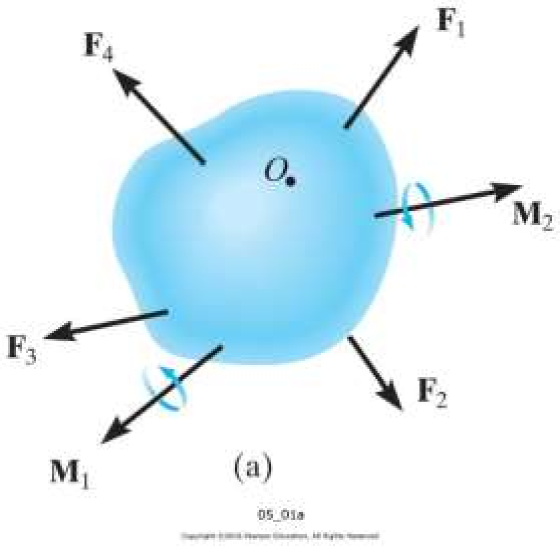
PHYS 170 Section 101
November Midterm Review
November 6, 2018

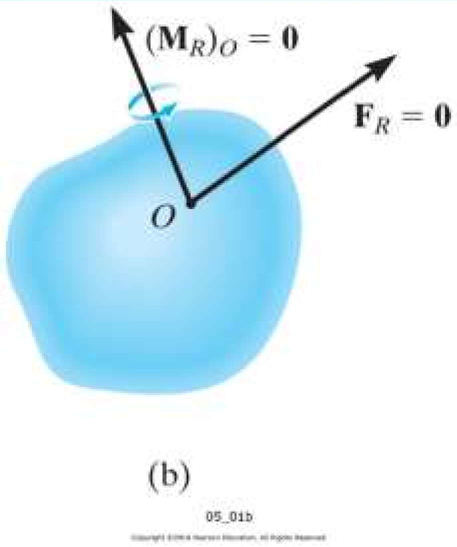
WARNING / DISCLAIMER

The instructor does not guarantee that the following review is complete. In particular, concepts and/or equations pertinent to the midterm *may* have been omitted below.

CHAPTER 5
EQUILIBRIUM OF A RIGID BODY

Conditions for Rigid Body Equilibrium





$$\mathbf{F}_R = \sum \mathbf{F} = \mathbf{0}$$
$$(\mathbf{M}_R)_O = \sum \mathbf{M}_O = \mathbf{0}$$

- A body is in equilibrium if the sum of the external forces acting on it vanishes and the sum of the moments about some point due to those forces added to all the couple moments also vanishes.

Free Body Diagrams & Support Reactions

A force is developed by a support that restricts the translation of its attached member

A couple moment is developed when rotation of the attached member is prevented

Will not be told which reactions are developed by which supports, so may want to include details of at least some of them on your information sheet, but also study available problems (lectures, homework, old exams, for other examples)

TABLE 5–2 Supports for Rigid Bodies Subjected to Three-Dimensional Force Systems

Types of Connection	Reaction	Number of Unknowns
(1)  cable		One unknown. The reaction is a force which acts away from the member in the known direction of the cable.
(2)  smooth surface support		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(3)  roller		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.

TABLE 5–2 Supports for Rigid Bodies Subjected to Three-Dimensional Force Systems

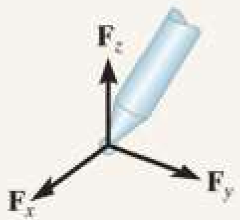
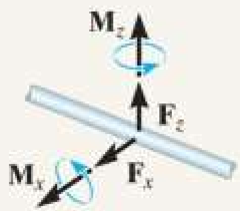
Types of Connection	Reaction	Number of Unknowns
(4)  ball and socket		Three unknowns. The reactions are three rectangular force components.
(5)  single journal bearing		Four unknowns. The reactions are two force and two couple-moment components which act perpendicular to the shaft. Note: The couple moments are *generally not applied* if the body is supported elsewhere. See the examples.

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TABLE 5–2 Continued

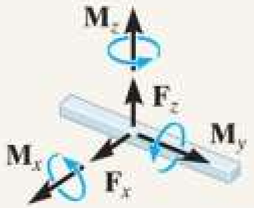
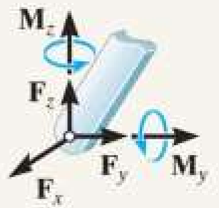
Types of Connection	Reaction	Number of Unknowns
(6)  single journal bearing with square shaft		Five unknowns. The reactions are two force and three couple-moment components. *Note*: The couple moments *are generally not applied* if the body is supported elsewhere. See the examples.
(7)  single thrust bearing		Five unknowns. The reactions are three force and two couple-moment components. *Note*: The couple moments *are generally not applied* if the body is supported elsewhere. See the examples.
(8)  single smooth pin		Five unknowns. The reactions are three force and two couple-moment components. *Note*: The couple moments *are generally not applied* if the body is supported elsewhere. See the examples.

