A Numerical Study of Boson Stars

Kevin Lai

Department of Physics and Astronomy
University of British Columbia
Vancouver BC
cwlai@physics.ubc.ca

Departmental Oral Defense 2004

July 13, 2004

THESIS SUPERVISOR: *M. Choptuik*
SUPERVISORY COMMITTEE: *M. Choptuik, K. Schleich, D. Scott, W. Unruh*

Outline

- Description and Motivation of Research
- Boson Stars and Critical Phenomena
- Boson Stars in *Spherical Symmetry*
- Boson Stars in *Axisymmetry*
- Summary and Conclusion

Description and Motivation of Research

- Numerical simulation of general relativistic boson stars in both spherically symmetric and axisymmetric spacetime within the framework of general relativity
  - *Time-independent*: construction of stationary solutions
  - *Time-dependent*: black hole critical phenomena
- Simulation use more straightforward numerical techniques: evolution of smooth initial data stay smooth
- Share some features as their fermionic counterparts, give insights and better understanding of fermionic stars
- Numerical work in the past using boson stars as the matter models give interesting results

Boson Stars

- Wheeler 1955:
  - (EM) *Geons*: self-gravitating objects whose constituent element is the EM field, the photonic configurations were given the name geons
- Kaup 1968, Ruffini & Bonazzola 1969 (in spherical symmetry):
  - *Klein-Gordon geons*: self-gravitating compact objects consists of massive complex scalar particles
  - Balance between attractive force of gravity and a pressure arising from the dispersive nature of wave
  - Variations of the original model include self-interacting boson stars, charged boson stars, and rotating boson stars
  - Mathematically described by the Einstein-Klein-Gordon system

Boson Stars

- Boson stars = *Stationary solutions to Einstein Klein-Gordon system*
- The action:

$$\int d^4x\sqrt{-g}\left[\frac{R}{16\pi}-\frac{1}{2}\left(\nabla_\mu\phi\,\nabla^\mu\phi^*+m^2\phi^*\phi\right)\right]$$

- EOM:

$$G_{\mu\nu}=8\pi T_{\mu\nu}$$

$$\nabla_\mu\nabla^\mu\phi-m^2\phi=0$$

where

$$T_{\mu\nu}=\frac{1}{2}\left(\nabla_\mu\phi^*\,\nabla_\nu\phi+\nabla_\mu\phi\,\nabla_\nu\phi^*\right)-\frac{1}{2}g_{\mu\nu}\left(\nabla^\alpha\phi^*\,\nabla_\alpha\phi+m^2\phi^*\phi\right)$$

