PHYS 170 Section 101
Final Exam Review
November 26, 2018

Announcements

- Modification of overall grading scheme:

  If your mark on the final exam is greater than your course mark computed using the original grading scheme, then your final grade will be equal to your mark on the final exam.

- Text sections referenced in syllabus, but not covered in class, and for which there will be no evaluation on final:

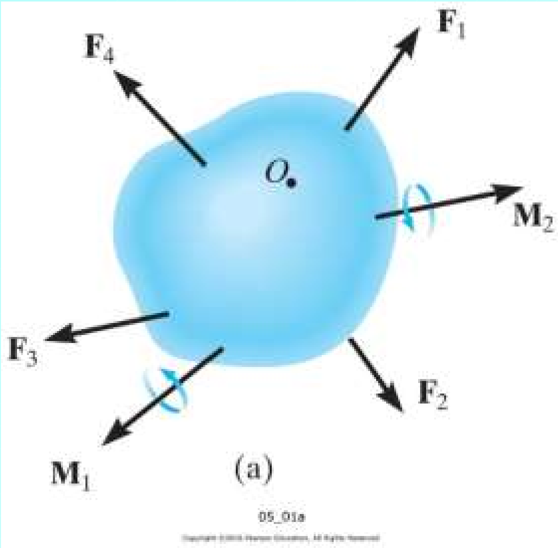
  5.7, 12.3, 14.4

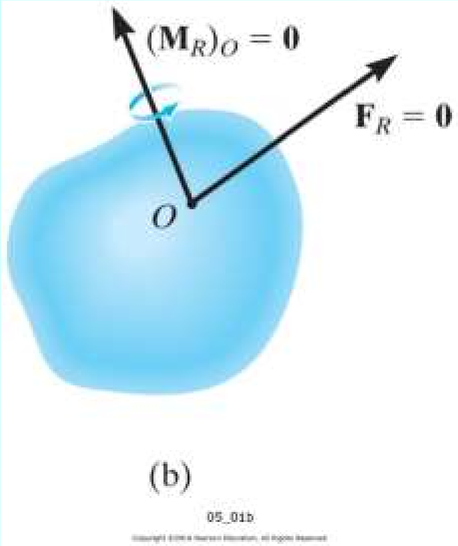
WARNING / DISCLAIMER

The instructor does not guarantee that the following review is complete. In particular, concepts and/or equations pertinent to the final exam *may* have been omitted below.

CHAPTER 5
EQUILIBRIUM OF A RIGID BODY

Conditions for Rigid Body Equilibrium





$$\mathbf{F}_R = \sum \mathbf{F} = \mathbf{0}$$

$$(\mathbf{M}_R)_O = \sum \mathbf{M}_O = \mathbf{0}$$

- A body is in equilibrium if the sum of the external forces acting on it vanishes and the sum of the moments about some point due to those forces added to all the couple moments also vanishes.

Free Body Diagrams & Support Reactions

A force is developed by a support that restricts the translation of its attached member

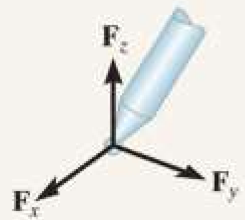
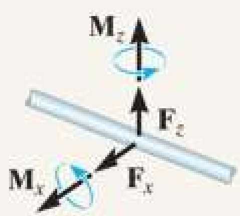
A couple moment is developed when rotation of the attached member is prevented

Will not be told which reactions are developed by which supports, so may want to include details of at least some of them on your information sheet, but also study available problems (lectures, homework, old exams, for other examples)

TABLE 5–2 Supports for Rigid Bodies Subjected to Three-Dimensional Force Systems

Types of Connection	Reaction	Number of Unknowns
(1)  cable		One unknown. The reaction is a force which acts away from the member in the known direction of the cable.
(2)  smooth surface support		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(3)  roller		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.

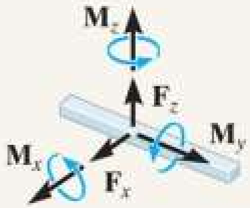
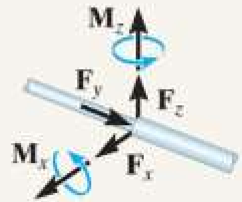
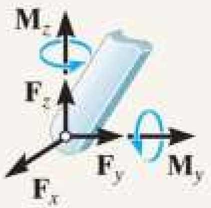
TABLE 5–2 Supports for Rigid Bodies Subjected to Three-Dimensional Force Systems

Types of Connection	Reaction	Number of Unknowns
(4)  ball and socket		Three unknowns. The reactions are three rectangular force components.
(5)  single journal bearing		Four unknowns. The reactions are two force and two couple-moment components which act perpendicular to the shaft. Note: The couple moments are *generally not applied* if the body is supported elsewhere. See the examples.

continued

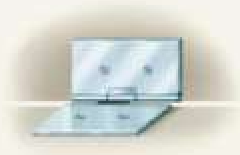
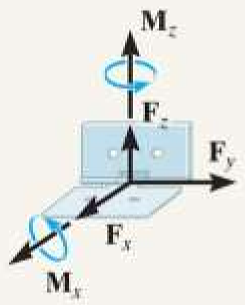
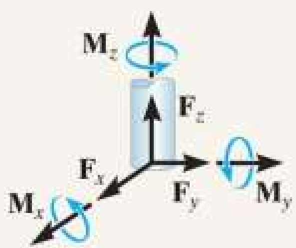
continued on next slide

TABLE 5–2 Continued

Types of Connection	Reaction	Number of Unknowns
(6)  single journal bearing with square shaft		Five unknowns. The reactions are two force and three couple-moment components. *Note*: The couple moments *are generally not applied* if the body is supported elsewhere. See the examples.
(7)  single thrust bearing		Five unknowns. The reactions are three force and two couple-moment components. *Note*: The couple moments *are generally not applied* if the body is supported elsewhere. See the examples.
(8)  single smooth pin		Five unknowns. The reactions are three force and two couple-moment components. *Note*: The couple moments *are generally not applied* if the body is supported elsewhere. See the examples.

continued on next slide

TABLE 5–2 Continued

Types of Connection	Reaction	Number of Unknowns
(9)  single hinge		Five unknowns. The reactions are three force and two couple-moment components. *Note*: The couple moments *are generally not applied* if the body is supported elsewhere. See the examples.
(10)  fixed support		Six unknowns. The reactions are three force and three couple-moment components.

Equations of Equilibrium

Vector equations of equilibrium

The vector conditions for equilibrium of a rigid body are

$$\sum \vec{F} = 0$$
$$\sum \vec{M}_O = 0$$

where $\sum \vec{F}$ is the vector sum of all the forces acting on the body and $\sum \vec{M}_O$ is the sum of any couple moments and the moments of all the forces about any point *O*.

