SIMULATION OF GALAXY COLLISIONS USING THE TOOMRE MODEL

Jordon Johnson
University of British Columbia, student #53972006
December 3, 2012

ABSTRACT

Second-order finite difference approximation (FDA) techniques were used to simulate the Toomre model of close interactions between galaxies. Collisions of co-planar galaxies produced striking tails and suggest that the direction of rotation of each galaxy is a significant factor in the tails' structure. Interactions between galaxies oriented perpendicularly to each other produced fragments of elliptical shell-like structures that suggest formation of small elliptical galaxies from collisions may be possible. While some parallel-plane interactions also produced elliptical shells, others formed structures that resembled the co-planar tails from specific observation angles. The directions of rotation of the galaxies were found to be significant in these cases as well.

I. INTRODUCTION

The origin and mechanics of large-scale structures in galaxies have been a topic of interest for decades. While some structures are thought to be a result of the formation of their respective galaxies, others, such as “tails” and “bridges,” are thought to be the result of interactions between nearby galaxies. With factors such as relative mass, relative orientation, and position, velocity and rotation vectors all influencing the outcome of galactic collisions, myriad large-scale structure types may be produced. Alar and Juri Toomre (Toomre 1972) used a simplified model of a galaxy to simulate a range of such interactions. While the number of particles used was fairly small, they were able to reproduce structures such as those observed in the galaxy pairs known as "the Mice" and "the Antennae." Modern computers provide the opportunity to generate larger-scale simulations.

Another subject of interest is the formation of giant elliptical galaxies from the collisions of a number of smaller galaxies. Despite the limitations of the Toomre model, applying it to a collision of a single pair of spiral galaxies may yield evidence of the formation of smaller elliptical galaxies by the same mechanism.

II. MATHEMATICAL FORMULATION

a) N-BODY ADAPTATION

The Toomre model is an adaptation of Newtonian mechanics for the n-body problem,

$$m_i \frac{d^2 \vec{r}_i}{dt^2} = G \sum_{j=1,\, j \neq i}^{N} \frac{m_i m_j}{r_{ij}^3} \vec{r}_{ij}, i=1,2,3 \ldots N, 0 \leq t \leq t_{max} \quad (1)$$

In the Toomre model, each galaxy is approximated by a massive core orbited by a number of massless test points; as a result, there are only two types of interactions: between the two galactic cores, and between each test point and both cores. This simplification reduces Eq. 1 to

$$m_I \frac{d^2 \vec{r}_I}{dt^2} = G \frac{m_I m_J}{r_{IJ}^3} \vec{r}_{IJ}, 0 \leq t \leq t_{max} \quad (2)$$

$$\frac{d^2 \vec{r}_i}{dt^2} = G \left(\frac{m_I}{r_{iI}^3} \vec{r}_{iI} + \frac{m_J}{r_{iJ}^3} \vec{r}_{iJ} \right), i=1,2,3 \ldots N, 0 \leq t \leq t_{max} \quad (3)$$

Where *I* and *J* indicate the cores, and *i* denotes the test points.

Eq. 2 and Eq. 3 can be separated into Cartesian components by substituting for r_{ij}:

$$r_{ij}^3 = \left[(x_j - x_i)^2 + (y_j - y_i)^2 + (z_j - z_i)^2 \right]^{3/2}$$

$$\vec{r}_{ij}=(x_j-x_i)\hat{x}+(y_j-y_i)\hat{y}+(z_j-z_i)\hat{z}$$

The total energy of the two galactic cores is given by

$$E(t)=T(t)+V(t) \quad (4)$$

where

$$T(t)=\frac{1}{2}m_I v_I^2+\frac{1}{2}m_J v_J^2 \quad (5)$$

$$V(t)=\frac{-G m_I m_J}{r_{IJ}} \quad (6)$$

It is worthy of note that, while *E(t)* is conserved for the two cores, the energies of the test particles are not conserved.

