PHYS 170 Section 101
Lecture 28
November 16, 2018

Nov 16—Announcements

- Evaluation of teaching assistants during next (final) tutorial, Tuesday, November 20
  - Bring lead pencil to fill out Scantron forms
  - Come prepared with comments as appropriate
- Will be happy to field questions concerning midterm 2 grading after class today

Lecture Outline/Learning Goals

- Finish worked example problem solved using principle of work and energy
- 14.5 Conservative Forces and Potential Energy
- Sample problem using principle of work and energy, potential energy
- 14.6 Conservation of Energy
- NOTE: 14.4 Power and Efficiency—self study, no exam problems on it

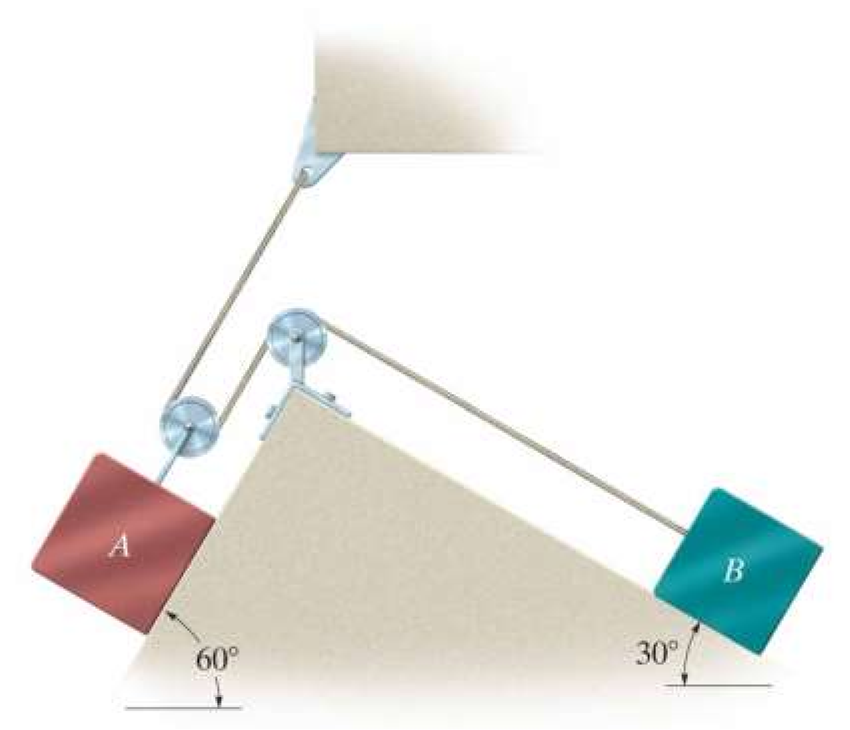
Problem 14-16 (Page 186, 13th edition)

Block B is given an initial speed down the plane. Block A moves up the plane. Both blocks eventually come to rest. The mass of A is 70 kg. The mass of B is 40 kg. The coefficient of kinetic friction between A and the inclined plane is 0.20. The coefficient of kinetic friction between B and the inclined plane is 0.05.

(1) Determine the initial speed of B in order that A travels 2 m up the plane before coming to rest.

(2) Determine the tension in the cord during the motion.

(3) Show that the tension force does not contribute to the total work done on the system.



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Work done by the tension force

Block A: $2T\Delta s_A$

Block B: $-T\Delta s_B$

Since we have $2\,\Delta s_A = \Delta s_B$ (equation (1)), it follows that the tension force does not contribute to the total work done on the system.

How many first year engineering students does it take to change a light bulb?

None. That's a second year subject.

How many second year engineering students does it take to change a light bulb?

Will this question be on the final examination?

How many electrical engineers does it take to change a light bulb?

None. They simply redefine darkness as the industry standard.