Scalar equations of equilibrium

In Cartesian vector form, the equations of equilibrium become

$$\sum \vec{F} = \sum F_x \vec{i} + \sum F_y \vec{j} + \sum F_z \vec{k} = 0$$

$$\sum M_O = \sum M_x \vec{i} + \sum M_y \vec{j} + \sum M_z \vec{k} = 0$$

The $\vec{i}$, $\vec{j}$, and $\vec{k}$ components are independent of one another, so these are equivalent to the six scalar equations:

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum F_z = 0$$

$$\sum M_x = 0 \qquad \sum M_y = 0 \qquad \sum M_z = 0$$

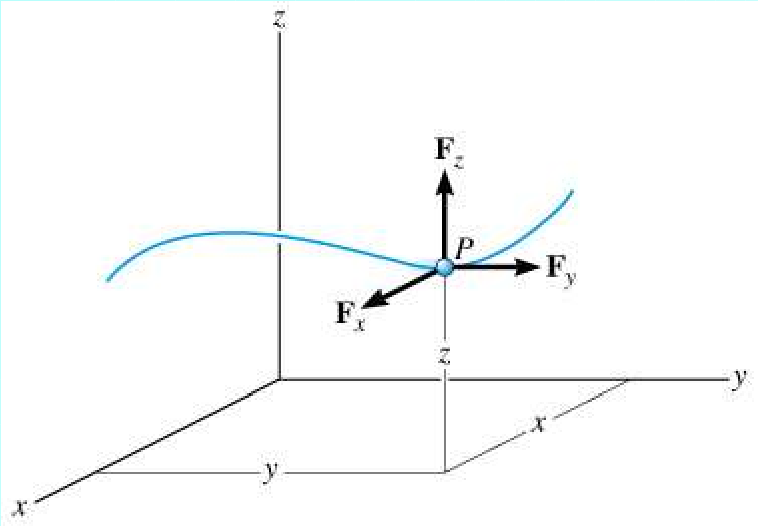
These equations can be used to solve for at most six unknowns shown on the free body diagram.

CHAPTER 13
KINETICS OF A PARTICLE: FORCE AND ACCELERATION

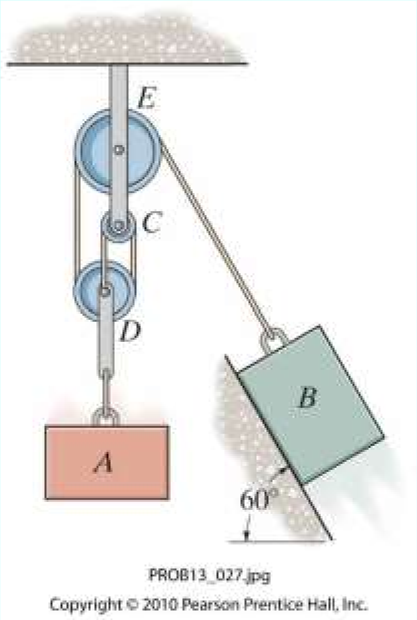
- Worked in different coordinate systems
  - Rectangular (Cartesian) coordinates
  - Normal / tangential coordinates
  - Cylindrical (polar) coordinates
- Restricted our problem solving to two dimensional problems
- Fundamental equation: Newton's 2nd law for a particle of mass *m*

$$\boxed{\sum \mathbf{F} = m\mathbf{a}}$$

Equations of Motion: Rectangular Coordinates



$$\Sigma F_x = m a_x$$
$$\Sigma F_y = m a_y$$
$$\Sigma F_z = m a_z$$



Absolute Dependent Motion

$$a_B = 3a_A$$

(Example only!!)

Relative-Motion of Two Particles Using Translating Axes

Position

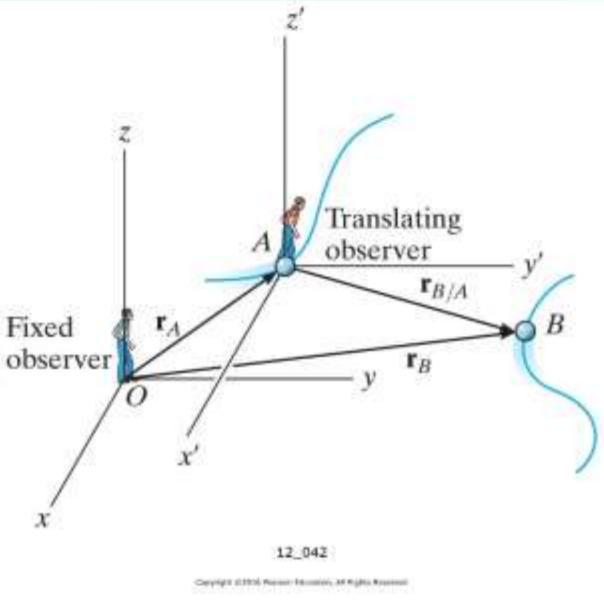
$$\vec{r}_{B/A} = \vec{r}_B - \vec{r}_A$$
$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$$

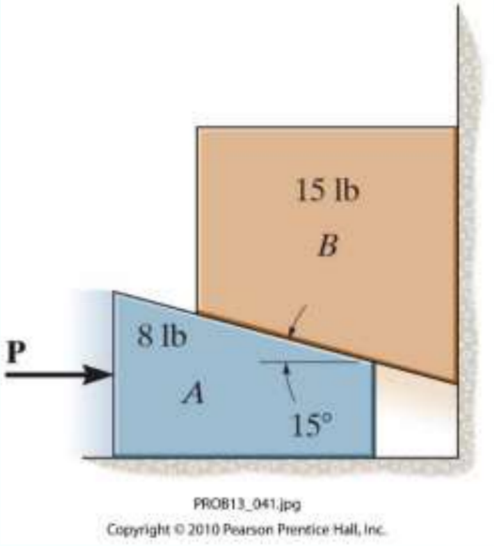
Velocity

$$\vec{v}_{B/A} = \vec{v}_B - \vec{v}_A$$
$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$