b) FINITE DIFFERENCE APPROXIMATION

The continuum time domain $0 \le t \le t_{max}$ is discretized into units Δt. It is convenient to determine the spacing via a level parameter, *l*, such that

$$n_t=2^l+1$$
$$\Delta t=\frac{t_{max}}{n_t-1}=\frac{t_{max}}{2^l}$$
$$t^n=(n-1)\Delta t, n=1,2,3...n_t$$

where t^n refers to the n^th^ time step. Using Taylor series, approximations to the derivatives of arbitrary functions can be found. The second-order centred FDA for first and second derivatives of $x_i(t)$ are

$$\frac{x_i^{n+1}-x_i^{n-1}}{2\Delta t}=\left.\frac{d x_i(t)}{dt}\right|_{t=t^n}+O(\Delta t^2) \quad (7)$$
$$\frac{x_i^{n+1}-2x_i^n+x_i^{n-1}}{\Delta t^2}=\left.\frac{d^2 x_i(t)}{dt^2}\right|_{t=t^n}+O(\Delta t^2) \quad (8)$$

where $x_i^n = x_i(t^n)$ (Choptuik 2012). The $O(\Delta t^2)$ terms are the error in the FDA. It is expected that halving Δt will reduce the error by a factor of four, so that for some function *f* at level *l*:

$$\frac{4(f^{l+1}-f^{l})}{f^{l}-f^{l-1}}=1 \quad (9)$$

c) NON-DIMENSIONALIZATION

A set of mass, length, and time units can be chosen such that constant coefficients are set to desired values without changing the mechanics of the system (Choptuik). For this application, it is convenient to set both the gravitational constant, G, and the mass of an "average" galaxy to 1.

III. METHODODOLGY

a) PROGRAMMING

FDA code was written in Octave, a programming language largely compatible with Matlab. Octave was chosen partly due to its facility with whole-array operations, which simplifies the code significantly. To improve code readability, a number of Octave functions were defined; their code can be found in the Appendix. The position output is formatted by Matt Choptuik's *nbodyout* function to be viewed using xfpp3d.

The program code is able to receive input for a large number of parameters for each galaxy, including mass, position and velocity in three dimensions, and two orientation angles for the disk. Using that input, it populates each galaxy with "stars" having positions and velocities in the x-y plane, rotates the disk to the appropriate orientation and places it in the specified location. Due to axial symmetry in the disks, only two rotation angles are needed; rotations about the y- and z-axis were chosen. Once the galaxies are populated with initial conditions, the code performs the FDA and returns the positions and velocities of all bodies over the specified time frame.

To simplify the calculations, both G and the mass of the "typical" galaxy were set equal to 1. In addition, it was noted that a star too close to its core could be ejected violently before the galactic collision due to the error in the calculations; therefore, the disk of each galaxy had a minimum radius set to half of the galaxy's total radius. Initial core positions and velocities were also chosen such that the cores did not come too close to each other.

b) CONVERGENCE TESTING

As the error in a second-order FDA is reduced by a factor of four when Δt is halved, the comparison of error values for different discretization levels can be used as a test of FDA code correctness. The program code was tested under the controlled conditions of two galaxies of equal mass in circular orbits about their common centre of mass.

Two types of convergence testing were performed. For the galactic cores, the conservation of energy of the system was tested. Taking the total energy of the system given in Eqns. 4-6, the error in calculated energy for the two cores is $dE(t) = E(t) - E(0)$. For the stars, errors in position over time were assessed. A star was chosen arbitrarily, and the x-component of its positions were compared at different discretization levels.

