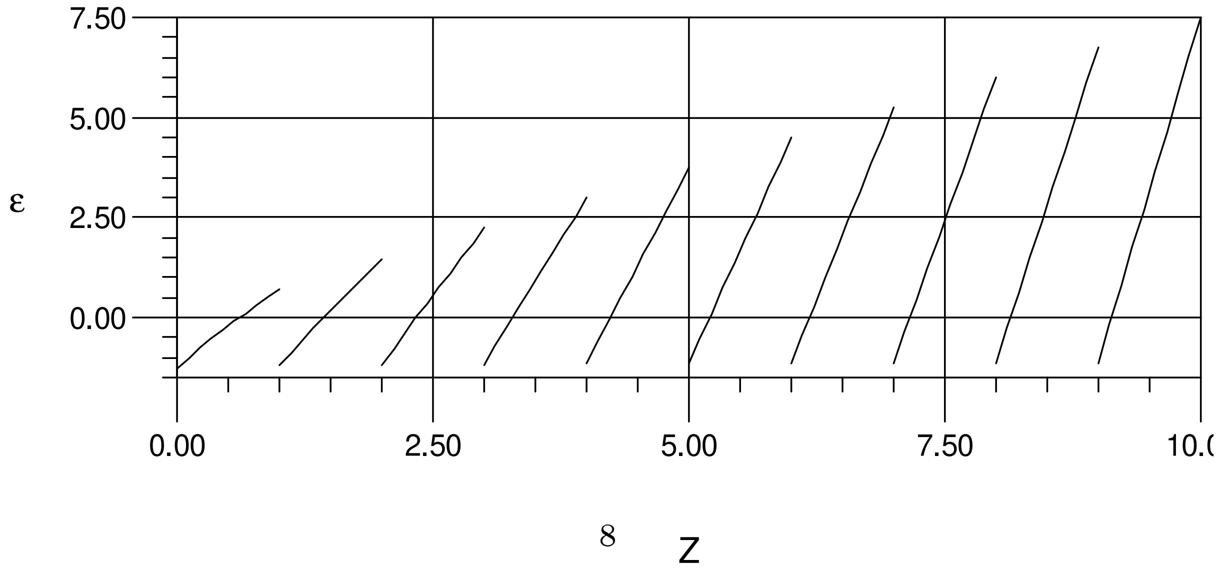
(in 2D, $\langle \rho_{min} \rangle = -\frac{5}{3\pi t_0^2}$; in 4D ,$\langle \rho_{min} \rangle = -\frac{15}{2\pi^2 t_0^4}$).

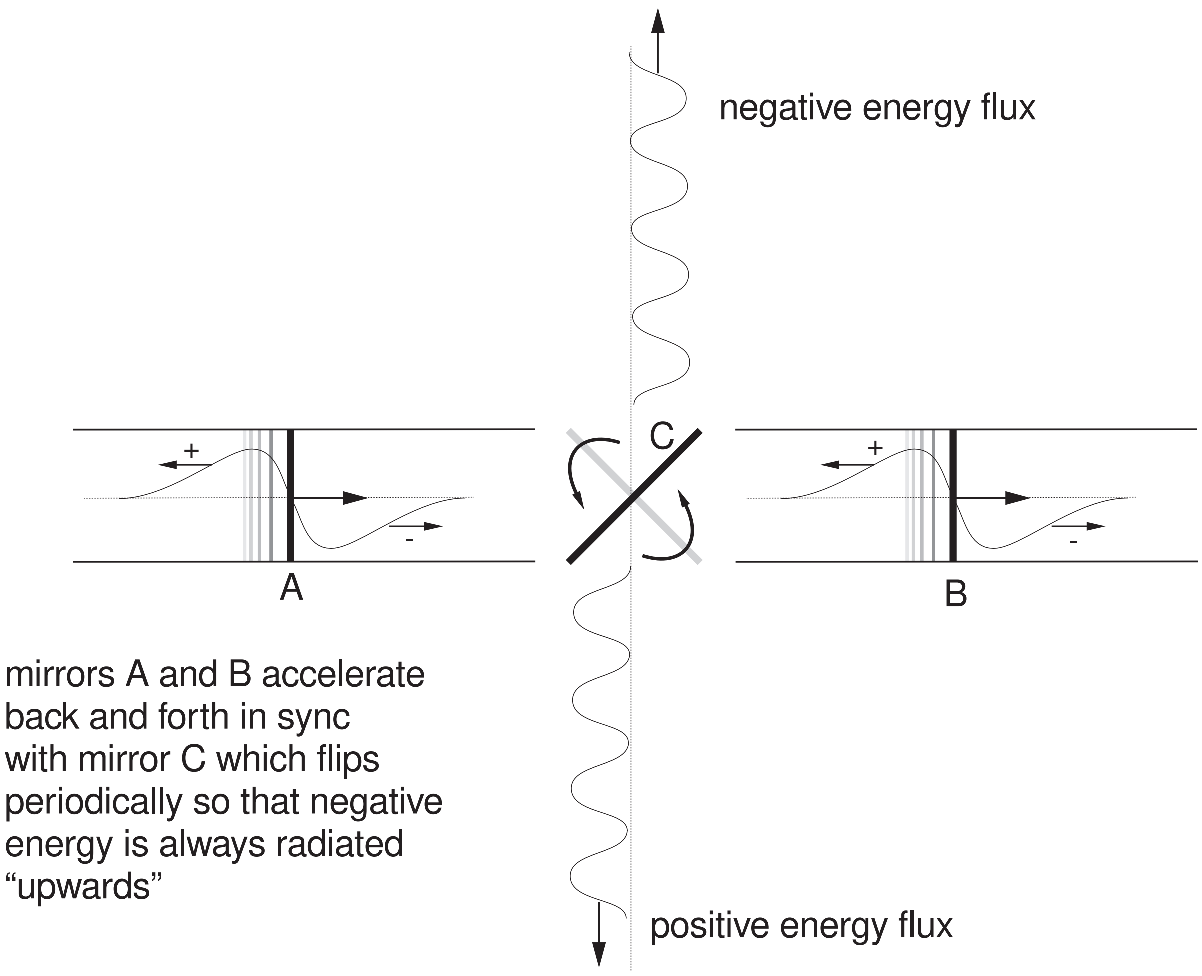
Also, limit the duration of the positive pulse to last a time t_0. Then a lower bound estimate for ϵ can be obtained by setting $a = t_0/2$, $a_{max} = 0$ and $b = t_1$.