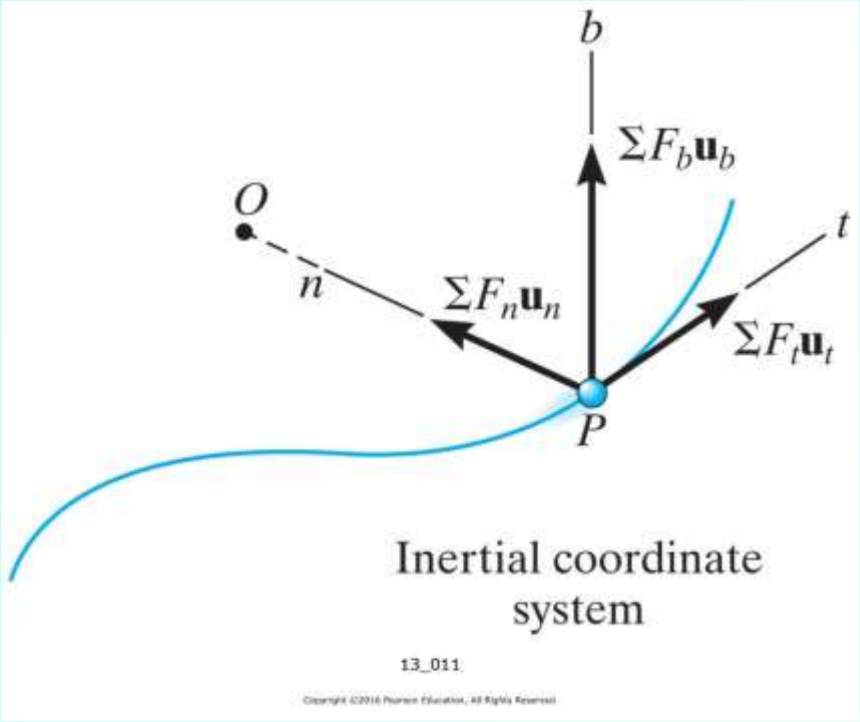
Acceleration

$$\vec{a}_{B/A} = \vec{a}_B - \vec{a}_A$$
$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$





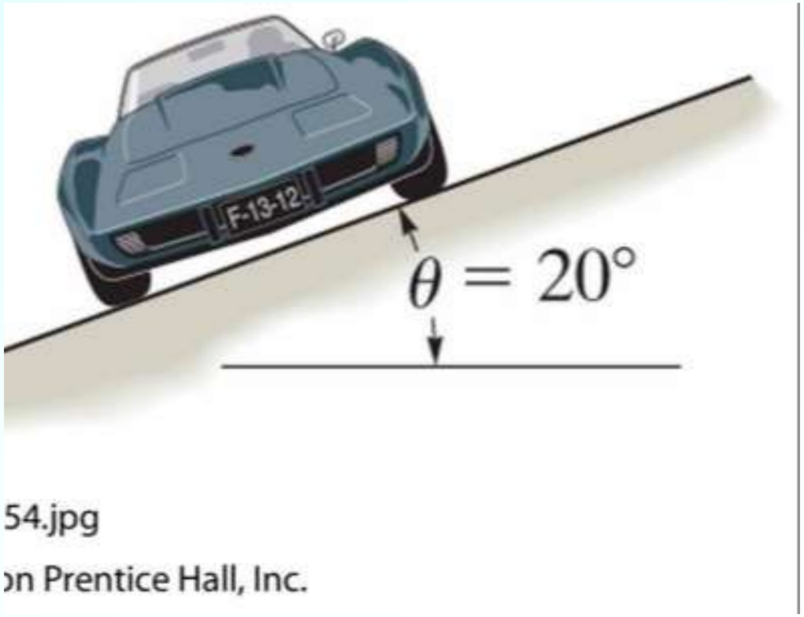
Equations of Motion: Normal and Tangential Coordinates



$$\sum F_t = ma_t = m\dot{v}$$

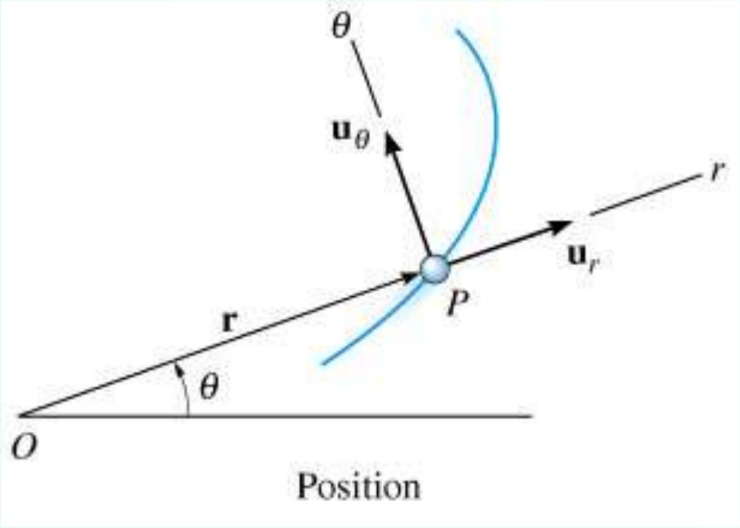
$$\sum F_n = ma_n = m\frac{v^2}{\rho}$$

$$\sum F_b = 0$$



Equations of Motion: Cylindrical Coordinates

RECALL: Ch 12 – Polar components: Planar Motion (2D)

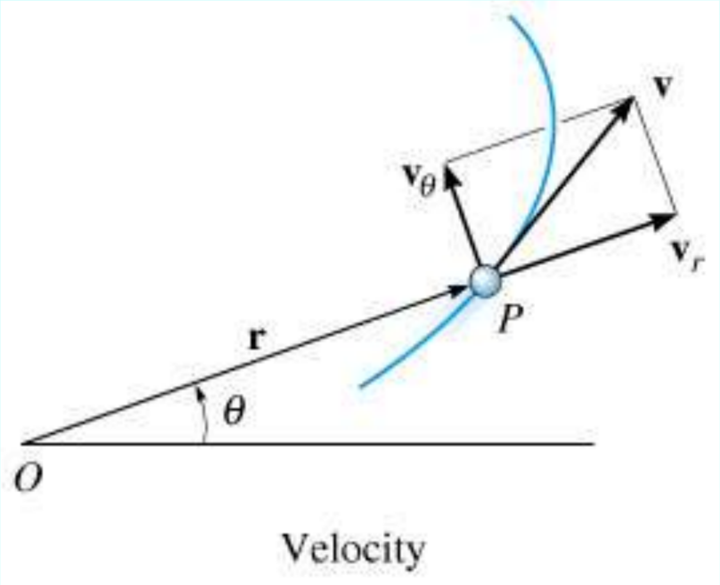


POSITION

$$\mathbf{r} = r\,\mathbf{u}_r$$

Note that, in general, r, θ coordinates of particle will be functions of time, i.e.