IV. NUMERICAL EXPERIMENTS

Each of the numerical experiments was performed by running the Octave program code described in Section III with 500 test points per galaxy. The galaxies were set equidistant to the origin of the observer reference frame, with velocities of equal magnitude and opposite direction, in order to keep the centre of mass at the origin.

a) CO-PLANAR INTERACTION

For co-planar interactions between equally massive galaxies, the significant parameters are positions, velocities and directions of rotation. Keeping all other parameters constant, and assigning the values 1 and -1 to right-handed and left-handed rotations, respectively, the simulation was performed for three different combinations of rotation vectors: {1,1}, {1,-1}, and {-1,-1}. The combination {-1,1} is a rotation of {1,-1} by π about the axis normal to the common galactic plane and, therefore, is not considered.

b) PERPENDICULAR DISK ORIENTATIONS

Co-planar interactions are not likely to produce significant structures outside the common plane of the galactic disks; the next logical step in exploring the more general three-dimensional case is to orient the disks at different angles relative to each other. Of particular interest is whether the remains of the collision are elliptical in structure. The most obvious scenario to examine is that of one disk perpendicular to the other. Since the collisions simulated are not head-on, the axial vector of the rotated disk does not remain co-linear with (or perpendicular to, depending on the axis of rotation used) the velocity vector of its core throughout the collision.

For these simulations, the disk of one galaxy was rotated by $\pi/2$ about the y-axis. The case where the disk is also rotated about the z-axis was not simulated due to time constraints. As before, all parameters except the rotations of the disks about their centres were kept constant. In this case, the rotation combination {-1,1} is no longer equivalent to one of the others; therefore, it is included in the simulations performed.

c) PARALLEL-PLANE INTERACTIONS

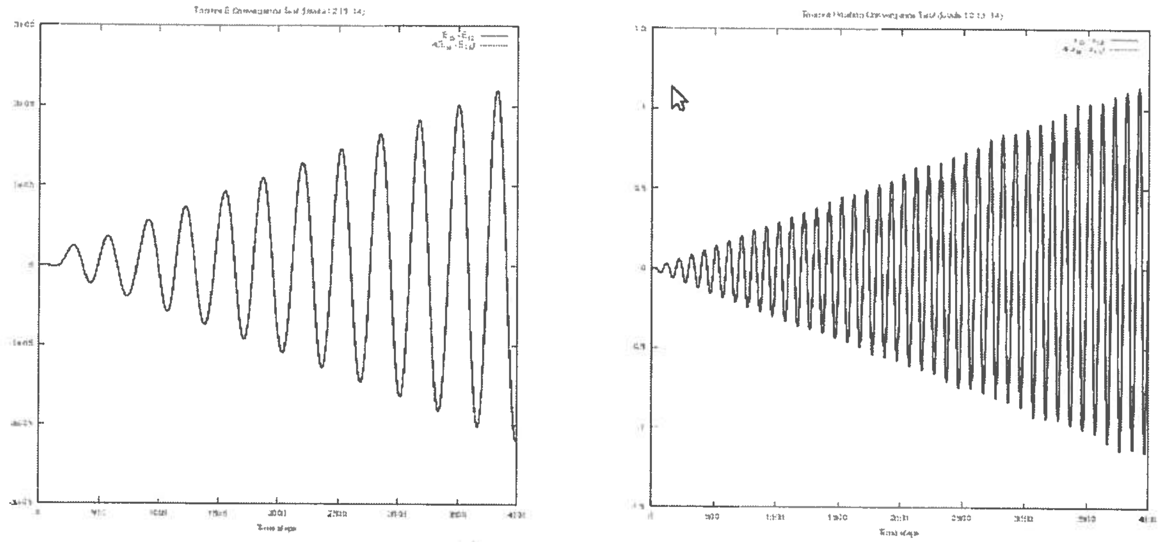
Another step in investigating galactic interactions in three dimensions is setting the disks' initial conditions in parallel planes. As with the perpendicular disk orientations, verifying evidence of elliptical structure is of considerable interest. However, also of interest in the parallel-plane case are any similarities in structure that might arise to the co-planar interactions.

Due to additional symmetries, the only rotation combinations considered are {1,1} and {1,-1}. However, due to the initial positions and velocities chosen for the galaxies, there may be structural differences between the case where the galaxies' initial velocity vectors are normal to the galactic planes and the case where the velocity vectors lie in-plane. Simulations for both scenarios were performed.

V. RESULTS

a) CONVERGENCE TESTING

As indicated by Fig. 1, the program code meets the criteria for convergence to second order in both the energy and position tests.