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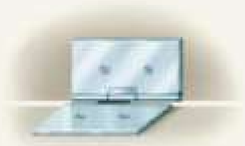
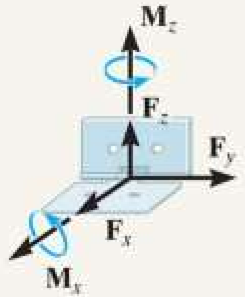
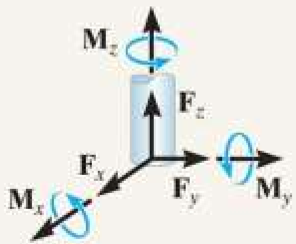
TABLE 5–2 Continued

<table>
<tr><th>Types of Connection</th><th>Reaction</th><th>Number of Unknowns</th></tr>
<tr><td>(9)

single hinge</td><td>
</td><td>Five unknowns. The reactions are three force and two couple-moment components. Note: The couple moments are generally not applied if the body is supported elsewhere. See the examples.</td></tr>
<tr><td>(10)

fixed support</td><td>
</td><td>Six unknowns. The reactions are three force and three couple-moment components.</td></tr>
</table>

Equations of Equilibrium

Vector equations of equilibrium

The vector conditions for equilibrium of a rigid body are

$$\sum \vec{F} = 0$$

$$\sum \vec{M}_O = 0$$

where $\sum \vec{F}$ is the vector sum of all the forces acting on the body and $\sum \vec{M}_O$ is the sum of any couple moments and the moments of all the forces about any point *O*.

Scalar equations of equilibrium

In Cartesian vector form, the equations of equilibrium become

$$\sum \vec{F} = \sum F_x \vec{i} + \sum F_y \vec{j} + \sum F_z \vec{k} = 0$$

$$\sum M_O = \sum M_x \vec{i} + \sum M_y \vec{j} + \sum M_z \vec{k} = 0$$

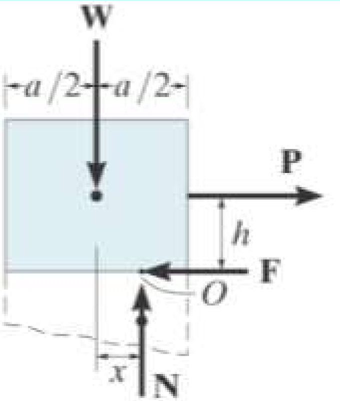
The $\vec{i}$, $\vec{j}$, and $\vec{k}$ components are independent of one another, so these are equivalent to the six scalar equations:

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum F_z = 0$$

$$\sum M_x = 0 \qquad \sum M_y = 0 \qquad \sum M_z = 0$$

These equations can be used to solve for at most six unknowns shown on the free body diagram.

CHAPTER 8
FRICTION



Resultant Normal and Frictional Forces

EQUILIBRIUM

- Assume block is in equilibrium and consider its FBD as in the figure to the left
- Observe
  1. Friction force **F** acts tangentially to surface and **opposes P**
  2. **N** acts normally (perpendicular) to surface, and is directed upwards to balance **W**

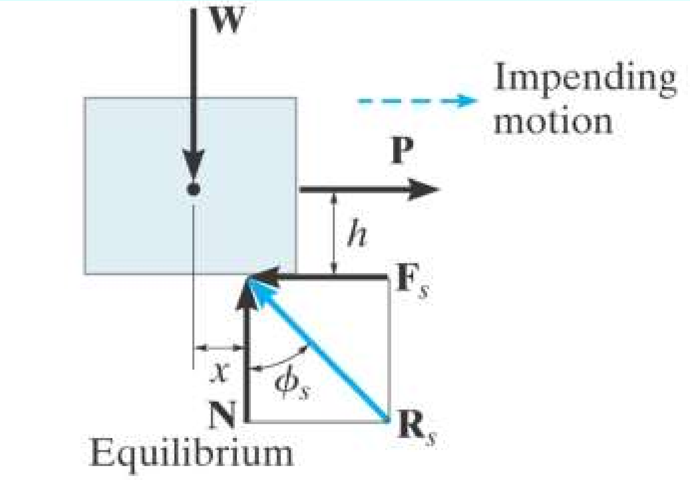
- We can determine *where* **N** acts by considering moment equilibrium about point O

$$Wx = Ph \qquad \text{(CCW and CW moments balance)}$$

or, solving for x

$$x = \frac{Ph}{W}$$

- **Important special case:** When $x=a/2$, the normal forces acts at the right corner of the block, and block is on verge of tipping (i.e. on verge of not being in moment equilibrium)



IMPENDING MOTION

- Depending on circumstances (e.g. h small, contact surfaces "slippery") frictional force **F** may not be large enough to balance **P**
- In this case, the block *slips* (translates) before it tips

- Imagine slowly increasing P from 0 – then F also slowly increases, but only to some **maximum value**, F_s
- F_s is known as the **limiting static frictional force**; any further increase in P causes the block to slip
- The following experimentally determined formula relates the limiting frictional force to the normal force

$$F_s = \mu_s N$$

where μ_s is known as the **coefficient of static friction**

Key Characteristics of Dry Friction

- Frictional force, **F**, acts tangentially to contact surfaces and opposes relative motion (or tendency for motion)

- Maximum static friction force, F_s , can usually be assumed to be **independent of contact area**