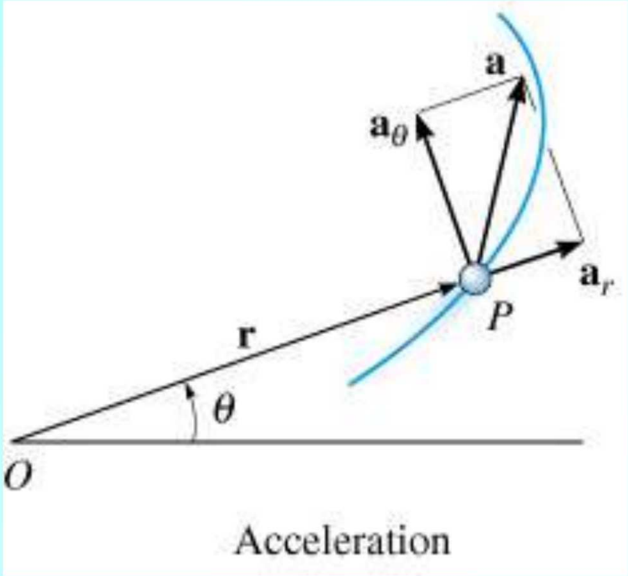
$$r = r(t)$$
$$\theta = \theta(t)$$



VELOCITY

$$\mathbf{v} = \dot{r}\,\mathbf{u}_r + r\dot{\theta}\,\mathbf{u}_\theta$$

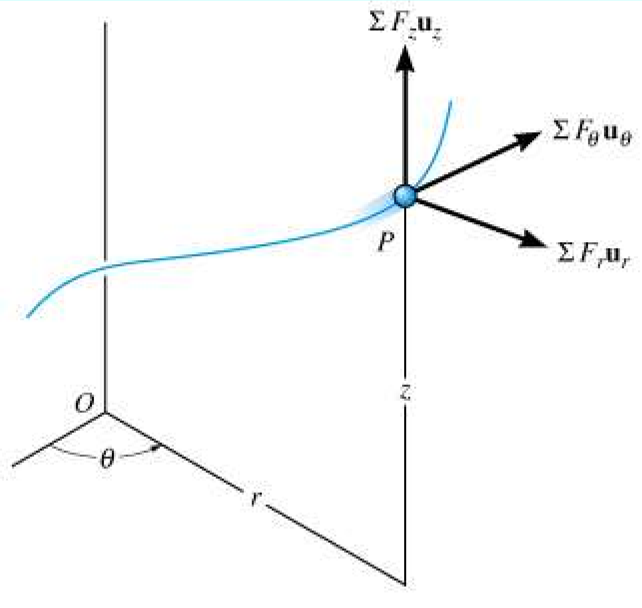
Polar components: Planar Motion (2D) [continued]



ACCELERATION

$$\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\mathbf{u}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{u}_\theta$$

EQUATIONS OF MOTION in cylindrical coordinates

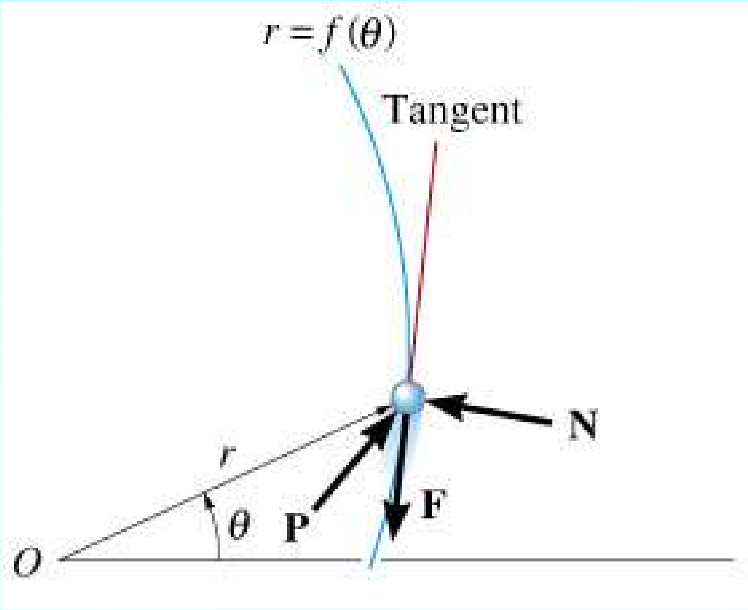


Inertial coordinate system

$$\Sigma F_r = m a_r$$
$$\Sigma F_\theta = m a_\theta$$
$$\Sigma F_z = m a_z$$

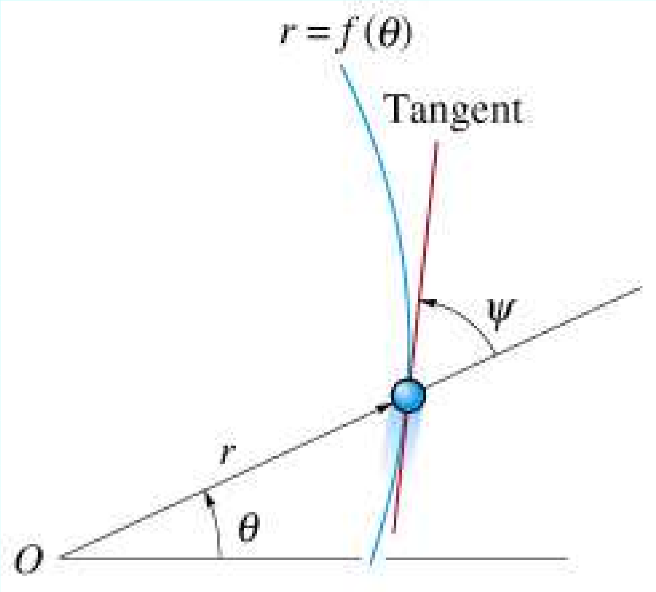
- Only dealt with problems in which the motion was restricted to 2D, i.e. to the $r - \theta$ plane. In this case, only the first two of the above equations apply

TANGENTIAL AND NORMAL FORCES



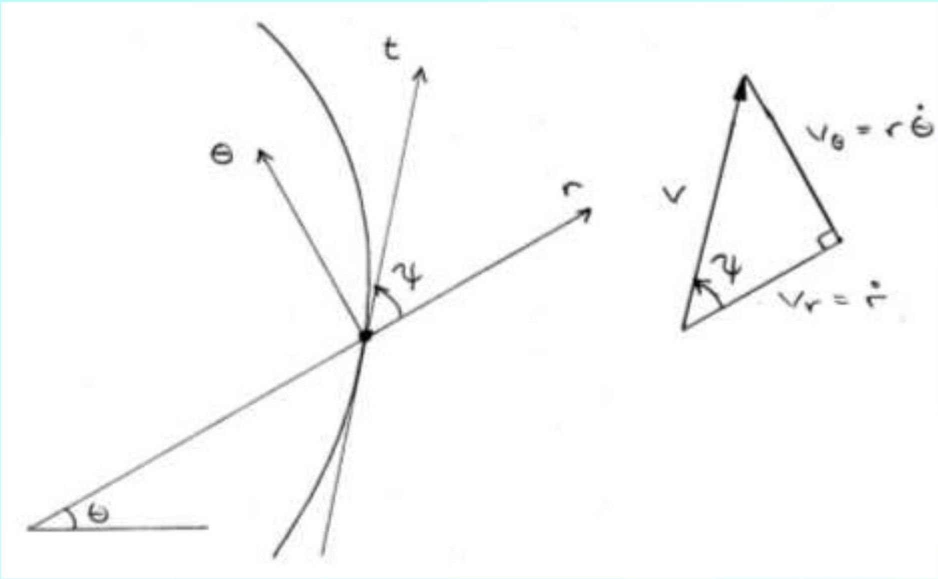
Many of the problems in this part of the course have the following features