14.5 Conservative Forces and Potential Energy

CONSERVATIVE FORCE

- **Definition:** A force is called **conservative** if the work that it does on a particle as the particle moves from one point to another is **independent of the path** that the particle travels (i.e. the work done depends only on the location of the initial and final points)

- **Examples of conservative forces:**

  - **Weight:** Work done by weight is **independent of the path** since it only depends on the relative vertical displacement of the initial and final points

  - **(Elastic) Spring force:** The work done by an elastic spring is **independent of the path** since it only depends on how much the spring is extended (compressed) as the particle to which the spring is attached moves from its initial point to its final point

CONSERVATIVE FORCE (continued)

- **Examples of non-conservative forces:**
  - **(Kinetic) friction:** This is the definitive example of a non-conservative force. The work done by such a force is clearly path dependent in the sense that given an initial point and a final point, the longer the path taken by the particle in moving between the two points, the more (negative) work will be done by the frictional force. As mentioned in the last lecture, some fraction of this work will be converted into heat.

POTENTIAL ENERGY

- One way of defining **energy** is the **capacity to do work**

- When discussing the dynamics of particles, we generally identify two types of energy

  1. **KINETIC ENERGY:** Energy associated with motion

  2. **POTENTIAL ENERGY:** Energy associated with position

     - Specifically, energy that is associated with the position of a particle measured relative to a fixed datum or reference plane

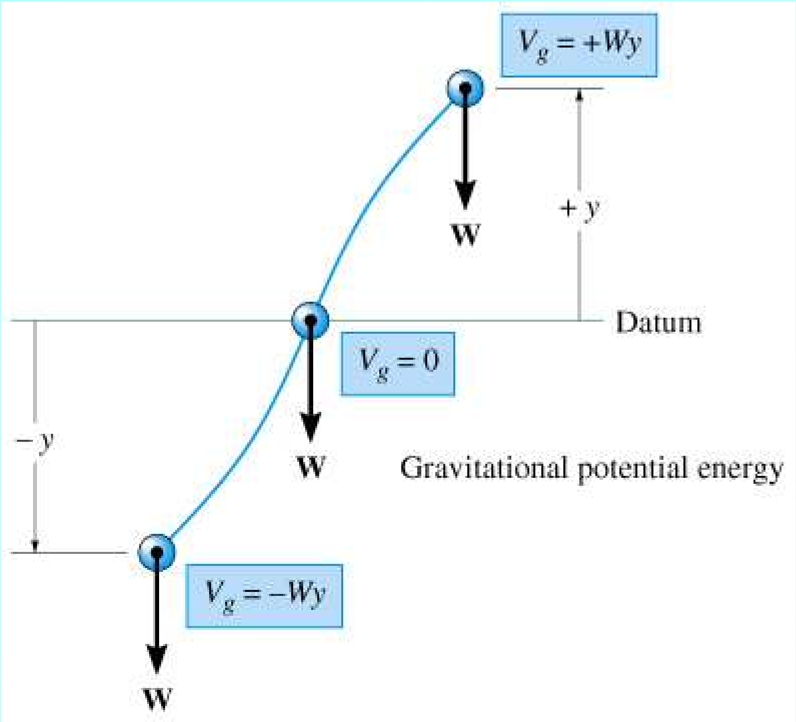
     - This type of energy is a measure of how much work a conservative force does on a particle, when the particle moves **from** a specific location **to** the datum

POTENTIAL ENERGY (continued)

- In basic studies of dynamics, such as ours, we generally encounter two types of potential energy

  1. **Gravitational potential energy**

  2. **Elastic potential energy** (potential energy associated with compression or elongation of an elastic spring)