- Generally, $F_s > F_k$ for any two surfaces in contact

- When slipping/sliding is **about to occur**, we have $F_s = \mu_s N$

- When slipping/sliding **is occurring**, we have $F_k = \mu_k N$

Types of Friction Problems

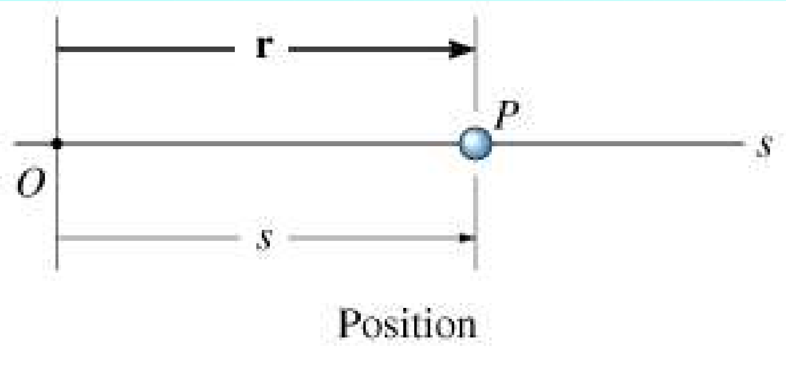
1. **EQUILIBRIUM**
   - Total number of unknowns must equal total number of equilibrium eqns.
   - Once frictional forces determined must check that $F \leq \mu_s N$

2. **IMPENDING MOTION AT ALL POINTS**
   - Total number of unknowns will equal the total number of equilibrium eqns. plus the total number of available frictional equations

3. **IMPENDING MOTION AT SOME POINTS**
   - Total number of unknowns will be less than the total number of available equilibrium equations, plus the total number of available frictional equations or conditional equations (such as tipping)
   - Will be more than one possibility for motion/impending motion and part of the solution of the problem is to determine which motion occurs, or is about to occur.

CHAPTER 12
KINEMATICS OF A PARTICLE

Rectilinear Kinematics: Continuous Motion



Position coordinate: s

Position vector: $\mathbf{r}$

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} \qquad \mathbf{a} = \frac{d\mathbf{v}}{dt} \qquad a\,ds = v\,dv$$

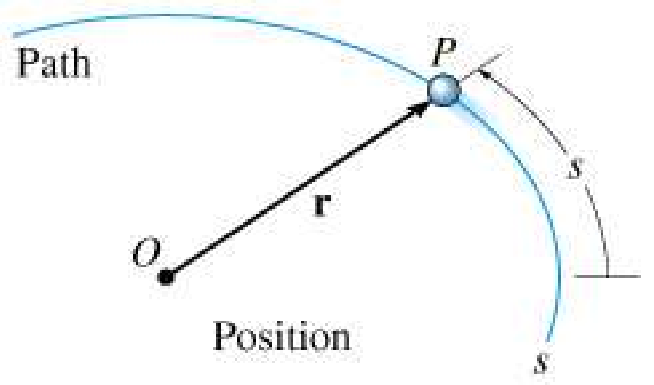
Constant acceleration, a_c

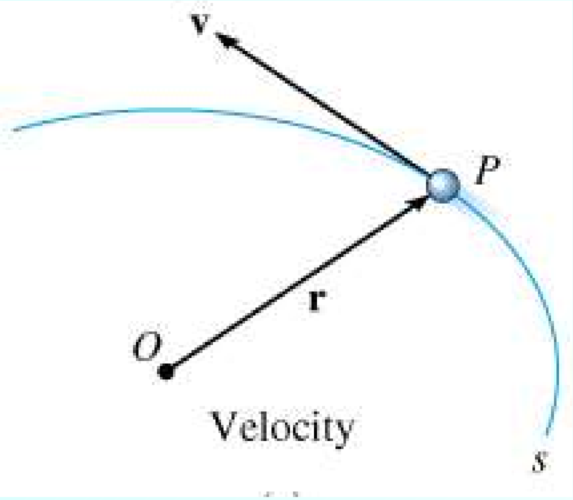
$$v = v_0 + a_c t$$

$$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$v^2 = v_0^2 + 2a_c(s - s_0)$$