Boson Stars

- *Boson Stars Ansatz:*
  - 1D case

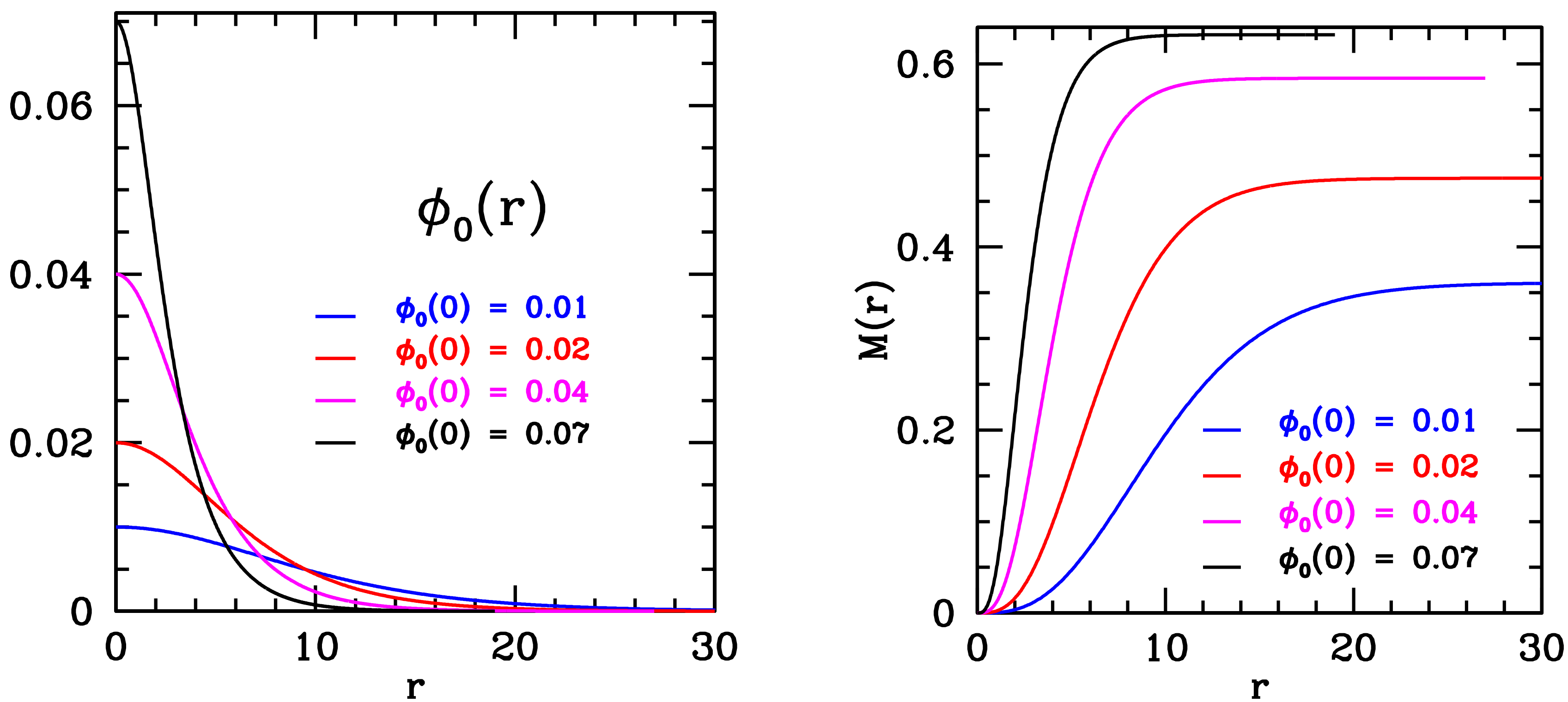
$$\phi(t,r) = \phi_0(r)e^{-i\omega t}$$

  - 2D case

$$\phi(t,r,\theta,\varphi) = \phi_0(r,\theta)e^{-i(\omega t + k\varphi)}$$

Boson Stars

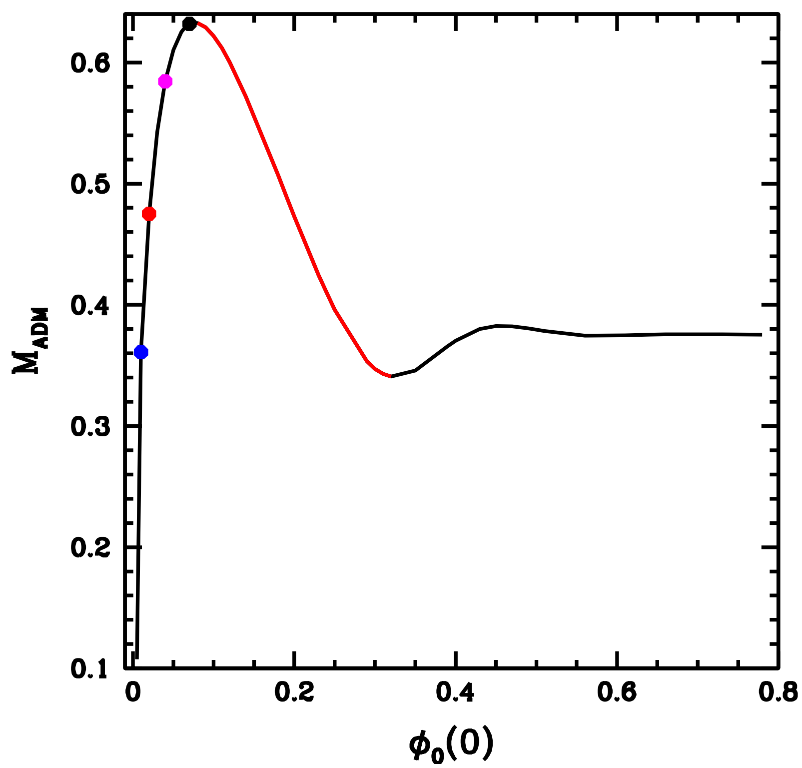
- Typical solutions in spherical symmetry:



- Parametrized by central value; Compact configuration with exponential tail

Boson Stars

- Family of stationary solutions in spherical symmetry:



Critical Phenomena

- Threshold of black hole formation: *Families* of solutions of a *dynamical* gravitating system parametrized by initial conditions

- Choose a single parameter p that has the properties:

1. For small p, the dynamics is regular, no black hole forms (subcritical)
2. For large p, gravitational collapse, a black hole forms (supercritical)

- There exist an intermediate critical value $p^\star$ that sits at the threshold of black hole formation (critical solution)

Critical Phenomena

- Critical solutions have additional symmetry:

  1. *Type I*: static/periodic (continuous/discrete time-translational),
  2. *Type II*: continuously/discretely self-similar (scaling symmetry)

- Mass of black hole forms:

  - *Type I*: finite
  - *Type II*: infinitesimal

- In Type I collapse:

  - Scaling law: $\tau(p) \sim -\gamma \ln |p - p^\star|$
  - γ independent of which particular family of initial data
  - One unstable mode, γ related to the growth factor (Lyapunov exponent)

Boson Stars in Spherical Symmetry

- ADM formalism
- Maximal-isotropic coordinate

$$ds^2 = \left(-\alpha^2 + \psi^4\beta^2\right) dt^2 + 2\psi^4\beta\, dt\, dr + \psi^4 \left(dr^2 + r^2 d\Omega^2\right)$$

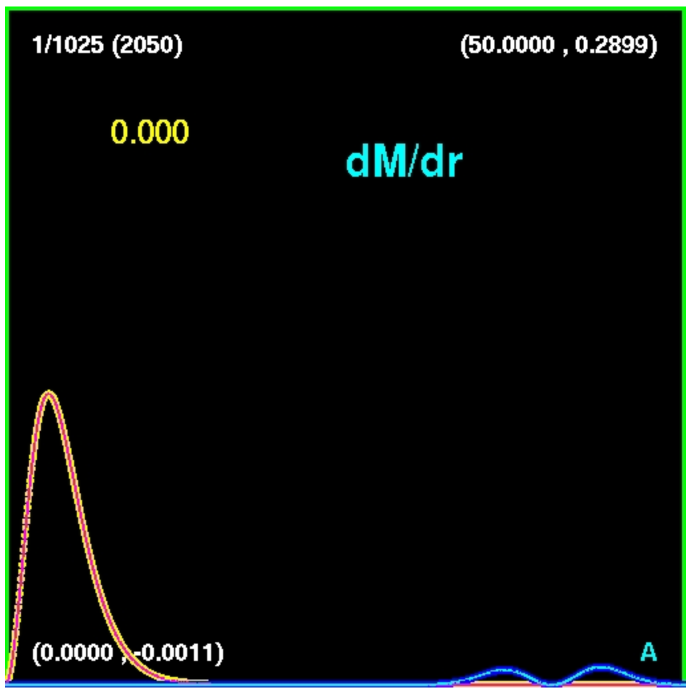
- Boson star driven to criticality by imploding a spherical shell of real massless scalar field:

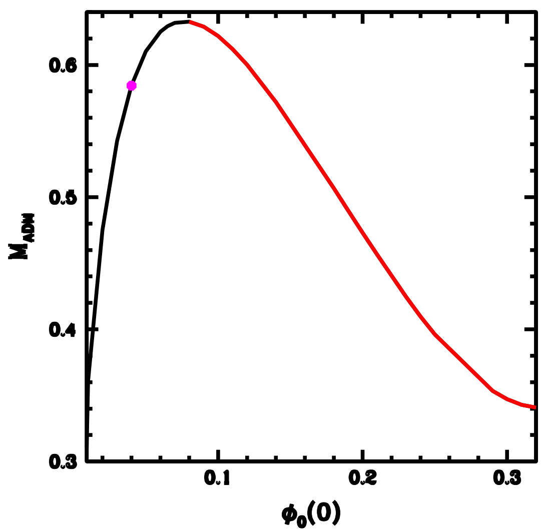
$$\phi_3(0,r) = A_3 \exp\left[-\left(\frac{r - r_0}{\sigma}\right)^2\right]$$