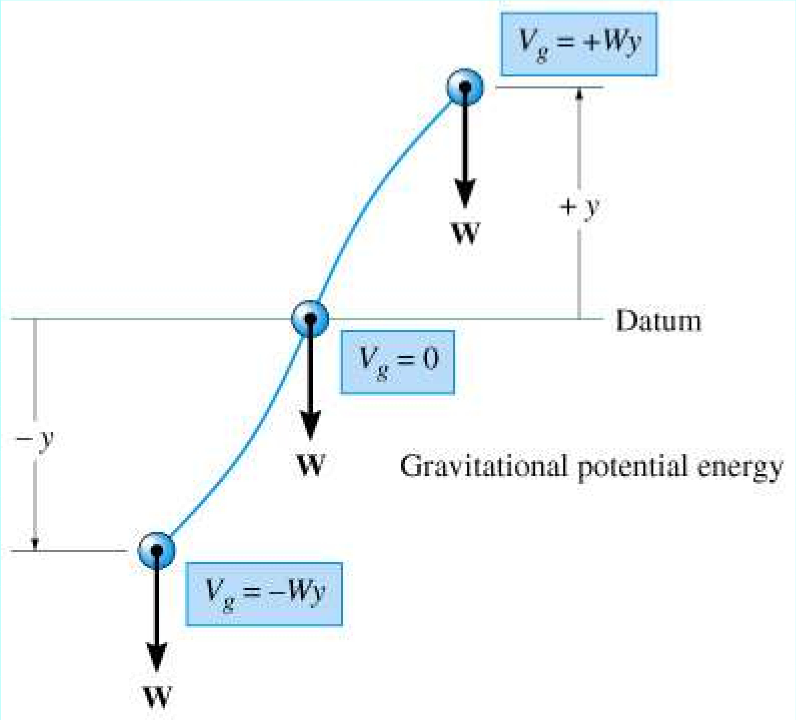
GRAVITATIONAL POTENTIAL ENERGY

- Consider the figure at left; when the particle is at the top position, it has a vertical displacement +*y* above the datum (which can be placed at an arbitrary vertical location)

- Its weight **W**, thus has **positive** potential energy, V_g , since **W** will do **positive** work on the particle if the particle moves to the datum

- Similarly, when the particle is at the bottom position its weight has **negative** potential energy (i.e. $V_g < 0$) since **W** will do **negative** work on the particle if the particle moves to the datum

- If the particle is **at** the datum, then we have $V_g = 0$



GRAVITATIONAL POTENTIAL ENERGY

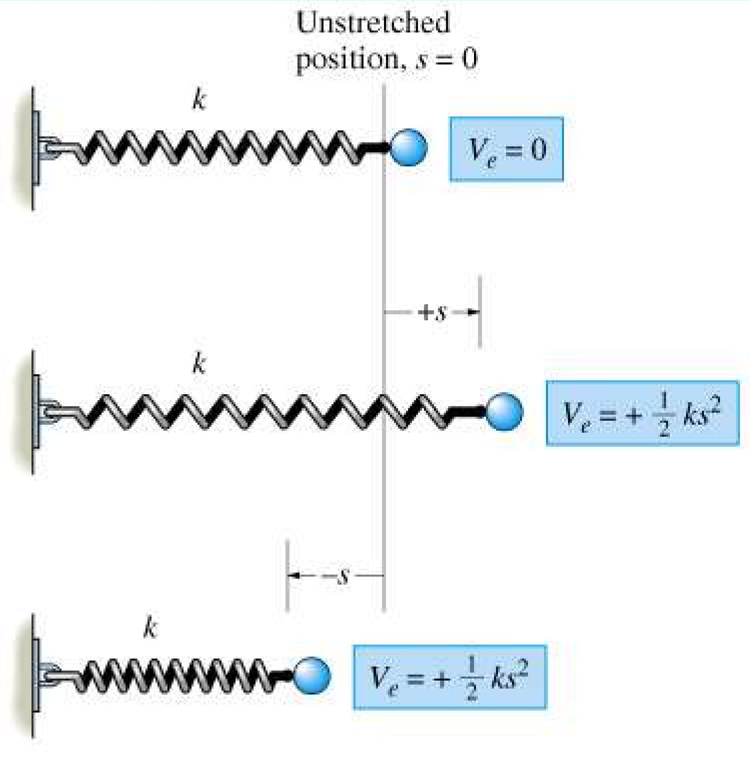
- In general then, assuming that ***y* is positive upward** (and that the vertical displacement is small compared to the radius of the Earth, so we can assume that g is constant), the gravitational potential energy, V_g , of a particle of weight W is