$Z \equiv \frac{\Delta t}{t_0} = \frac{t1 - t_0/2}{t_0}$; so $0 \leq Z \leq \frac{x_{max}-1}{2}$ and $3 \leq x \leq x_{max} + 2$.



Lower bound estimates for the quantum interest ϵ as a function of pulse separation $z = \Delta t/t_0$ in 2D (above) and 4D (below) Minkowski spacetime. Ten curves are plotted for values of z_{max} from 1 to 10; the range of each curve is $[z_{max} - 1, z_{max}]$. The larger z_{max} the less negative energy was sampled, allowing greater separation of the fluxes.





A pair of mirrors (A and B) accelerate back and forth in tandem, radiating negative energy in the direction of increasing accelerating and positive energy in the opposite direction. Between these two mirrors is a third mirror (C) that periodically flips so that the negative energy parts of the flux generated by mirrors A and B are always reflected in the same direction. Such a device could in principle produce a steady flow of negative energy, *if* it were possible to reflect just the negative part of the mirror flux.

In light of quantum interest:

- What about the negative energy in the Casimir effect? Seems to be a different “kind” of negative energy – renormalization induced rather than an interference effect between positive energy quanta; and apparently there are no problems in using it to violate the 2^{nd} law of thermodynamics.

- What about black hole evaporation? Does spacetime curvature remove some of the restrictions faced by negative energy fluxes in Minkowski spacetime? Or is a black hole more like an “evaporating Casimir box” with the event horizon forming the (one way) boundary?

To address these questions, look at a massless scalar field in 2D curved spacetime:

(A. Ottewill and S. Takagi, *Prog. Theor. Phys.* **79** (1988) 429;
W.R. Walker, *Class. Quantum Grav.* **2** L37 (1985);
and W. Israel, private communication)

$$ds^2 = -e^{2\lambda(u,v)}dudv.$$

Using the trace anomaly, $T^{\alpha}_{\alpha} = \frac{1}{24\pi}R$, and the conservation equations we can find the complete stress energy tensor

$$T_{\alpha\beta} = \frac{1}{12\pi}\left(\lambda_{;\alpha\beta} + \lambda_{,\alpha}\lambda_{,\beta} - g_{\alpha\beta}(\Box\lambda + \frac{1}{2}(\nabla\lambda)^2\right)$$
$$+F(u)u_{,a}u_{,b} + G(v)v_{,a}v_{,b}.$$

e.g. for 2D Schwarzschild $e^{2\lambda} = f(r) = 1 - \frac{2m}{r}$, $du = dt - \frac{dr}{f}$ and $dv = dt + \frac{dr}{f}$; then F=G=0 gives the Boulware state.

Can a mirror that is slowly lowered in a black hole spacetime produce a persistent out-flux of negative energy, violating quantum interest in the asymptotically flat region of the spacetime?

Flux from a moving mirror:

$$F_c = -\frac{1}{12\pi}\frac{da}{d\tau}. \quad \text{(as seen by a comoving observer)}$$

Decompose the proper acceleration a into a "kinematic" part a_k and a "gravitational" part, a_g:

$$a = a_g + a_k,$$

where

$$a_k = \frac{d\chi}{d\tau}, \quad \chi \text{ is the "boost angle"},$$

$a_g = \lambda_{,\alpha} n^\alpha, \quad \vec{n}$ is the vector normal to the mirror's world line

Then

$$F(u) = -\frac{1}{12\pi} e^{2\chi_c + \lambda_c} \frac{d}{d\tau}\left(e^\lambda a_k\right).$$

Thus a $\chi = constant$ mirror trajectory (constant proper velocity) produces no flux – the negative energy observed by the comoving observer is a result of motion through the Boulware state.

In fact, using the results of Fewster and Teo

(C.J. Fewster and E. Teo, *Bounds on negative energy densities in static space-times*, gr-qc/9812032)

it is easy to show that massless scalar fields on the background metric $ds^2 = -e^{2\lambda(u,v)}dudv$ satisfy the quantum inequalities, and thus quantum interest, modulo the conformal "blueshift" factor:

$$\langle \rho_{min} \rangle_{2D} = e^{-2\lambda} \langle \rho_{min} \rangle_{2D,Minkowski}.$$

(but note that ρ is measured *relative* to the Boulware ground state).