- Family parameter: A_3

Boson Stars in Spherical Symmetry

- Marginally subcritical evolution with $\phi_0(0) = 0.04$

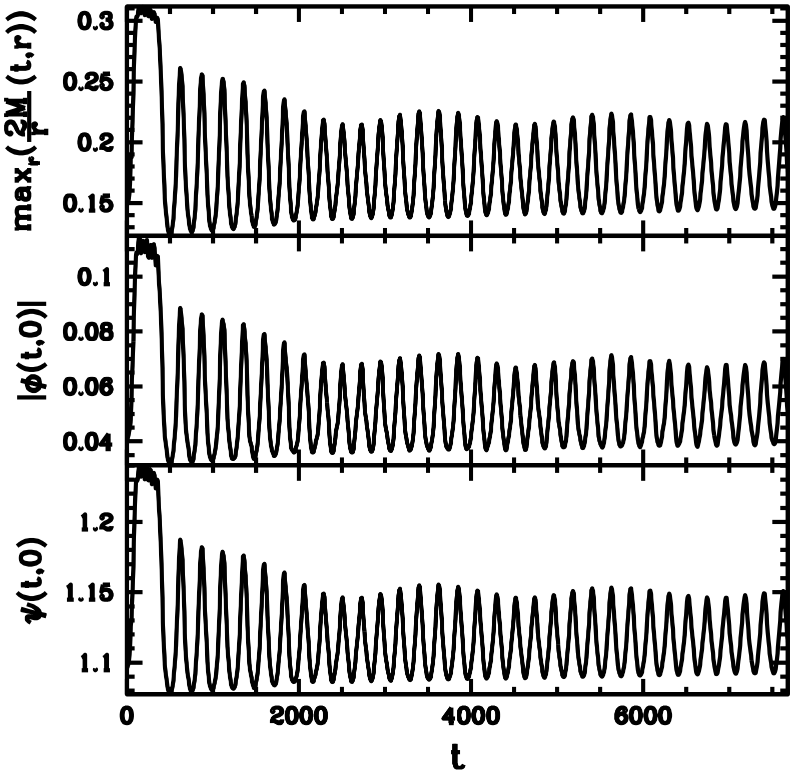




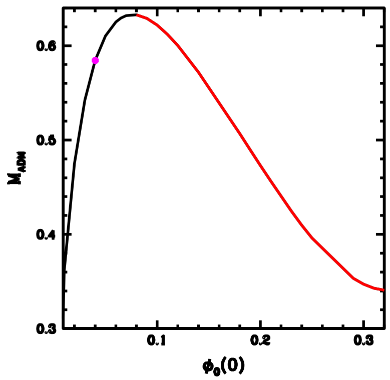
$\partial M(t,r)/\partial r$ vs r

Boson Stars in Spherical Symmetry

- Final fate of subcritical evolution $\phi_0(0) = 0.04$

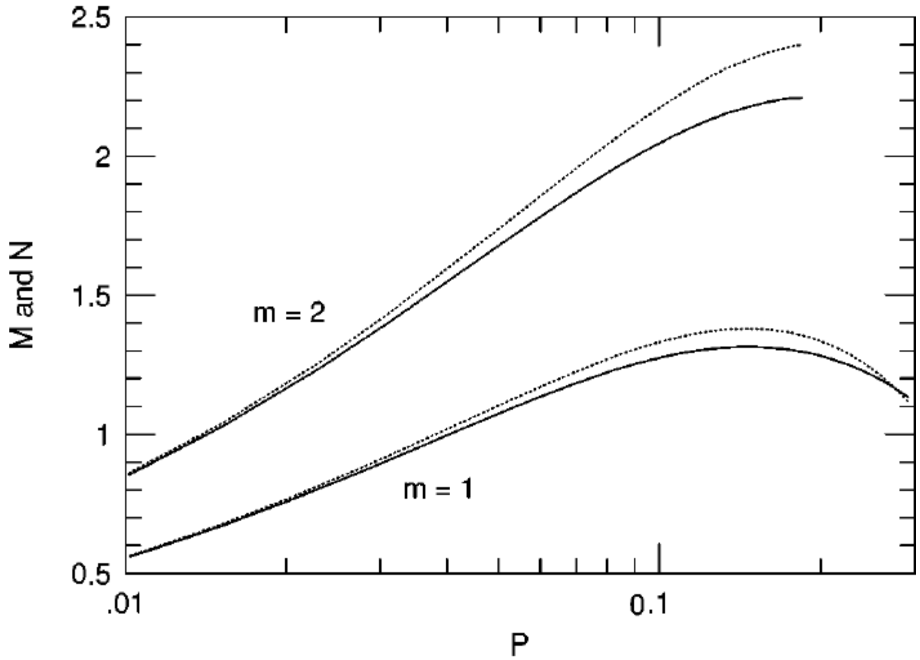


- Fundamental radial mode σ_0^2:
  - Simulation result: 0.0013
  - Perturbative analysis: 0.0014



Boson Stars in Axisymmetry

The Initial Value Problem for Rotating Boson Stars



$$\boldsymbol{k \longrightarrow m}$$

$$\phi_0(r,\theta)e^{-i(\omega t + m\varphi)}$$

- Silveira & de Sousa (1995): Newtonian
- Schunck & Mielke (1996): Relativistic
- Yoshida & Eriguchi (1997): Relativistic

Boson Stars in Axisymmetry

The Initial Value Problem for Rotating Boson Stars

- Stationary, axisymmetric spacetime:

$$ds^2 = -\alpha^2 dt^2 + \psi^4 r^2 \sin^2\theta e^{2\sigma}\left(\beta dt + d\varphi\right)^2 + \psi^4 d\Omega^2$$

- Ansatz:

$$\phi(t, r, \theta, \varphi) = \phi_0(r, \theta) e^{-i(\omega t + k\varphi)}$$

  - k is an integer
  - Family parametrized by $\partial^k \phi_0(0, \theta)/\partial r^k$

Boson Stars in Axisymmetry

The Initial Value Problem for Rotating Boson Stars

- Coupled nonlinear quadratic eigenvalue problem:

$$\phi_{0,rr} + \cdots + \left[\left(\frac{\omega - \beta k}{\alpha} \right)^2 - m^2 \right] \psi^4 \phi_0 = 0$$

$$\psi_{,rr} + \cdots = 0$$

$$\beta_{,rr} + \cdots = 0$$

$$\alpha_{,rr} + \cdots = 0$$

$$\sigma_{,rr} + \cdots = 0$$

Boson Stars in Axisymmetry

The Initial Value Problem for Rotating Boson Stars

- Numerical Strategies:

  multigrid eigenvalue solver on *compactified* domain:

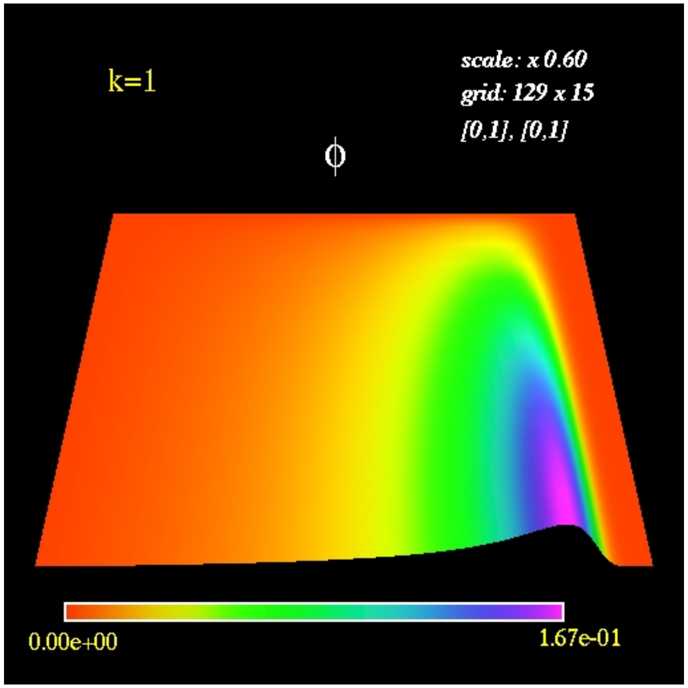
$$\zeta = \frac{r}{1+r}$$
$$s = \cos\theta$$

  - Efficient
  - Dirichlet outer boundary conditions

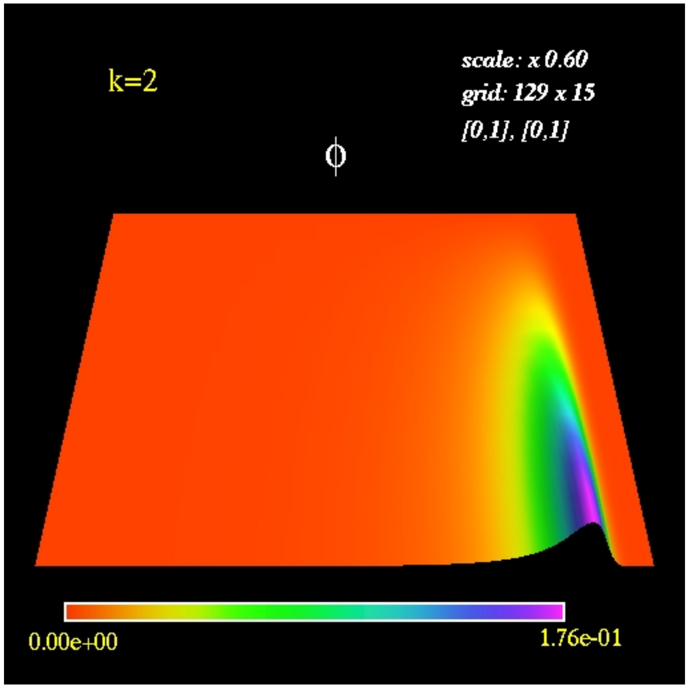
Boson Stars in Axisymmetry

The Initial Value Problem for Rotating Boson Stars

- Typical solutions:



$k = 1, \phi_0(0.5, 1) = 0.03$

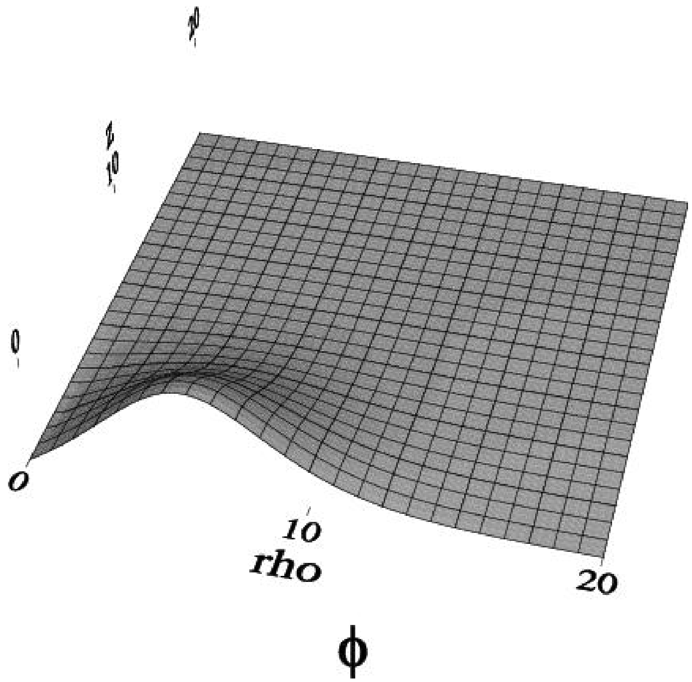


$k = 2, \phi_0(0.875, 1) = 0.16$

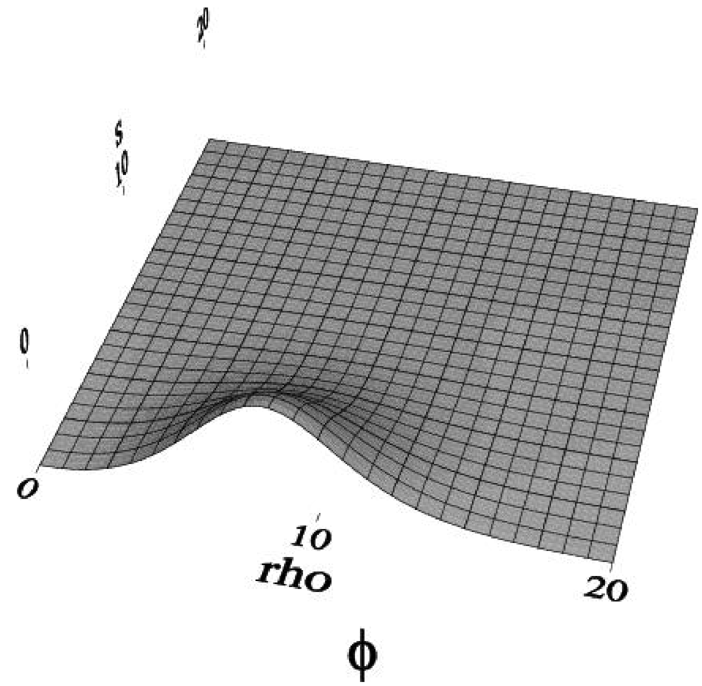
Boson Stars in Axisymmetry

The Initial Value Problem for Rotating Boson Stars

- Typical solutions (solutions have regularity problem)