$$V_g = Wy$$

or

$$V_g = mgy$$

where m is the mass of the particle



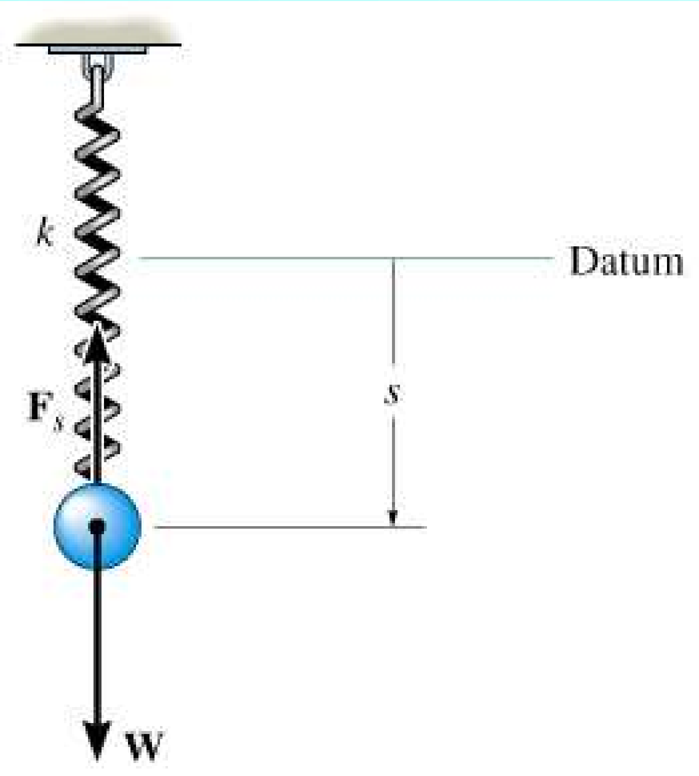
Elastic potential energy

ELASTIC POTENTIAL ENERGY

- An elastic (ideal) spring that is compressed or elongated a distance *s* from its equilibrium position has an **elastic potential energy**, V_s , given by

$$V_s = +\frac{1}{2}ks^2$$

- Note that V_s is **always positive** since whether the spring is compressed or elongated from equilibrium, it will do positive work on a particle attached to it, as the particle returns to the datum (i.e. as the spring returns to its unstretched/equilibrium position)



POTENTIAL FUNCTION (TOTAL POTENTIAL ENERGY)

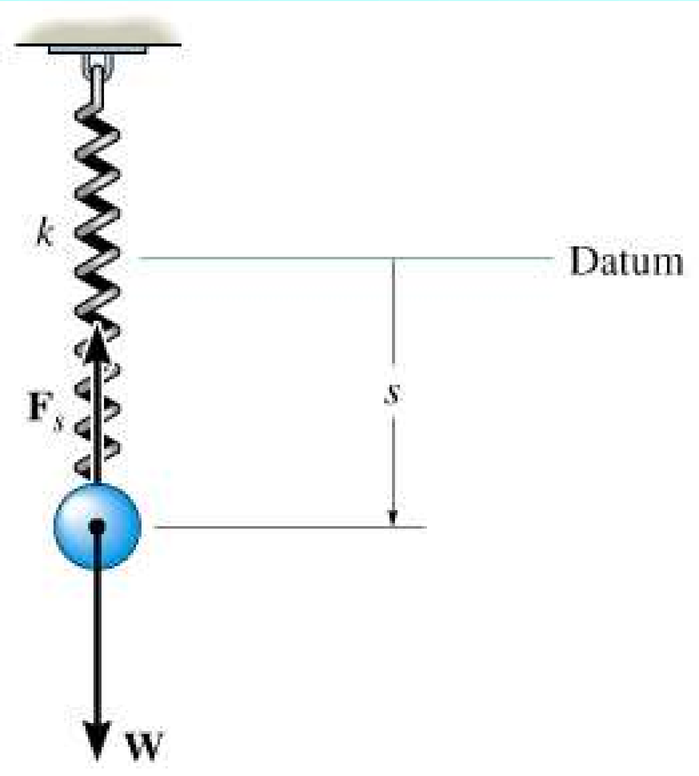
- In a general case, a particle can have both gravitational and elastic forces acting on it, as shown in the figure.