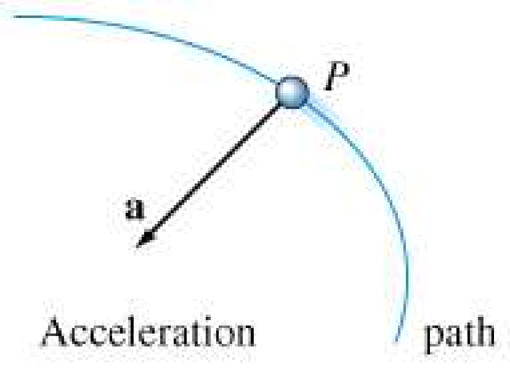
General Curvilinear Motion





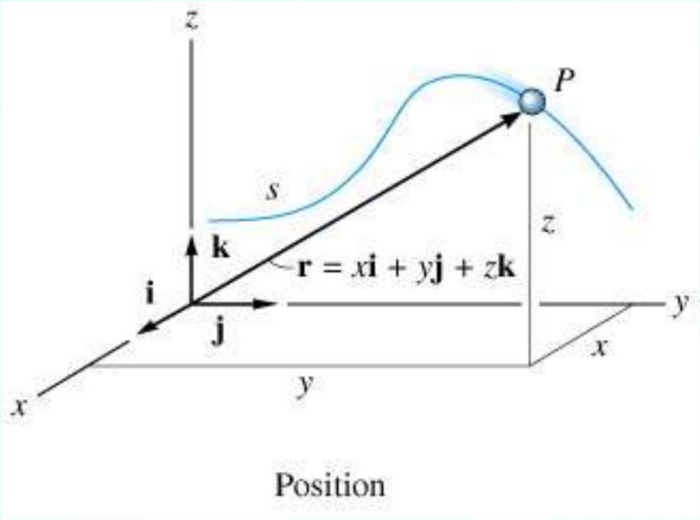
Position vector: **r**

$$\mathbf{v} = \frac{d\mathbf{r}}{dt}$$

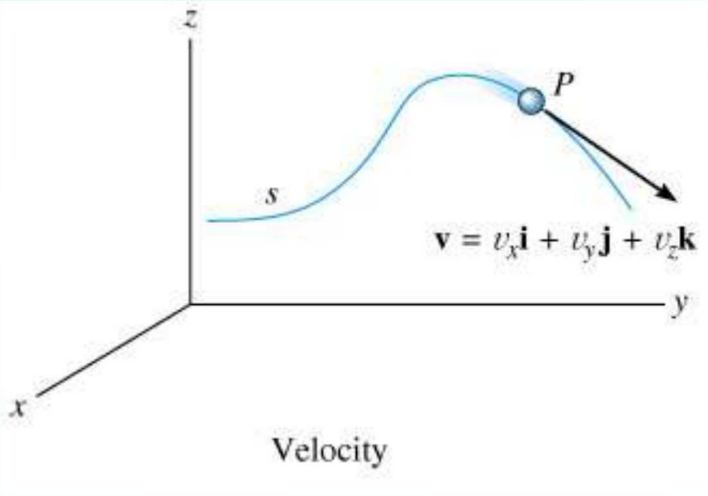


$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d^2\mathbf{r}}{dt^2}$$

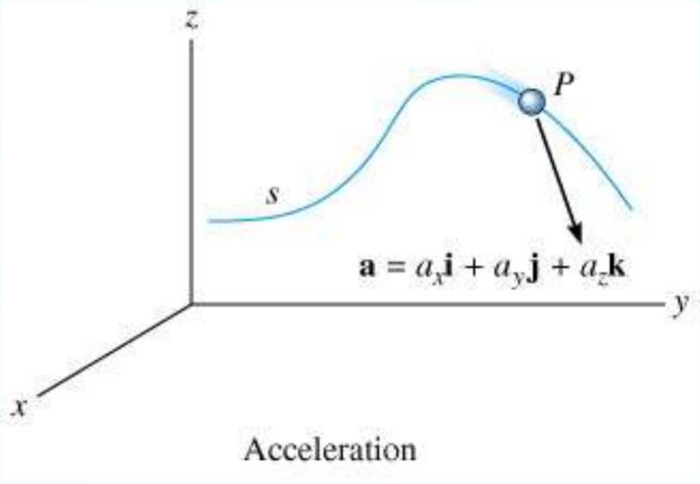
Curvilinear Motion: Rectangular Components



$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$



$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = v_x\,\mathbf{i} + v_y\,\mathbf{j} + v_z\,\mathbf{k}$$



$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = a_x\,\mathbf{i} + a_y\,\mathbf{j} + a_z\,\mathbf{k}$$

$$a_x = \dot{v}_x = \ddot{x} = \frac{d^2x}{dt^2} = \frac{dv_x}{dt}$$

$$a_y = \dot{v}_y = \ddot{y} = \frac{d^2y}{dt^2} = \frac{dv_y}{dt}$$

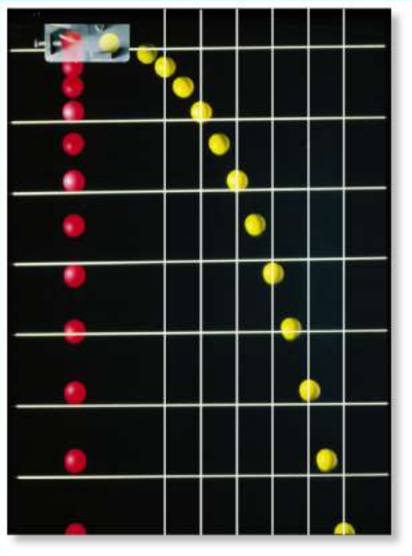
$$a_z = \dot{v}_z = \ddot{z} = \frac{d^2z}{dt^2} = \frac{dv_z}{dt}$$

Projectile Motion

$$v = v_0 + a_c t: \qquad v_x = (v_0)_x$$

$$x = x_0 + v_0 t + \frac{1}{2} a_c t^2: \qquad x = x_0 + (v_0)_x t$$

$$v^2 = v_0^2 + 2a_c(s - s_0): \qquad v_x = (v_0)_x$$



$$v = v_0 + a_c t: \qquad v_y = (v_0)_y - gt$$

$$y = y_0 + v_0 t + \frac{1}{2} a_c t^2: \qquad y = y_0 + (v_0)_y t - \frac{1}{2} g t^2$$