$k = 1, \phi_0(0.5, 1) = 0.03$

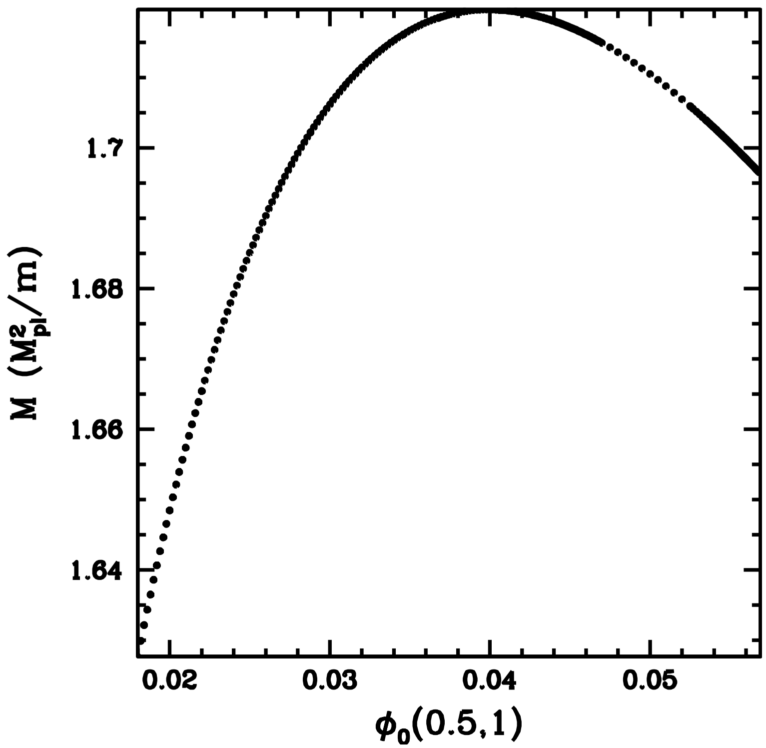


$k = 2, \phi_0(0.875, 1) = 0.16$

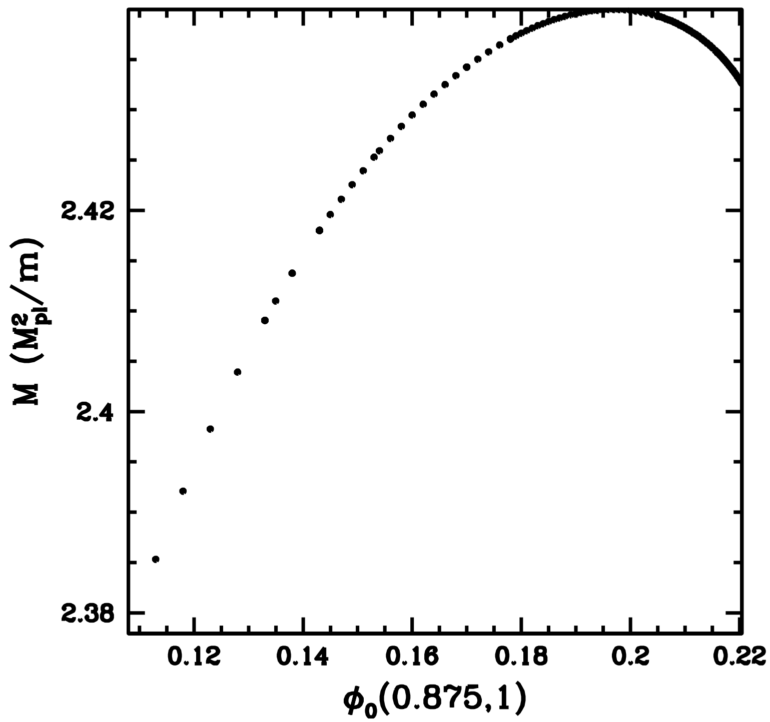
Boson Stars in Axisymmetry

The Initial Value Problem for Rotating Boson Stars

- Families of solutions for $k = 1$ and $k = 2$:



$k = 1$



$k = 2$

Boson Stars in Axisymmetry

Dynamics of Non-rotating Boson Stars

- $(2+1)+1$ formalism
- The metric:

$$ds^2 = -\alpha^2 dt^2 + \psi^4 \left[(\beta^\rho dt + d\rho)^2 + (\beta^z dt + dz)^2 + e^{2\rho\bar{\sigma}} \rho^2 d\varphi^2 \right]$$

- Massive complex scalar field:

$$T^{\phi}_{\mu\nu} \equiv \left[(\nabla_\mu \phi \nabla_\nu \phi^* + \nabla_\nu \phi \nabla_\mu \phi^*) - g_{\mu\nu} \left(\nabla^\alpha \phi \nabla_\alpha \phi^* + m^2 |\phi|^2 + 2\lambda |\phi|^4 \right) \right]$$

- Massless real scalar field:

$$T^{\phi_3}_{\mu\nu} \equiv 2 \nabla_\mu \phi_3 \nabla^\mu \phi_3 - g_{\mu\nu} \nabla^\alpha \phi_3 \nabla_\alpha \phi_3$$

Boson Stars in Axisymmetry

Dynamics of Non-rotating Boson Stars

Head-on Collisions of Boson Stars

- Initial data: Two copies of boson stars interpolated from spherically symmetric solutions $\phi_0(0) = 0.02$

