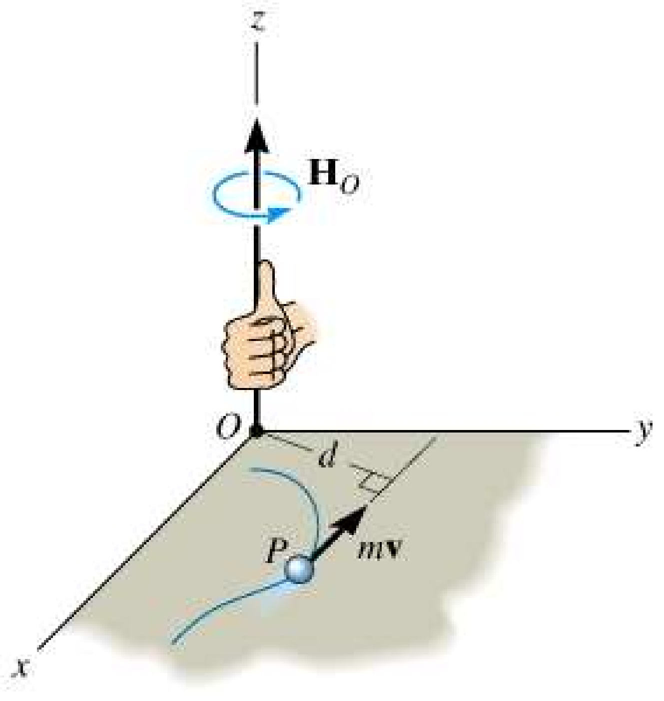
PHYS 170 Section 101
Lecture 34
November 30, 2018

Lecture Outline/Learning Goals

- Worked example using conservation of angular momentum



Angular momentum: Scalar Formulation

- Consider a particle *P*, moving on a curve in the *x*-*y* plane, with linear momentum $m\mathbf{v}$, as shown in the figure

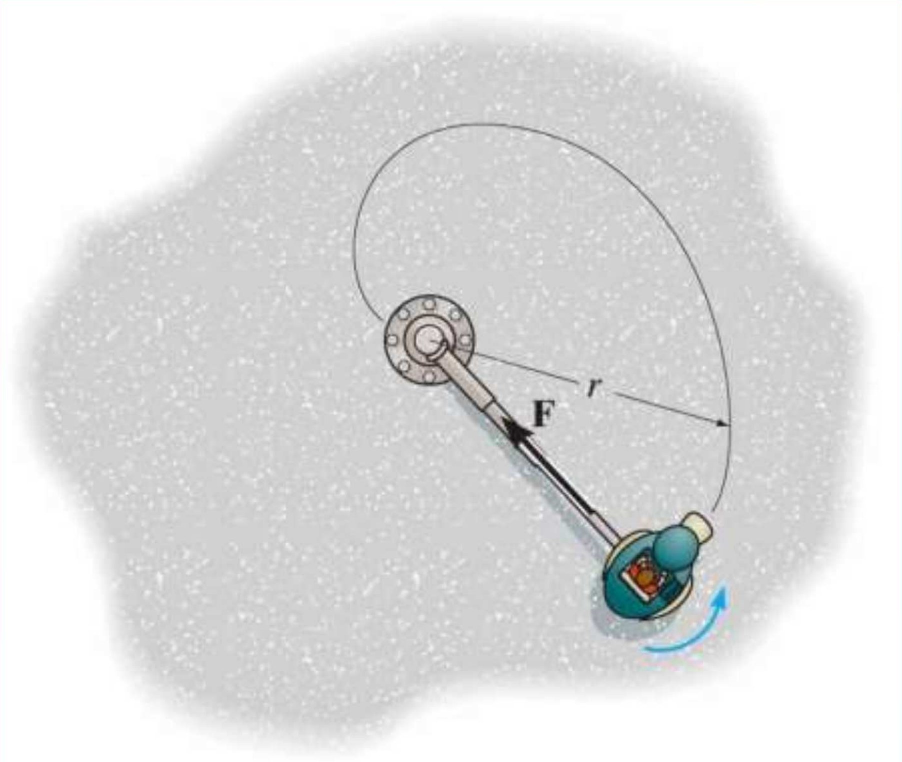
- Then the **magnitude** of the particle's angular momentum about *O* is given by

$$(H_O)_z = (d)(mv)$$

where *d* is the moment arm (perpendicular distance) from *O* to the line of action of $m\mathbf{v}$

- The **direction** of the particle's angular momentum is perpendicular to the *x*-*y* plane (i.e. along the *z*-axis), with a sense given by the right hand rule, where the fingers of the right hand curl in the direction of the rotation of $m\mathbf{v}$ about the *z*-axis

Problem 15-109 (Page 275, 13th edition)