(a) Comparison of energy error values. (b) Comparison of position error values.

Figure 1. Results of convergence tests. The near-coincidence of the two curves plotted in each graph indicates convergence.

b) CO-PLANAR INTERACTIONS

Fig. 2 shows the different structures produced by co-planar collisions with different combinations of disk rotations.

(a) Rotations = {1,1} (b) Rotations = {1,-1} (c) Rotations = {-1,-1}

Figure 2. Structures produced by close interactions of co-planar galaxies with varying rotations.

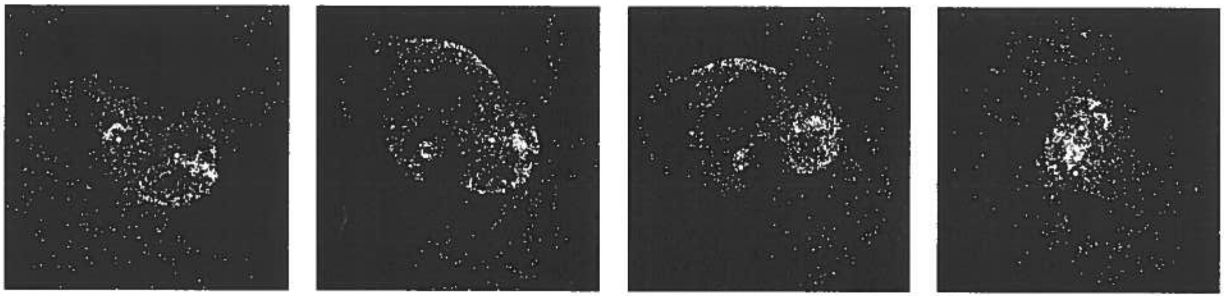
Animations for each interaction in Fig. 2 can be found online:

(a) http://laplace.physics.ubc.ca/Students/jordon/collision4.mpg
(b) http://laplace.physics.ubc.ca/Students/jordon/collision5.mpg
(c) http://laplace.physics.ubc.ca/Students/jordon/collision6.mpg

While all three interactions produced interesting structures – both in the shapes of the tails and in the spaces devoid of stars – of particular note are the "crossed tails" shown in Fig. 2(b); similar structures are seen in the Antennae. Also of note are the significant differences between the structures in each simulation; these results suggest that the direction of rotation of each galaxy has a considerable impact on the structures produced during the collision.

c) PERPENDICULAR DISK ORIENTATIONS

Fig. 3 shows the results of collisions with different combinations of disk rotations, where one of the disks is initially rotated by $\pi/2$ about the y-axis.



(a) Rotations = {1,1} (b) Rotations = {1,-1} (c) Rotations = {-1,-1} (d) Rotations = {-1, 1}

Figure 3. Structures produced by close interactions of galaxies at perpendicular orientations with varying rotations.

Animations for each collision in Fig. 3 can be found online:

(a) http://laplace.physics.ubc.ca/Students/jordon/collision8.mpg
(b) http://laplace.physics.ubc.ca/Students/jordon/collision9.mpg
(c) http://laplace.physics.ubc.ca/Students/jordon/collision10.mpg
(d) http://laplace.physics.ubc.ca/Students/jordon/collision11.mpg

The rotations of the galaxies seem less significant for these simulations than for the co-planar experiments. All four configurations formed what appear to be pieces of elliptical shells. While these results are insufficient to draw strong conclusions, they hint at the possibility of formation of small elliptical galaxies from single-pair collisions.