- Each star is boosted and translated:

$$\phi^{(1)}(\rho, z; p_z) \equiv \phi_0(\rho, z)\, e^{i p_z z}$$

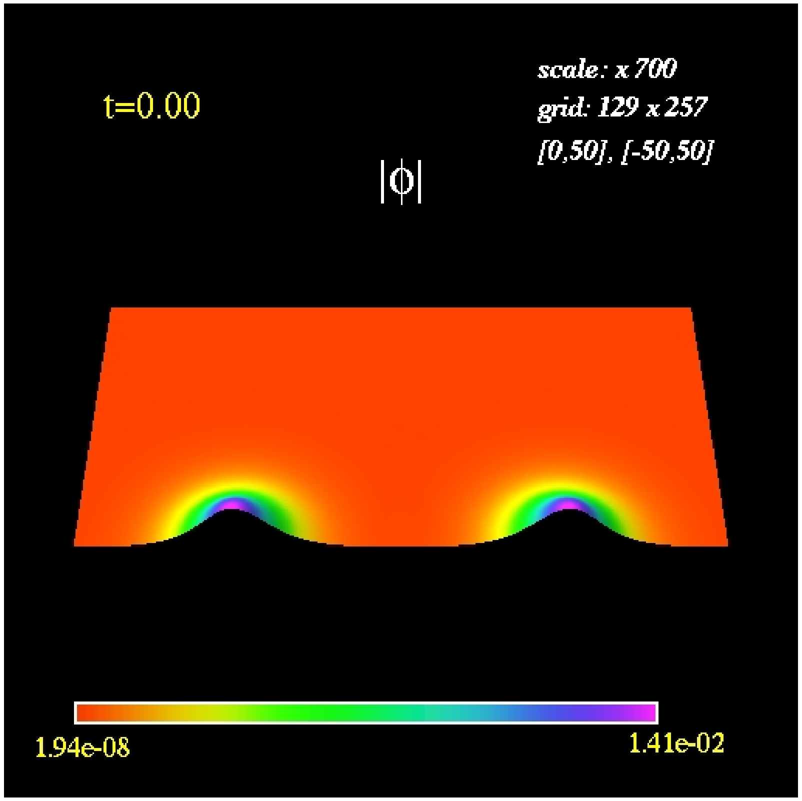
$$\phi^{(2)}(\rho, z; p_z) \equiv \phi_0(\rho, z)\, e^{-i p_z z}$$

Boson Stars in Axisymmetry

Dynamics of Non-rotating Boson Stars

Head-on Collisions—Solitonic Behaviour

- Initial setup: stars centered at $(0, -25)$ and $(0, 25)$, $p_z = 0.4$

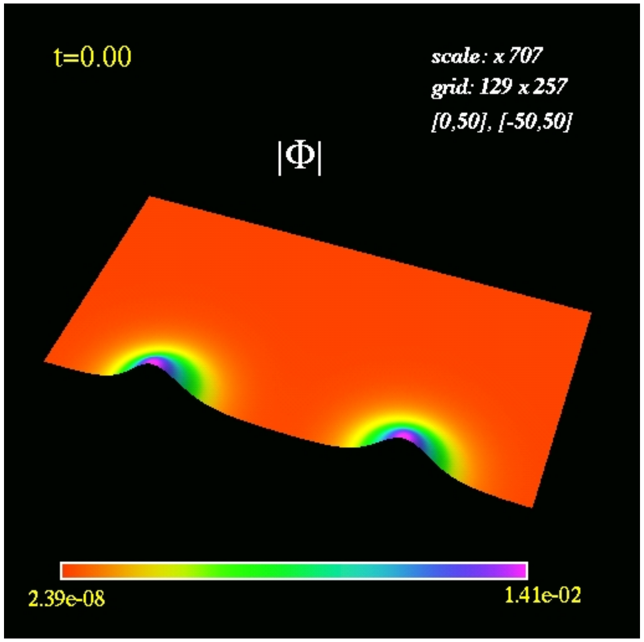


Boson Stars in Axisymmetry

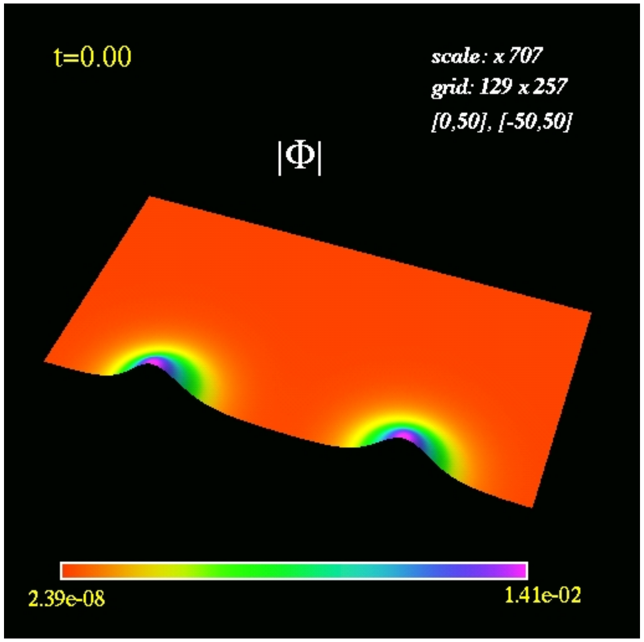
Dynamics of Non-rotating Boson Stars

Head-on Collisions—Critical Behaviour

- Initial setup: stars centered at $(0, -25)$ and $(0, 25)$, $p_z \approx 0.21, \lambda = 1$
- Family parameter: p_z