- They are natural to treat in polar coordinates
- The path of the particle is specified (constrained motion)
- Some of the forces on the particle act in the **normal** or **tangential** directions



- We thus need to be able to determine the orientation of the (t,n) coordinate system with respect to the (r,θ) system
- As shown in the figure, the tangent to the particle path will form an angle ψ with the **extended** radial line

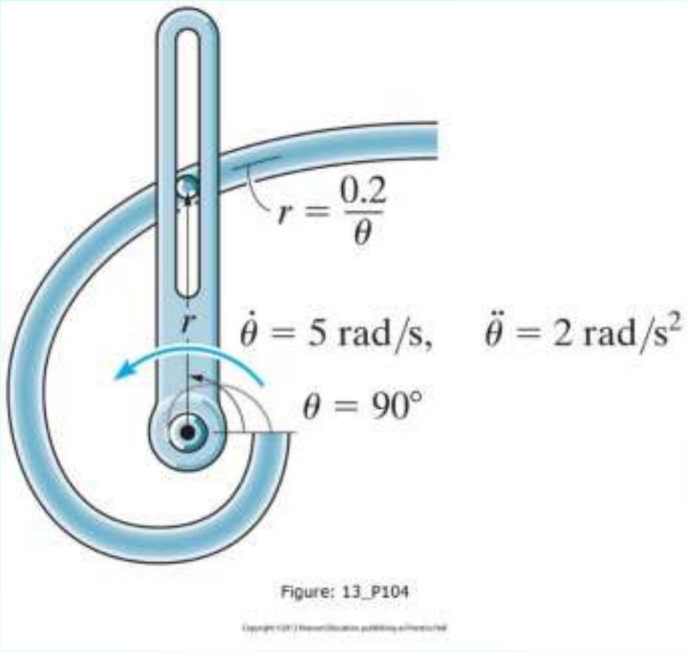
TANGENTIAL AND NORMAL FORCES (continued)

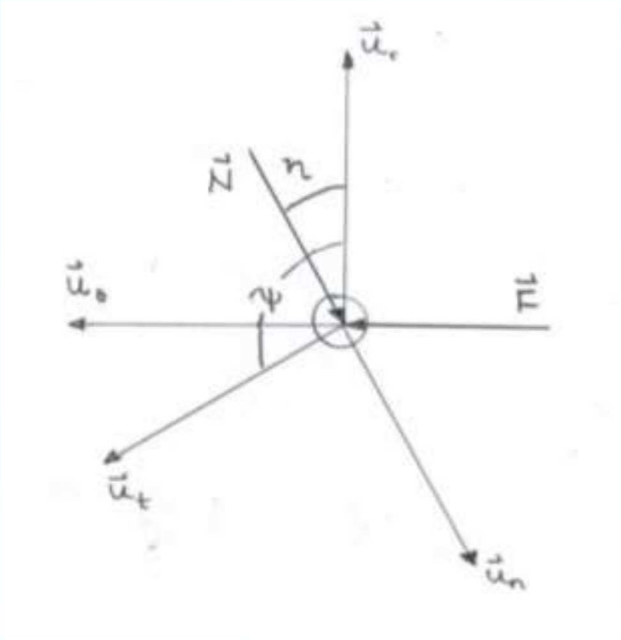


- Showed that

$$\tan\psi = \frac{r\dot{\theta}}{\dot{r}}$$

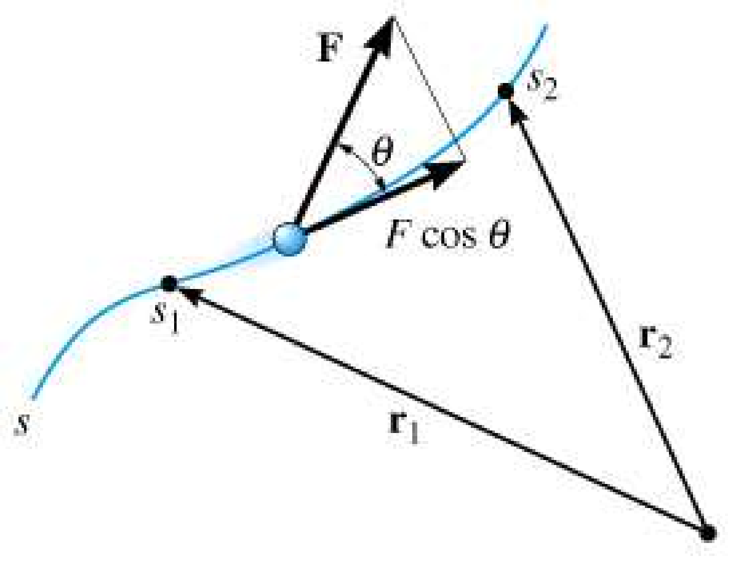
$$\tan\psi = \frac{r}{dr/d\theta}$$





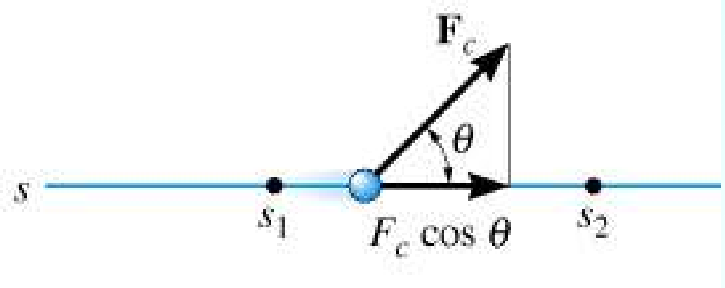
CHAPTER 14
KINETICS OF A PARTICLE: WORK AND ENERGY