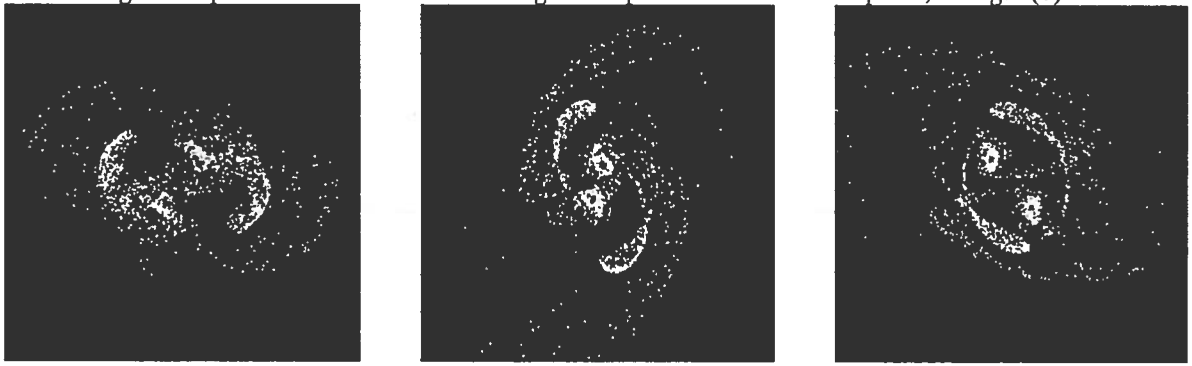
d) PARALLEL-PLANE INTERACTIONS

Fig. 4 shows the results of simulated parallel-plane collisions with different initial velocity directions and combinations of disk rotations.

(a) Rotations = {1,1}, velocities normal to galactic planes

(b) Rotations = {1,-1}, velocities normal to galactic planes

(c) Rotations = {1,-1}, velocities in-plane, cf Fig. 2(b)



(d) Rotations = {1,1}, velocities in-plane

(e) Rotations = {1,1}, velocities in-plane, cf Fig. 2(a)

(f) Rotations = {1,1}, velocities in-plane, cf Fig. 2(c)

Figure 4. Parallel-plane interactions with varying initial velocity directions and rotation combinations.

Animations for each interaction in Fig. 4 can be found online:

(a) http://laplace.physics.ubc.ca/Students/jordon/collision12.mpg
(b) http://laplace.physics.ubc.ca/Students/jordon/collision13.mpg
(c) http://laplace.physics.ubc.ca/Students/jordon/collision15.mpg
(d), (e), (f) http://laplace.physics.ubc.ca/Students/jordon/collision14.mpg

For the simulations where the initial galactic velocities are normal to the planes of their disks, elliptical shell-like structures were formed. As with the perpendicular interactions, these results hint at the possibility of the formation of elliptical galaxies.

For the simulations where the initial velocities are in-plane, the tails showed striking resemblances to their co-planar counterparts. The {1,-1} rotation combination (Fig. 4(c)) produced crossed tails similar to those formed in the coplanar {1,-1} interaction (Fig. 2(c)). Depending on the point of view, the {1,1} combination resembled both the coplanar {1,1} interaction (Fig. 4(e) cf Fig. 2(a)) and the coplanar {-1,-1} interaction (Fig. 4(f) cf Fig. 2(c)). The {1,1} tails also gave the appearance of a bridge from another angle of observation (Fig. 4(d)).

VI. CONCLUSIONS

The directions of rotation of colliding galaxies seem to play a significant part in the structure of the tails formed. Parallel-plane interactions in three dimensions seem to produce structures with similar cross-sections to their two-dimensional counterparts.

While elliptical structures can be formed from a single-pair collision, it is suspected that a more complete numerical model of a galaxy will be required to produce strong evidence of small-scale formation of elliptical galaxies.

A considerable amount of additional research is possible using the program code. While examples of both co-planar, perpendicular and parallel-plane interactions have been observed, many additional variations of the initial conditions may yield interesting results. In addition, running the simulations with much larger numbers of test points, while time-intensive, may provide additional information on the structures already observed. Also, the inclusion of a parameter to reduce the gravitational force at

```
  PE  = -( M(1)*M(2)/magr(1,2) );

end % function
```

REFERENCES

Toomre, A., & Toomre, J. (1972). Galactic Bridges and Tails. *The Astrophysical Journal, 178*(3), 623-666.

Choptuik, M. (2012). *Physics 210: Intro Computational Physics: Course Notes & Resources* [Website]. Retrieved from http://laplace.physics.ubc.ca/210/Notes.html