$$v^2 = v_0^2 + 2a_c(s - s_0): \qquad v_y^2 = (v_0)_y^2 - 2g(y - y_0)$$

$$y(x) = a(x - x_0)^2 + b(x - x_0) + y_0$$

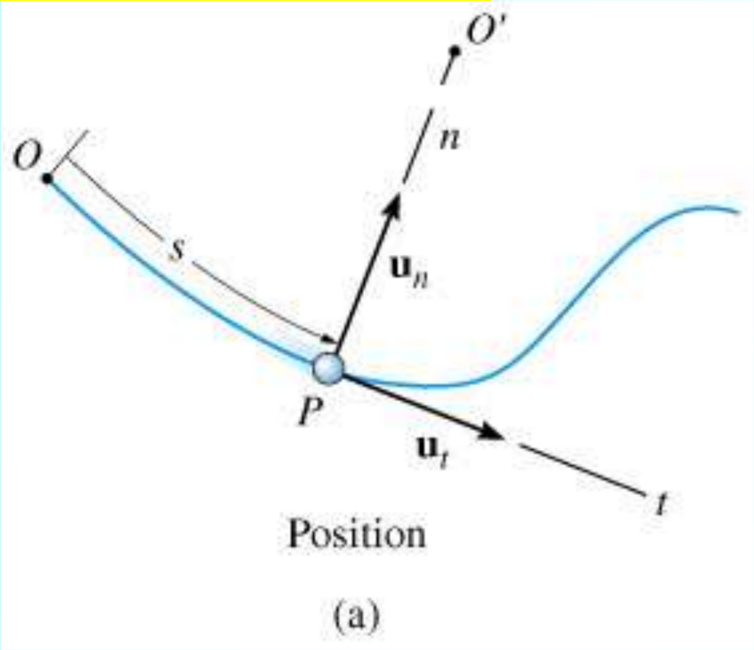
$$a = -\frac{g}{2v_0^2 \cos^2 \theta_0}$$

$$b = \tan \theta_0$$

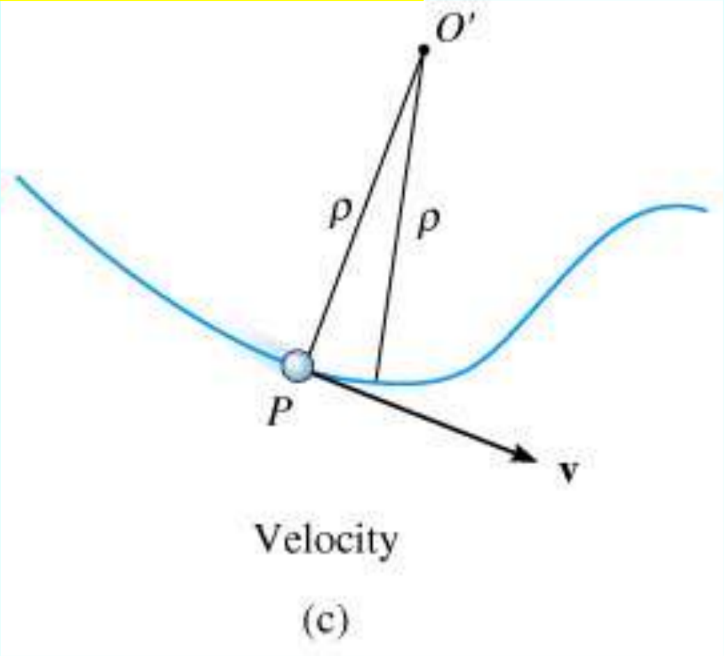
Trajectory equation

Curvilinear Motion: Normal & Tangential Components

Center of curvature, O'