Supercritical evolution

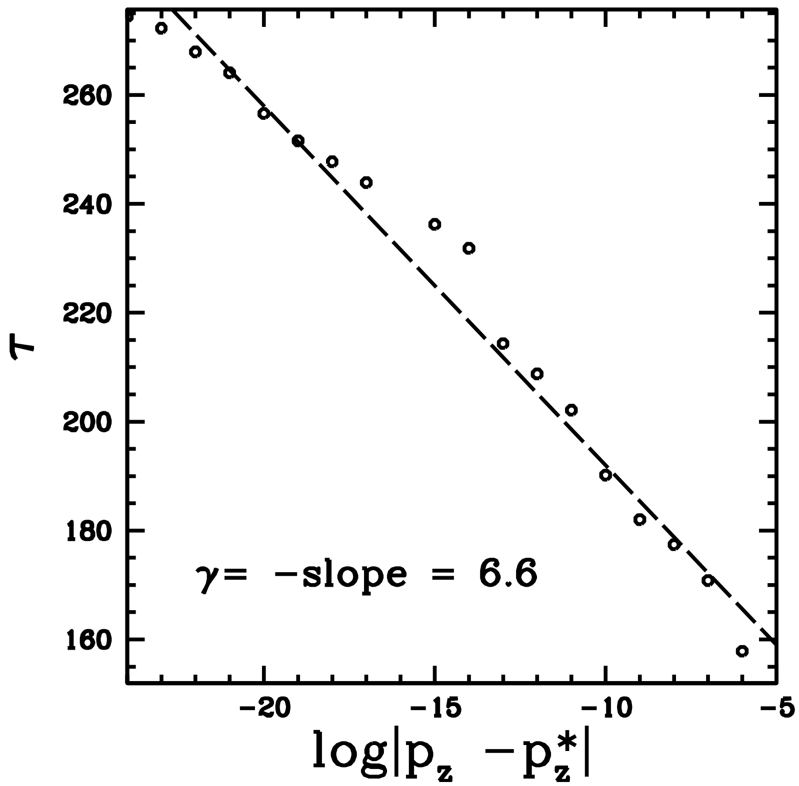


Subcritical evolution

Boson Stars in Axisymmetry

Dynamics of Non-rotating Boson Stars

Head-on Collisions—Critical Behaviour



- Scaling laws for near critical evolutions of *head-on boson star collisions*

$$\tau = -\gamma \log |p_z - p_z^*|$$

Summary and Conclusion

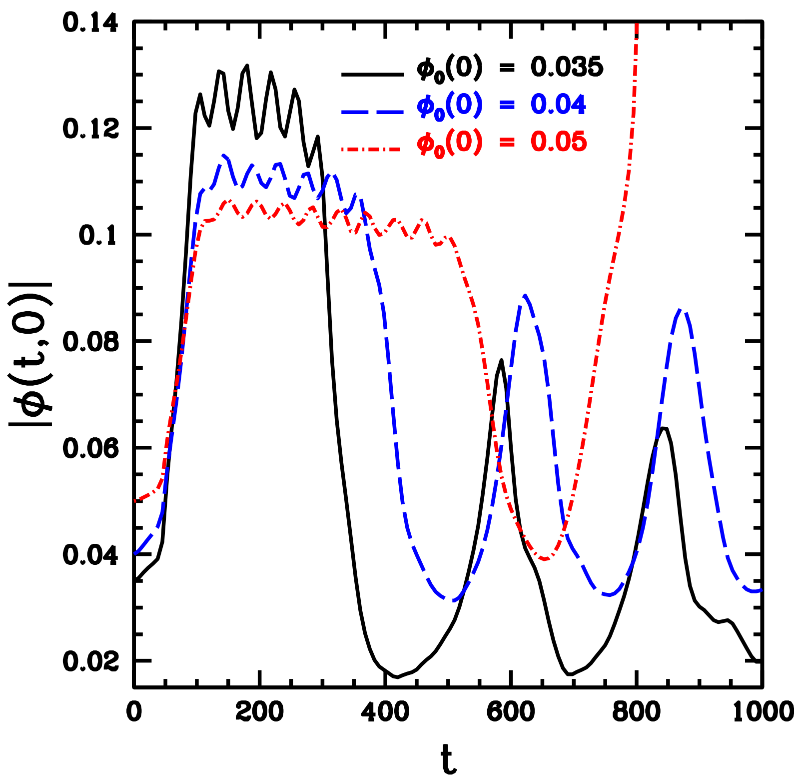
- Numerical simulations of general relativistic boson stars in *spherically symmetric* and *axisymmetric spacetime*
- Spherical Symmetry:
  - End states of Type I marginally subcritical evolution can be a *stable boson star executing large amplitude oscillations*
  - Perturbation analysis suggests that these oscillations are excitations of the fundamental normal modes

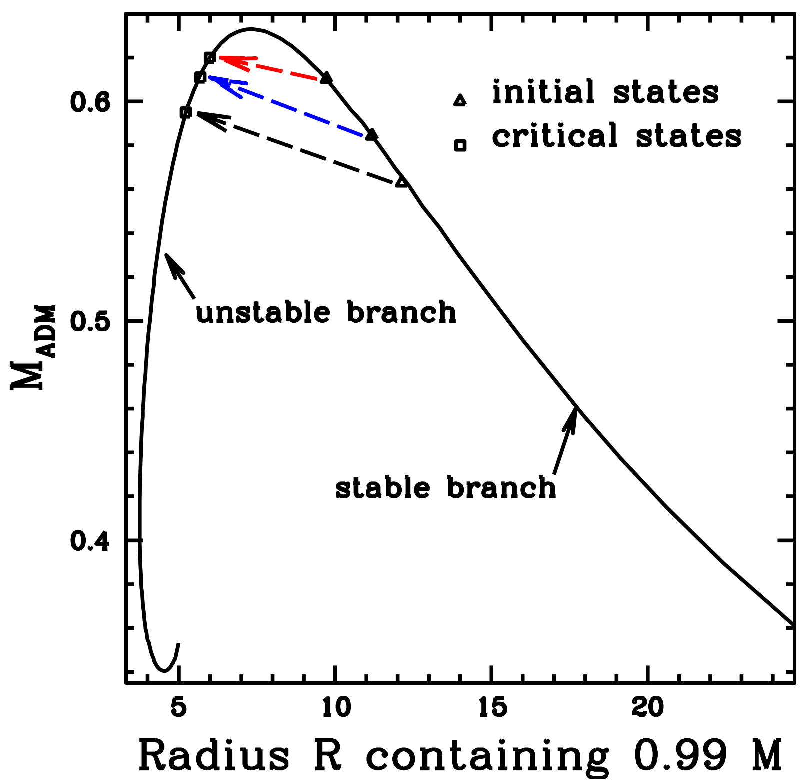
Summary and Conclusion

- Axisymmetry:
  - Based on an eigenvalue multigrid method, we developed an efficient algorithm for constructing the equilibrium configurations of *rotating boson stars*
  - Families of solutions for $k = 1$ and $k = 2$ were found
  - We encountered regularity problems which may be resolved in the future
  - We demonstrated existence of *Type I critical phenomena in axisymmetry* for
    - Binary boson stars collisions
    - Perturbation by an aspherical real scalar field