WORK OF A VARIABLE FORCE



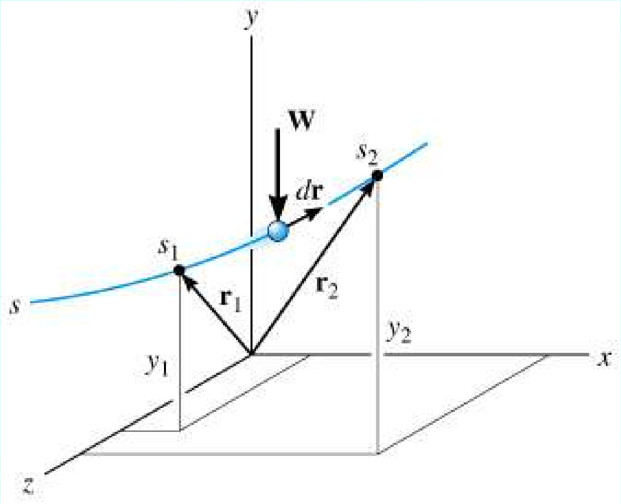
$$U_{1-2} = \int_{\mathbf{r}_1}^{\mathbf{r}_2} \mathbf{F} \cdot d\mathbf{r} = \int_{s_1}^{s_2} F \cos\theta \, ds$$

WORK OF A CONSTANT FORCE ALONG A STRAIGHT LINE



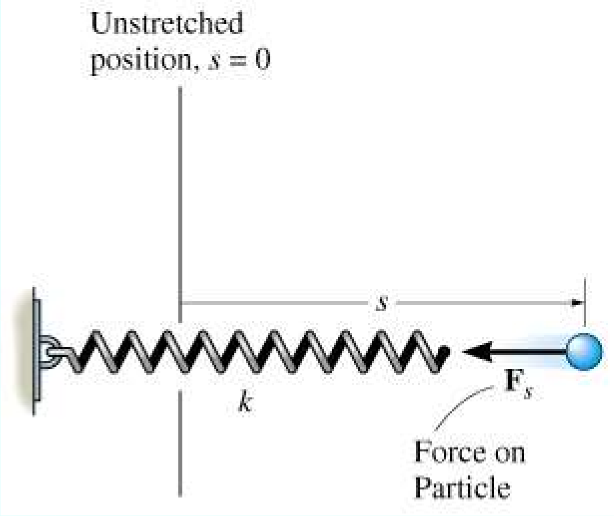
$$U_{1-2} = F_c \cos\theta (s_2 - s_1)$$

WORK OF A WEIGHT



$$U_{1-2} = -W\Delta y$$

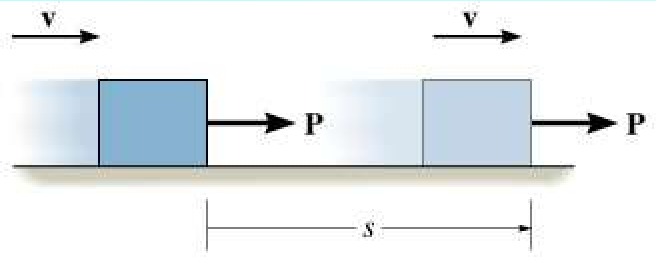
WORK OF A SPRING FORCE

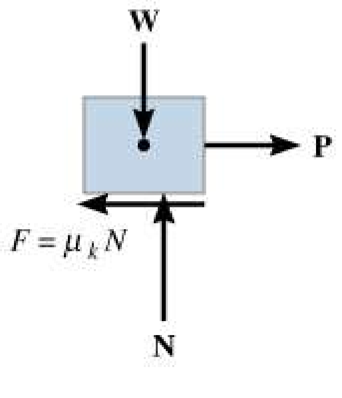


Work done by the spring force on the particle

$$U_{1-2} = -\left(\frac{1}{2}ks_2{}^2 - \frac{1}{2}ks_1{}^2\right)$$

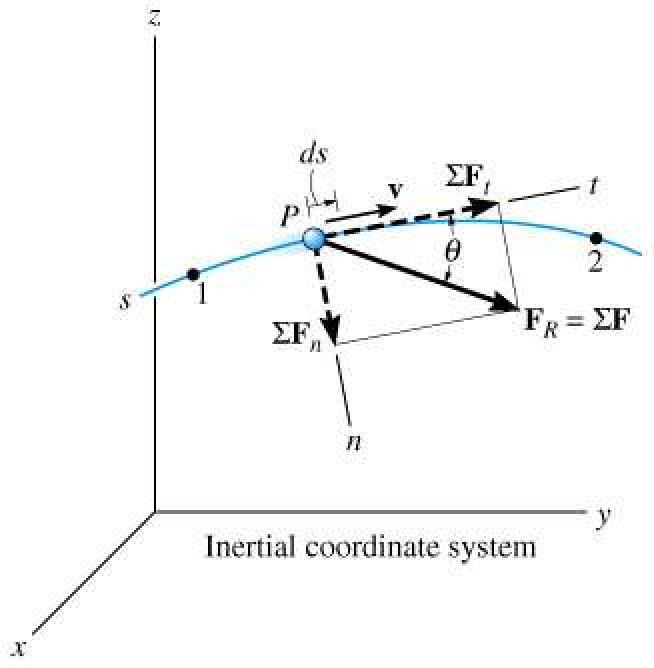
WORK OF FRICTION CAUSED BY SLIDING





$$U_{1-2} = -\mu_k N s$$

PRINCIPLE OF WORK & ENERGY



$$\Sigma U_{1-2} = \frac{1}{2} m v_2^{\,2} - \frac{1}{2} m v_1^{\,2}$$

$$T_1 + \Sigma U_{1-2} = T_2$$

Applies to a system of particles as well as a single particle. For a system, there are implicit sums over all the particles in the various terms of the equations.