Radius of curvature, ρ

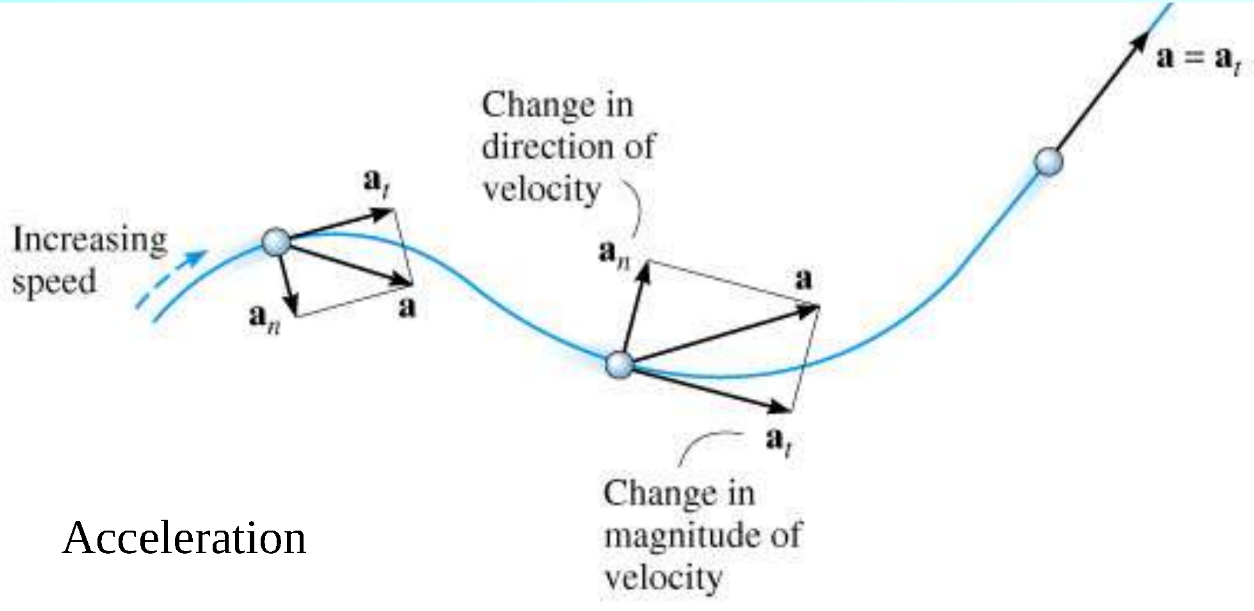


Position: $s = s(t)$ as measured from O

$$\mathbf{v} = v\mathbf{u}_t$$

$$v = \frac{ds}{dt} = \dot{s}$$

Curvilinear Motion: Normal & Tangential Components

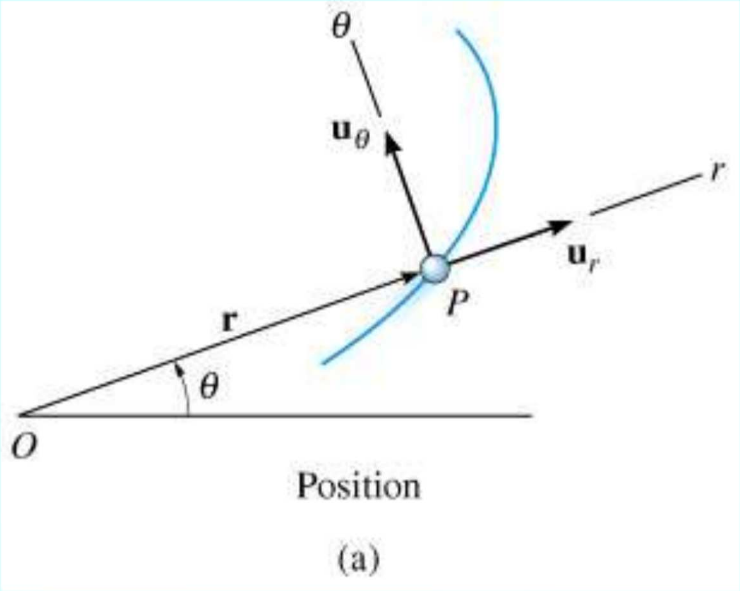


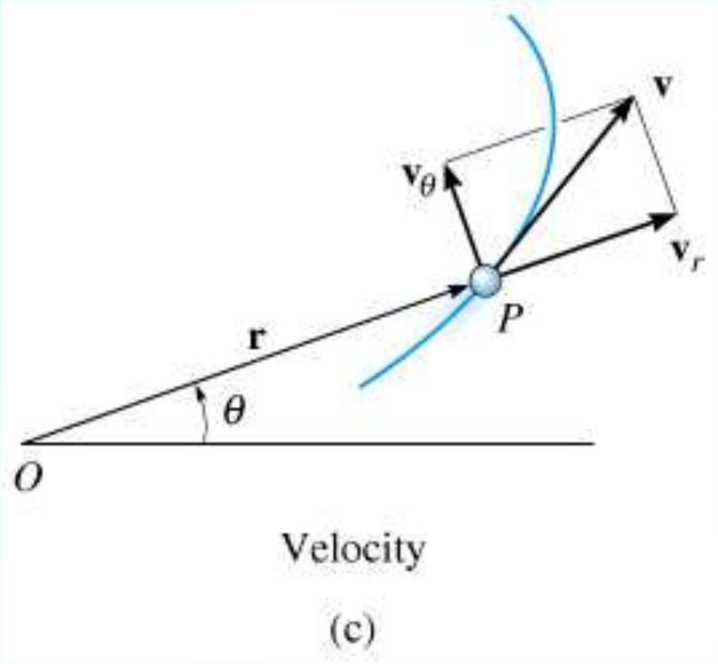
$$\mathbf{a} = a_t \mathbf{u}_t + a_n \mathbf{u}_n$$

$$a_t = \dot{v} \qquad \text{or} \qquad a_t\, ds = v\, dv$$

$$a_n = \frac{v^2}{\rho}$$

Curvilinear Motion: Cylindrical (Polar) Coordinates





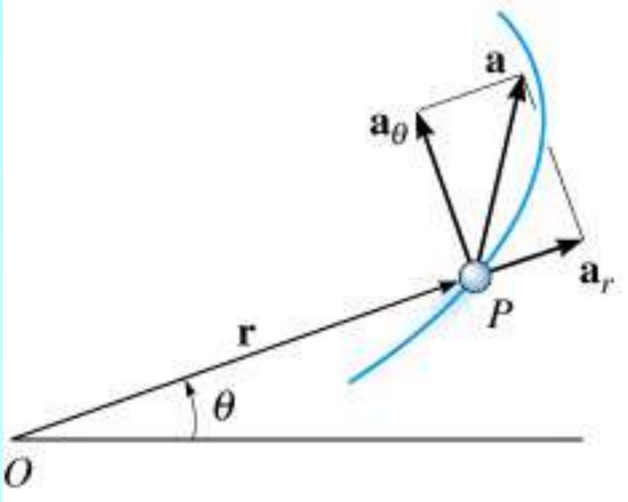
$$\mathbf{r} = r\,\mathbf{u}_r$$

$$\mathbf{v} = v_r\,\mathbf{u}_r + v_\theta\,\mathbf{u}_\theta$$

$$v_r = \dot{r}$$

$$v_\theta = r\,\dot{\theta}$$

Curvilinear Motion: Cylindrical (Polar) Coordinates



Acceleration

(e)