Boson Stars in Spherical Symmetry

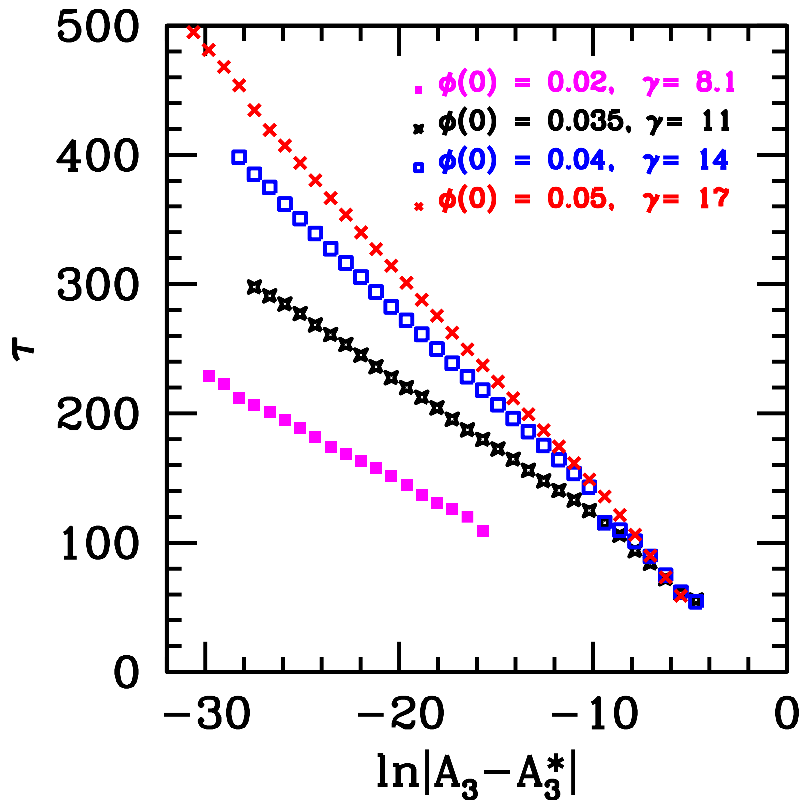
- Transition of perturbed boson stars in critical evolutions





Boson Stars in Spherical Symmetry

- Lifetime scaling laws for critically perturbed boson stars



Boson Stars in Axisymmetry

Dynamics of Non-rotating Boson Stars

Perturbation by an Aspherical Real Scalar Field

- Initial data: Boson stars interpolated from spherically symmetric solutions $\phi_0(0) = 0.02$

- Real scalar field takes the form of a "generalized gaussian"

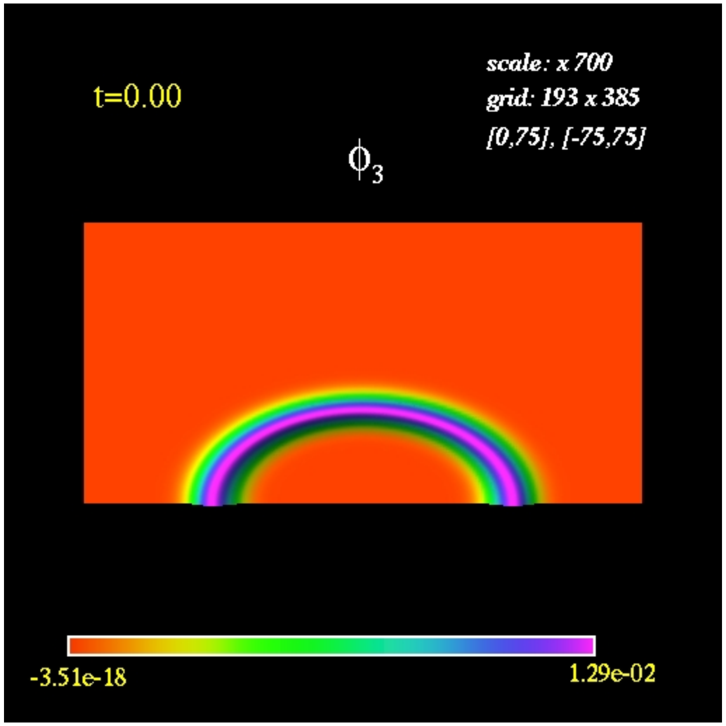
$$\phi_3(0, \rho, z) = A_3 \exp\left[- \left(\frac{\sqrt{(\rho - \rho_0)^2 + \epsilon (z - z_0)^2} - R_0}{\Delta} \right)^2 \right]$$

- Family parameter: A_3

Boson Stars in Axisymmetry

Dynamics of Non-rotating Boson Stars

Perturbation by an Aspherical Real Scalar Field

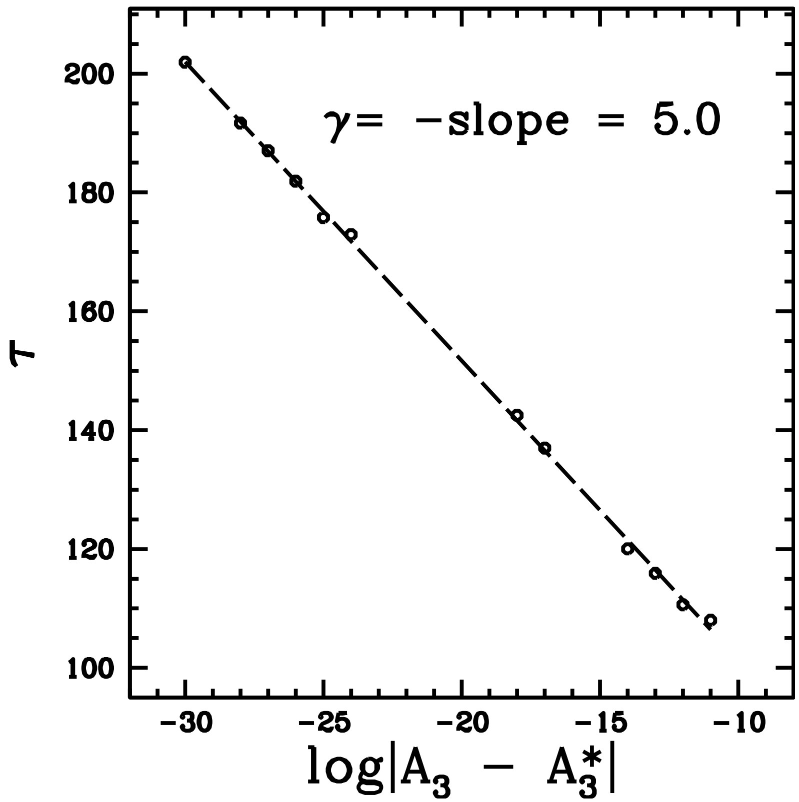


- Top view of an aspherical real scalar field ϕ_3 with amplitude A_3 for driving boson stars to criticality

Boson Stars in Axisymmetry

Dynamics of Non-rotating Boson Stars

Perturbation by an Aspherical Real Scalar Field



- Scaling laws for near critical evolutions of boson stars perturbed by an *aspherical real scalar field*:

$$\tau = -\gamma \log |A_3 - A_3^*|$$