Conservative Forces and Potential Energy

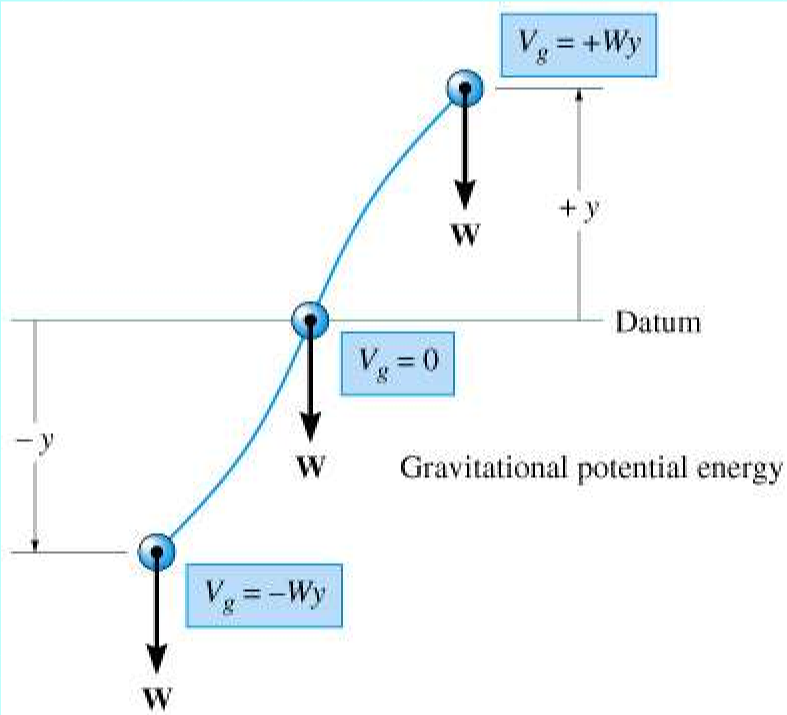
CONSERVATIVE FORCE

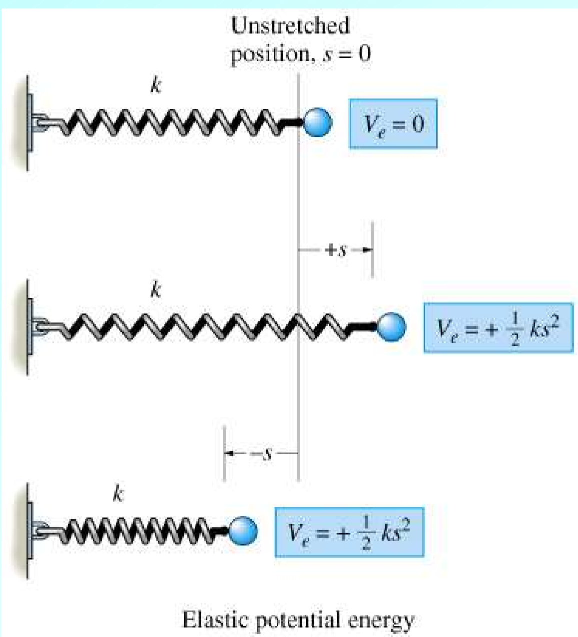
- **Definition:** A force is called **conservative** if the work that it does on a particle as the particle moves from one point to another is **independent of the path** that the particle travels (i.e. the work done depends only on the location of the initial and final points)

- **Examples of conservative forces**

  - **Weight**

  - **(Elastic) Spring force**





GRAVITATIONAL POTENTIAL ENERGY

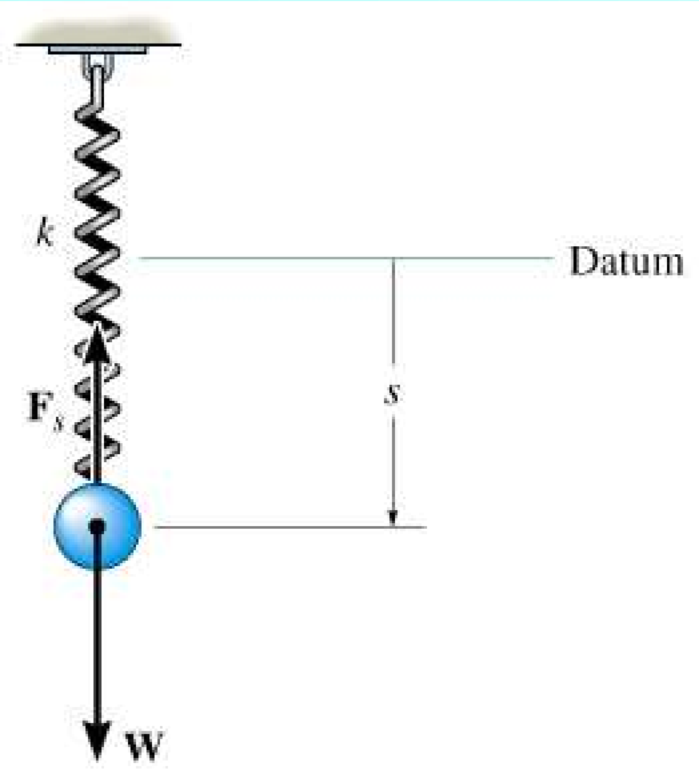
- Assuming that y is positive, the gravitational potential energy, V_g, of a particle of weight W is

$$V_g = mgy$$

ELASTIC POTENTIAL ENERGY

- An elastic (ideal) spring that is compressed or elongated a distance s from its equilibrium position has an elastic potential energy, V_s, given by

$$V_s = +\frac{1}{2}ks^2$$



POTENTIAL FUNCTION (TOTAL POTENTIAL ENERGY)

Potential function, *V*

$$V = V_g + V_s$$

Relation to work

$$U_{1-2} = V_1 - V_2$$

Conservation of Energy

- For a particle acted on only by conservative forces, the sum of the particle's kinetic and potential energy is **constant** during the motion of the particle

$$T_1 + V_1 = T_2 + V_2$$

- For a system of particles acted on only by conservative forces we have

$$\Sigma T_1 + \Sigma V_1 = \Sigma T_2 + \Sigma V_2$$

CHAPTER 15
KINETICS OF A PARTICLE: IMPULSE AND MOMENTUM

Linear Momentum (defn): For a particle with mass, m, and velocity, $\mathbf{v}$ (as measured in an inertial frame), the linear momentum, $\mathbf{L}$, of the particle is

$$\boxed{\mathbf{L} = m\mathbf{v}}$$

Impulse (defn): For a generally time-dependent force and a time interval $t_1 \leq t \leq t_2$ over which the force acts, the linear impulse over that interval is

$$\boxed{\mathbf{I} = \int_{t_1}^{t_2} \mathbf{F}(t)\, dt}$$

Principle of Impulse and Momentum (single particle)

$$\boxed{m\mathbf{v}_1 + \sum \int_{t_1}^{t_2} \mathbf{F}\, dt = m\mathbf{v}_2}$$

Principle of Impulse and Momentum (system of particles)

$$\Sigma m_i(\mathbf{v}_i)_1 + \Sigma\int_{t_1}^{t_2} \mathbf{F}_i\, dt = \Sigma m_i(\mathbf{v}_i)_2$$

Conservation of Linear Momentum for a System of Particles

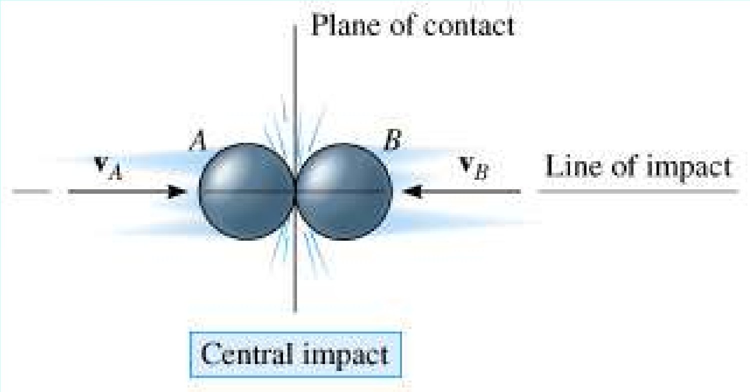
- If no external impulses act on a system of particles (no external forces, for example), then linear momentum for the system is conserved

$$\Sigma m_i(\mathbf{v}_i)_1 = \Sigma m_i(\mathbf{v}_i)_2$$

- Scalar form (can apply conservation in one or more coordinate directions when there are no external impulses in that direction