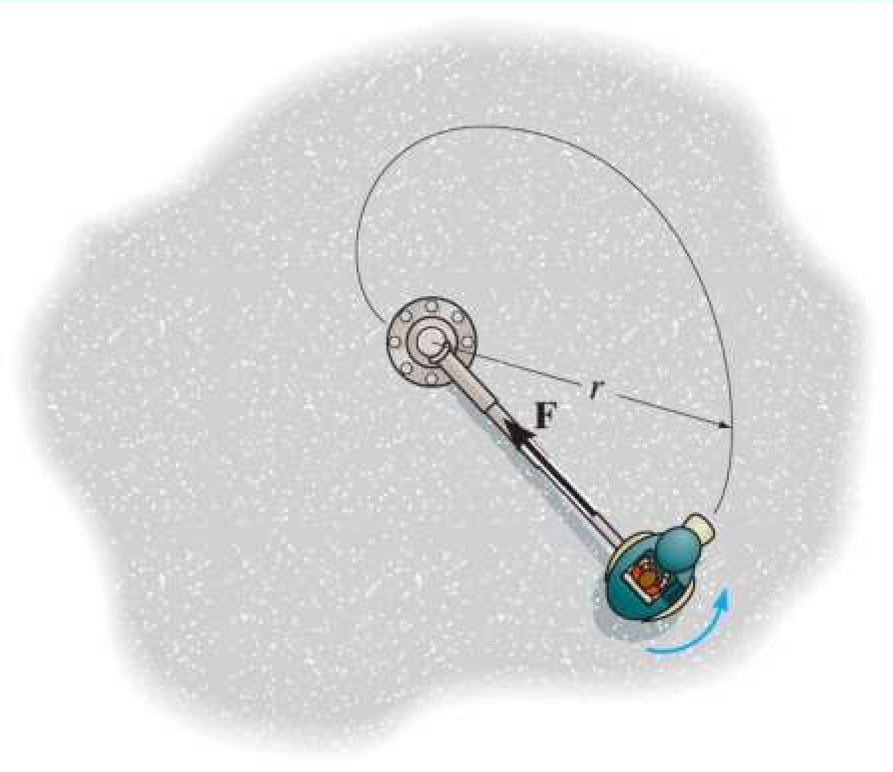


PROB15_105.jpg

The 150 lb car of an amusement park ride is connected to a rotating telescopic boom. When $r = 15$ ft, the car is moving on a horizontal circular path at 30 ft/s

(1) The boom is shortened by 3 ft/s. Determine the speed of the car when $r = 10$ ft

(2) Determine the work done by the force $\vec{F}$ when the boom is shortened from 15 ft to 10 ft



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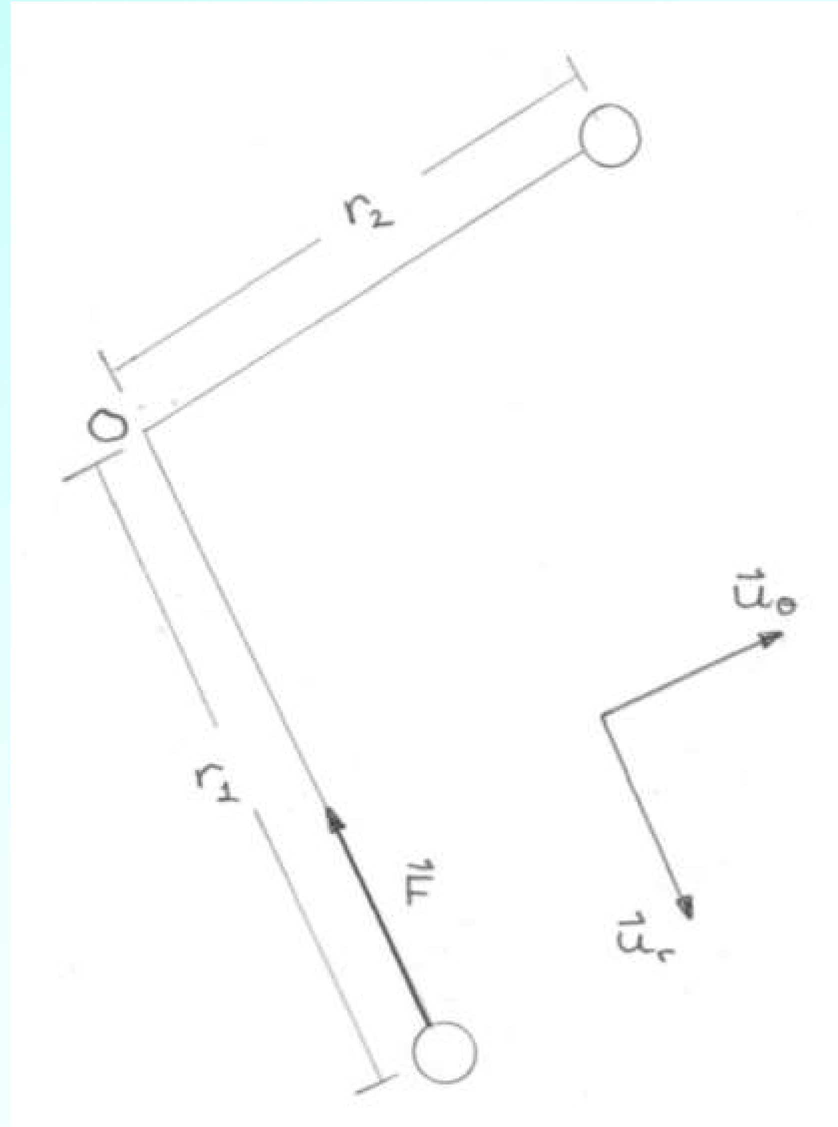

