

Source file: bvp1d4.f

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c=====
c   Solves 1-d linear boundary value problem
c
c       u''(x) = f(x) on x = [0,1]; u(0) = u0, u(1) = u1
c
c   using mixed fourth-order and second order finite
c   difference technique and LAPACK banded solver DGBSV.
c=====
program      bvp1d4

implicit    none

integer     i4arg

c-----
c   Domain extrema and maximum system size.
c-----
real*8      xmin,          xmax
parameter   ( xmin = 0.0d0, xmax = 1.0d0 )

integer     maxn
parameter   ( maxn = 2**19 + 1 )

c-----
c   Storage for discrete x-values, exact solution and
c   right hand side values.
c-----
real*8      x(maxn),      uexact(maxn),
&           f(maxn)

c-----
c   Number of lower and upper bands.
c-----
integer     kl,           ku
parameter   ( kl = 2,    ku = 2 )

c-----
c   Storage for LAPACK-banded-form of linear system,
c   right-hand-side of system and pivot vector,
c   for use with DGBSV.
c
c   Note that for pivoting purposes (row interchanges)
c   DGBSV requires an additional 'kl' rows of workspace.
c   Leading dimension of 'ab' is thus
c
c       ku + kl + kl + 1 = 7
c-----
integer     ldab
parameter   ( ldab = 7 )
real*8      ab(ldab,maxn), rhs(maxn)
integer     ipiv(maxn)

c-----
c   Other standard LAPACK parameters.
c-----
integer     nrhs,         info

c-----
c   Discretization level, size of system (# of discrete
c   unknowns) and output option.
c-----
integer     level,        n,          option

c-----
c   Storage for difference coefficients. Note: these
c   arrays have elements -2, -1, 0, 1 and 2.
c-----
real*8      cdd2(-2:2),   cdd4(-2:2), c0(-2:2)

c-----
c   Mesh spacing and related constants.
c-----
real*8      h,           hm2,         hm2by12

c-----
c   Other locals.
c-----
integer     i,           j,           k
real*8      rmserr

c-----
c   Argument parsing.
c-----
level = i4arg(1,-1)
if( level .lt. 0 ) go to 900
n = 2 ** level + 1
if( n .gt. maxn ) then

write(0,*) 'Insufficient internal storage'
stop
end if
option = i4arg(2,0)

c-----
c   Set up finite-difference 'mesh' (discrete x-values)
c   and difference coefficient arrays.
c-----
h       = 1.0d0 / ( n - 1 )
do j = 1 , n
x(j) = xmin + ( j - 1 ) * h
end do
x(n) = xmax

hm2     = 1.0d0 / ( h * h )
hm2by12 = hm2 / 12.0d0

c0(-2) = 0.0d0
c0(-1) = 0.0d0
c0( 0) = 1.0d0
c0( 1) = 0.0d0
c0( 2) = 0.0d0

cdd2(-2) = 0.0d0
cdd2(-1) = hm2
cdd2( 0) = -2.0d0 * hm2
cdd2( 1) = hm2
cdd2( 2) = 0.0d0

cdd4(-2) = -hm2by12
cdd4(-1) = 16.0d0 * hm2by12
cdd4( 0) = -30.0d0 * hm2by12
cdd4( 1) = 16.0d0 * hm2by12
cdd4( 2) = -hm2by12

c-----
c   Set up exact solution and right hand side vector.
c-----
call exact(uexact,f,x,n)

c=====
c   Set up banded system. Recall that for LAPACK
c   banded storage for LU decomposition
c
c   a( i , j ) -> ab( kl + ku + 1 + i - j , j )
c=====
c   i = 1: (Left boundary) u(1) = u_0
c-----
i = 1

do k = 0 , 2
j = i + k
ab(kl + ku + 1 + i - j,j) = c0(k)
end do
rhs(i) = uexact(i)

c-----
c   i = 2: 0(h^2) approximation of u''(x) = f(x)
c-----
i = 2

do k = -1 , 2
j = i + k
ab(kl + ku + 1 + i - j,j) = cdd2(k)
end do
rhs(i) = f(i)

c-----
c   i = 3, ..., n-2: 0(h^4) approximation of u''(x) = f(x)
c-----
do i = 3 , n - 2
do k = -2 , 2
j = i + k
ab(kl + ku + 1 + i - j,j) = cdd4(k)
end do
rhs(i) = f(i)
end do

c-----
c   i = n-1: 0(h^2) approximation of u''(x) = f(x)
c-----

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i = n - 1

do k = -2 , 1
  j = i + k
  ab(kl + ku + 1 + i - j,j) = cdd2(k)
end do
rhs(i) = f(i)
-----
c i = n: (Right boundary) u(n) = u_1
-----
i = n

do k = -2 , 0
  j = i + k
  ab(kl + ku + 1 + i - j,j) = c0(k)
end do
rhs(i) = uexact(i)