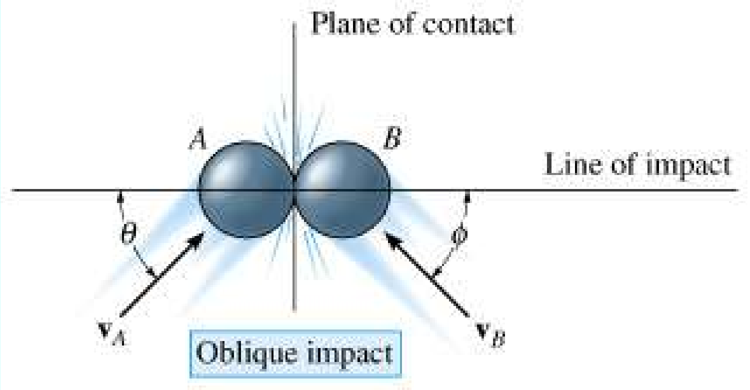
$$\Sigma m_i(v_{ix})_1 = \Sigma m_i(v_{ix})_2$$
$$\Sigma m_i(v_{iy})_1 = \Sigma m_i(v_{iy})_2$$
$$\Sigma m_i(v_{iz})_1 = \Sigma m_i(v_{iz})_2$$

Impact

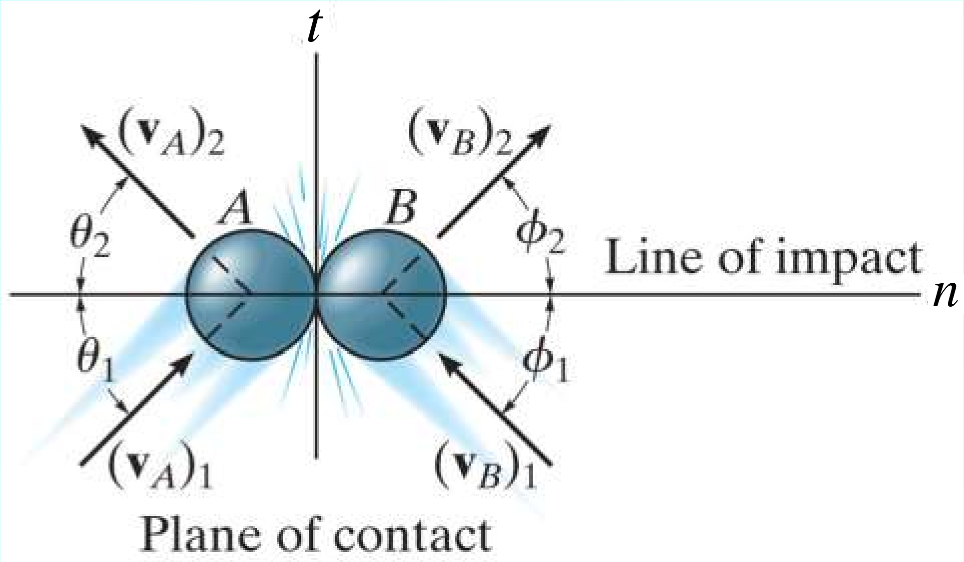


Coefficient of restitution

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1} = \frac{\text{relative velocity after impact}}{\text{relative velocity before impact}}$$



Oblique Impact



- Establish *n* axis (normal axis) along the line of impact and *t* axis (transverse axis) within plane of contact
- Impulsive forces of deformation and restitution act *only in the n direction*

- Momentum is conserved along the line of impact, so that

$$\sum mv_{1n} = \sum mv_{2n}$$

- Along the *t* axis, perpendicular to the line of impact, the momenta of particles *A* and *B* are individually conserved since no impulses act on either of them in this direction

Equations for Oblique Impact

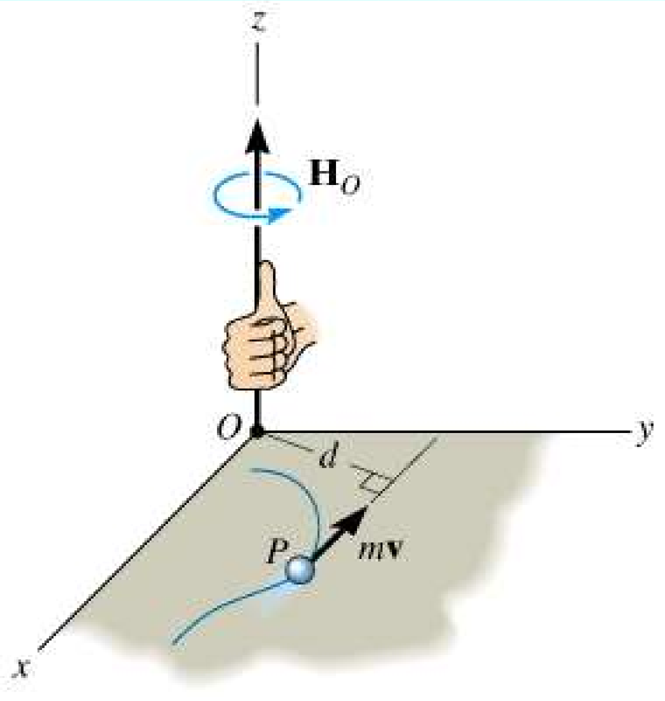
$$v_{A2n} = \frac{(m_A - em_B)v_{A1n} + m_B(1+e)v_{B1n}}{m_A + m_B}$$

$$v_{A2t} = v_{A1t}$$

$$v_{B2n} = \frac{(m_B - em_A)v_{B1n} + m_A(1+e)v_{A1n}}{m_B + m_A}$$

$$v_{B2t} = v_{B1t}$$

Angular Momentum



$$(H_O)_z = (d)(mv)$$

Principle of Angular Impulse and Momentum

$$\text{angular impulse} = \int_{t_1}^{t_2} \mathbf{M}_O\, dt$$

$$(\mathbf{H}_O)_1 + \Sigma \int_{t_1}^{t_2} \mathbf{M}_O\, dt = (\mathbf{H}_O)_2$$

Conservation of Angular Momentum

When the vector sum of angular impulses acting on a particle during a time interval vanishes (no angular impulse act on the particle, for example), then angular momentum of the particle is conserved

$$(\mathbf{H}_O)_1 = (\mathbf{H}_O)_2$$

More generally, angular momentum about some axis is conserved when there are no angular impulses acting about that axis