$$\mathbf{a} = a_r\,\mathbf{u}_r + a_\theta\,\mathbf{u}_\theta$$

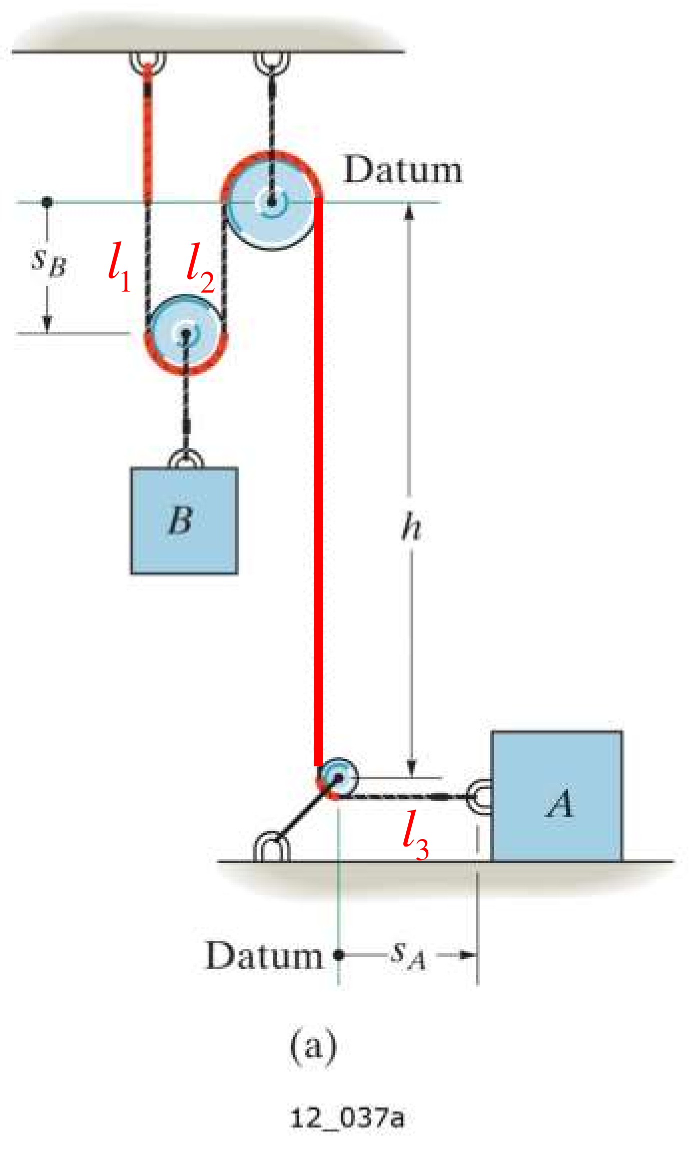
$$a_r = \ddot{r} - r\dot{\theta}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

Make sure that you can take an expression such as (example only!!)

$$r(\theta) = 2e^{\theta}\sin(\theta)$$

and compute $\dot{r}$ and $\ddot{r}$ in terms of θ, $\dot{\theta}$ and $\ddot{\theta}$



Absolute Dependent Motion: Analysis of Two Particles (Example)

Determine the relationship between v_A and v_B .

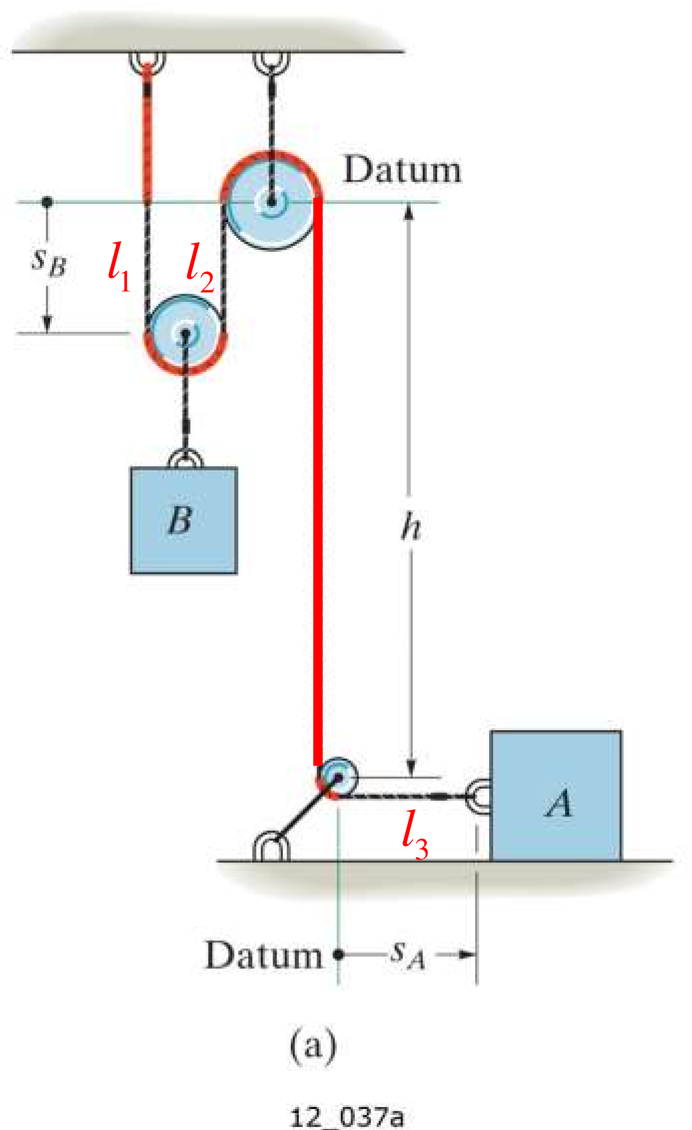
1. Label the non-constant length segments of the rope, l_1, l_2, l_3, etc.