=====
c Solve banded system.
=====

nrhs = 1
call dgbsv( n, kl, ku, nrhs, ab, ldab, ipiv, rhs, n,
& info )

if( info .eq. 0 ) then
-----
c Solver successful, output either (x_j, u_j) or
c (x_j, error_j) to stdout. Also compute rms error
c and output to standard error.
-----
rmserr = 0.0d0
do j = 1 , n
  if( option .eq. 0 ) then
    write(*,*) x(j), rhs(j)
  else
    write(*,*) x(j), (uexact(j) - rhs(j))
  end if
  rmserr = rmserr + (uexact(j) - rhs(j)) ** 2
end do
rmserr = sqrt(rmserr / n)
write(0,*) 'rmserr = ', rmserr
else
-----
c Solver failed.
-----
write(0,*) 'bvp1d4: dgbsv() failed, info = ', info
end if

stop

900 continue
write(0,*) 'usage: bvp1d4 <level> [<option>]'
write(0,*)
write(0,*) ' Specify option .ne. 0 for output'
write(0,*) ' of error instead of solution'
stop

end

=====
c Computes exact values for u(x) (unknown function)
c and f(x) (right hand side function). x array must
c have been previously defined.
=====
subroutine exact(u,f,x,n)

implicit none
integer n
real*8 u(n), f(n), x(n)

real*8 pi2
integer j

pi2 = 8.0d0 * atan(1.0d0)
do j = 1 , n
  u(j) = sin(pi2 * x(j))
  f(j) = -pi2 * pi2 * u(j)
end do

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return

end

Source file: sgi-output

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```

#####
# Building 'bvp1d4' and sample output on the SGIs
#####
sgii% pwd; ls
/usr/people/phys410/linsys/ex3
Makefile bvp1d4.f

sgii% make
f77 -g -64 -c bvp1d4.f
f77 -g -64 -L/usr/local/lib bvp1d4.o \
-lp410f -llapack -lblas -o bvp1d4

sgii% bvp1d4
usage: bvp1d4 <level> [<option>]

Specify option .ne. 0 for output
of error instead of solution

#####
# Note: compare with completely second-order 'bvp1d 4'
# which results in rms error of approximately 9.0E-03.
# These results are about 15 times better at this resolution
# (h = 1/16).
#####
sgii% bvp1d4 4
0.0000000000000000E+00 -1.0547118733938987E-14
6.2500000000000000E-02 0.3834724412118576
0.1250000000000000 0.7079302872941245
0.1875000000000000 0.9246563908935262
0.2500000000000000 1.000689732294703
0.3125000000000000 0.9244421766816860
0.3750000000000000 0.7075056502724236
0.4375000000000000 0.3828904610080090
0.5000000000000000 -3.1565329029368671E-15
0.5625000000000000 -0.3828904610080154
0.6250000000000000 -0.7075056502724303
0.6875000000000000 -0.9244421766816929
0.7500000000000000 -1.000689732294711
0.8125000000000000 -0.9246563908935345
0.8750000000000000 -0.7079302872941338
0.9375000000000000 -0.3834724412118676
1.0000000000000000 -2.4492935982947064E-16
rmserr = 5.8394829778013078E-04

#####
# Convergence test: Solve BVP on a sequence of levels,
# redirect stdout so that only overall RMS error appears
# on terminal. Rate of convergence is not as definitive
# as it was for the second order calculation, but clearly
# this method converges much more rapidly than the second
# order method.
#####
sgii% foreach level (4 5 6 7 8 9 10)
foreach? bvp1d4 $level > /dev/null
foreach? end
rmserr = 5.8394829778013078E-04
rmserr = 2.5181486533000874E-05
rmserr = 1.1531108217752065E-06
rmserr = 5.8557438669720841E-08
rmserr = 3.2465293440184777E-09
rmserr = 1.8966747918271887E-10
rmserr = 9.4215663095393918E-12

#####
# Making output files for subsequent plotting via gnuplot.
# See previous handout for 'bvp1d' for typical 'gnuplot'
# "script" files.
#####
sgii% bvp1d4 4 > out4
rmserr = 5.8394829778013078E-04
sgii% bvp1d4 4 1 > err4
rmserr = 5.8394829778013078E-04
sgii% bvp1d4 5 1 > err5
rmserr = 2.5181486533000874E-05

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```
sgii% bvp1d4 6 1 > err6
rmserr = 1.1531108217752065E-06
```

Source file: Makefile

```
.IGNORE:

F77_COMPILE = $(F77) $(F77FLAGS) $(F77CFLAGS)
F77_LOAD    = $(F77) $(F77FLAGS) $(F77LFLAGS)

.f.o:
    $(F77_COMPILE) *.f

EXECUTABLES = bvp1d4

all: $(EXECUTABLES)

bvp1d4: bvp1d4.o
    $(F77_LOAD) bvp1d4.o -lp410f -llapack $(LIBBLAS) -o bvp1d4

clean:
    rm *.o
    rm $(EXECUTABLES)

vclean: clean
    rm err[0-9]*
    rm out[0-9]*
    rm *.ps
```

Figure file: soln4.ps

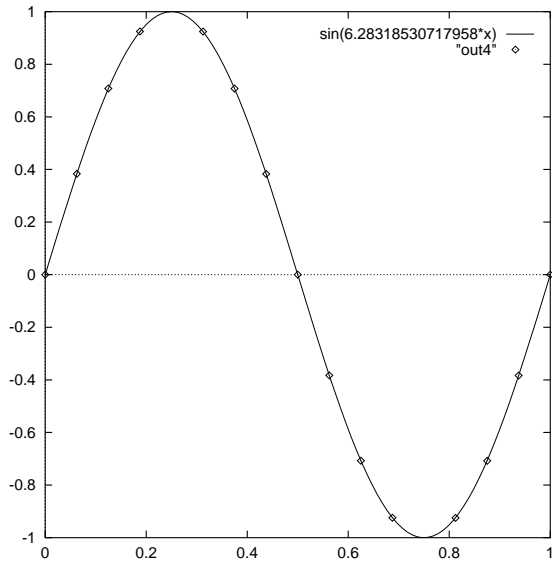


Figure file: err456.ps