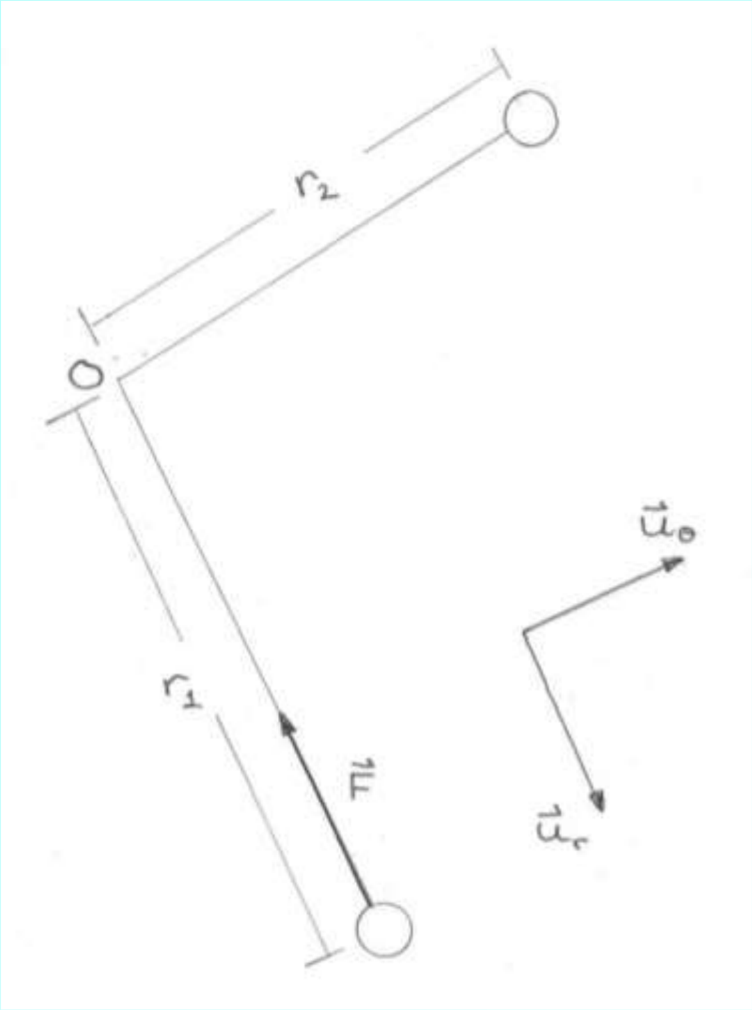

Solution strategy

Work in polar coordinates since there are velocity components in both the radial and transverse directions.

Radial component of linear momentum does not contribute to angular momentum about z axis (passing through O).

Use conservation of angular momentum about z axis with transverse momentum to compute transverse velocity component of shortened boom.

Compute speed of car when $r = 10$ ft from radial and transverse components of velocity.

Use energy balance equation to compute work done by force $\vec{F}$ in shortening boom.

Data

$$W = mg = 150 \text{ lb} \qquad g = 32.2 \text{ ft/s}^2$$

$$\vec{F} = -F\,\vec{u}_r$$

Kinematics

$$\vec{r} = r\,\vec{u}_r$$

$$\vec{v} = v_r\,\vec{u}_r + v_\theta\,\vec{u}_\theta$$

$$v_r = \dot{r} \qquad v_\theta = r\dot{\theta} \qquad v = \sqrt{v_r^2 + v_\theta^2}$$

Original circular path

$r_1 = 15 \text{ ft} \qquad v_{1r} = 0 \qquad v_{1\theta} = 30 \text{ ft/s}$

After boom is shortened

$r_2 = 10 \text{ ft} \qquad v_{2r} = -3 \text{ ft/s} \qquad v_{2\theta} = \ ??$

Conservation of angular momentum (only transverse component of velocity contributes, mass is constant so drops out of equation)

$$r_2 v_{2\theta} = r_1 v_{1\theta}$$

$$v_{2\theta} = \frac{r_1 v_{1\theta}}{r_2} = \frac{15(30)}{10} = 45 \text{ ft/s}$$

Speed of car when $r = 10$ ft

$$v_2 = \sqrt{{v_{2r}}^2 + {v_{2\theta}}^2} = \sqrt{(-3)^2 + 45^2} = 45.1 \text{ ft/s}$$

Work done by $\vec{F}$: Energy balance equation

$$\frac{1}{2}m{v_1}^2 + U_{1-2} = \frac{1}{2}m{v_2}^2$$

$$U_{1-2} = \frac{1}{2}m{v_2}^2 - \frac{1}{2}m{v_1}^2$$

$$= \frac{1}{2}\left(\frac{150}{32.2}\right)\left(45.1^2 - 30^2\right) \text{ lb}\cdot\text{ft} = 2.64 \text{ kip}\cdot\text{ft}$$


END OF COURSE MATERIAL!!