2. Write down the rope equation

$$l_1 + l_2 + l_3 = \text{constant}$$

3. Write down the path equations (when summed these should contain all of the l_i's)

$$l_1 = s_B$$
$$l_2 = s_B$$
$$l_3 = s_A$$



4. Sum the path equations

$$l_1 + l_2 + l_3 = s_A + 2s_B$$

5. Use the rope equation to eliminate the l_i's

$$\text{constant} = s_A + 2s_B$$

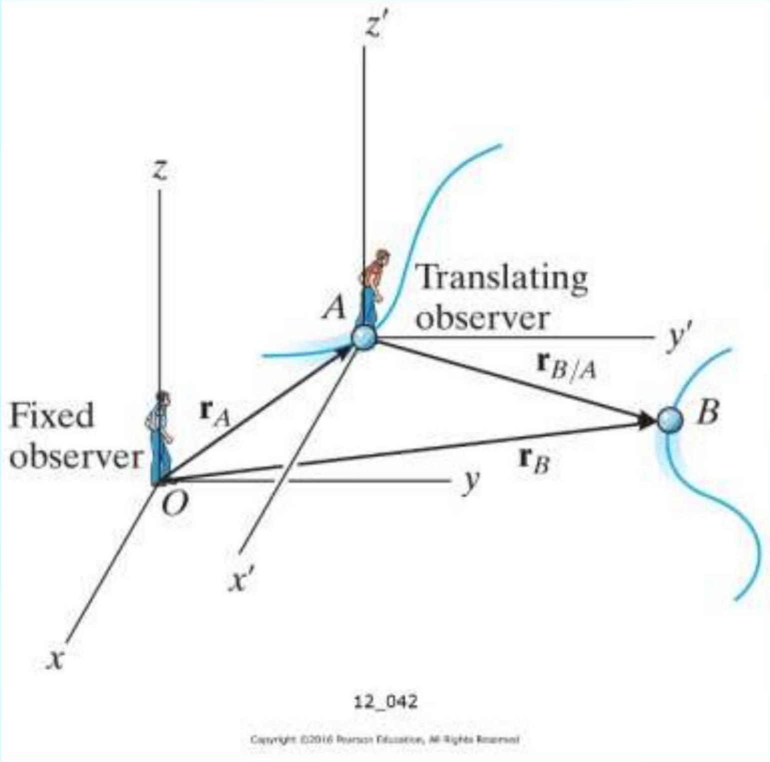
$$s_A + 2s_B = \text{constant}$$

6. Differentiate with respect to time to get the relationship between the velocities

$$v_A + 2v_B = 0$$

$$v_B = -\frac{1}{2}v_A$$

Relative Motion of Two Particles

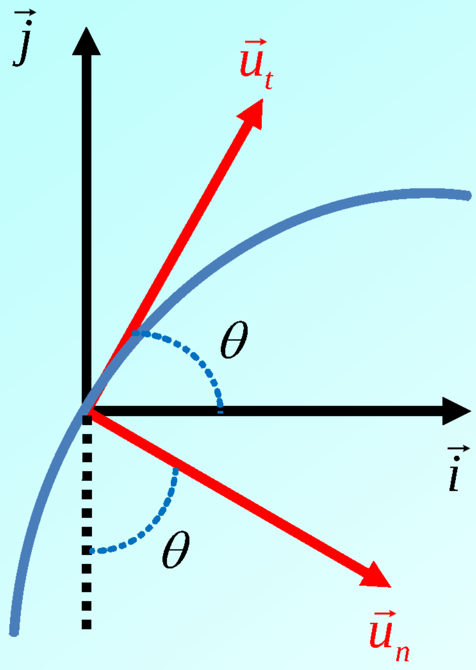


$$\vec{r}_{B/A} = \vec{r}_B - \vec{r}_A \qquad \vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$$

$$\vec{v}_{B/A} = \vec{v}_B - \vec{v}_A \qquad \vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$

$$\vec{a}_{B/A} = \vec{a}_B - \vec{a}_A \qquad \vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$

Relationship between tangential/normal and Cartesian unit vectors (example!!)



$$\vec{u}_t = \cos\theta\,\vec{i} + \sin\theta\,\vec{j}$$

$$\vec{u}_n = \sin\theta\,\vec{i} - \cos\theta\,\vec{j}$$

Be sure you can derive relationships such as these.