- In this case we can simply add V_g and V_s algebraically (they are both scalars), to get a **potential function**, V, defined by

$$V = V_g + V_s$$

- Bear in mind that both types of potential energy are defined with respect to their specific datum (arbitrary vertical position for V_g , equilibrium spring position for V_s)

- In general, if our particle has a location (x,y,z) in space, we will have $V = V(x,y,z)$, and hence the terminology potential function



POTENTIAL FUNCTION

Relation to work:

- The work U_{1-2} done by conservative forces (gravitational/elastic) when a particle moves from position 1 to position 2 can be expressed as the difference in the potential functions at the two points

$$U_{1-2} = V_1 - V_2$$

- For example, in the figure above, the gravitational and spring potential energies are being measured from a common datum, so that we have

$$\begin{aligned} V &= V_g + V_s \\ &= -Ws + \frac{1}{2}ks^2 \end{aligned}$$

POTENTIAL FUNCTION

Relation to work (continued)

- If the particle now moves from s_1 (below the datum) to s_2 (further below the datum), then we have

$$U_{1-2} = V_1 - V_2 = \left(-Ws_1 + \frac{1}{2}ks_1^2 \right) - \left(-Ws_2 + \frac{1}{2}ks_2^2 \right)$$

$$= W(s_2 - s_1) - \left[\frac{1}{2}ks_2^2 - \frac{1}{2}ks_1^2 \right]$$

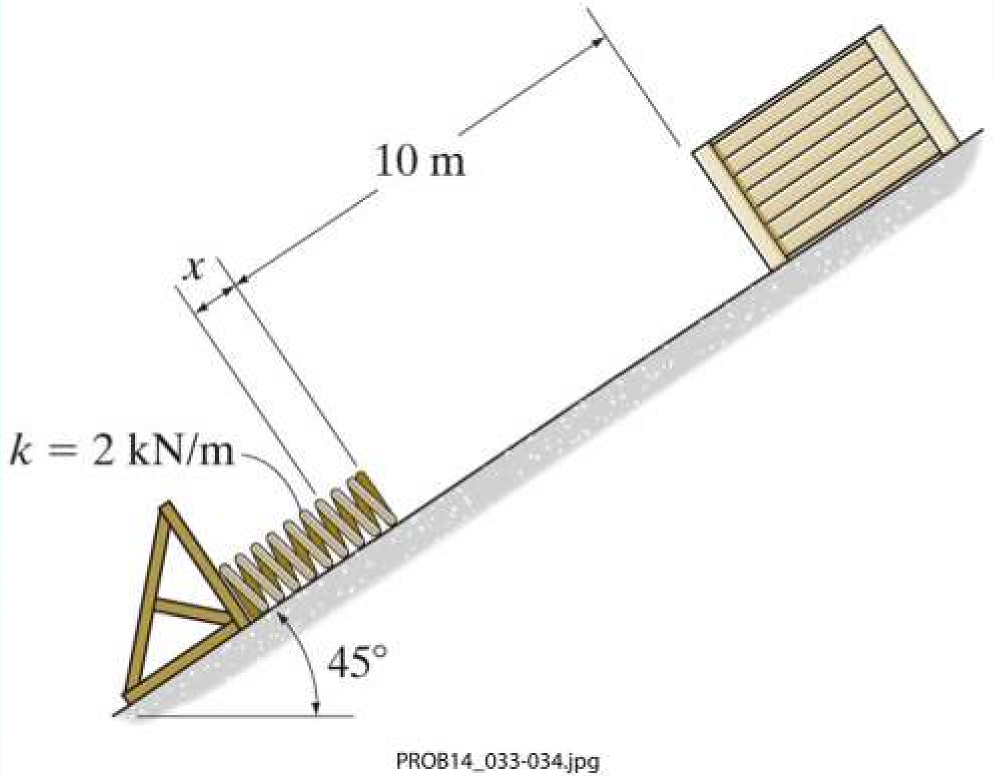
= (work done by gravitational force) + (work done by elastic force)

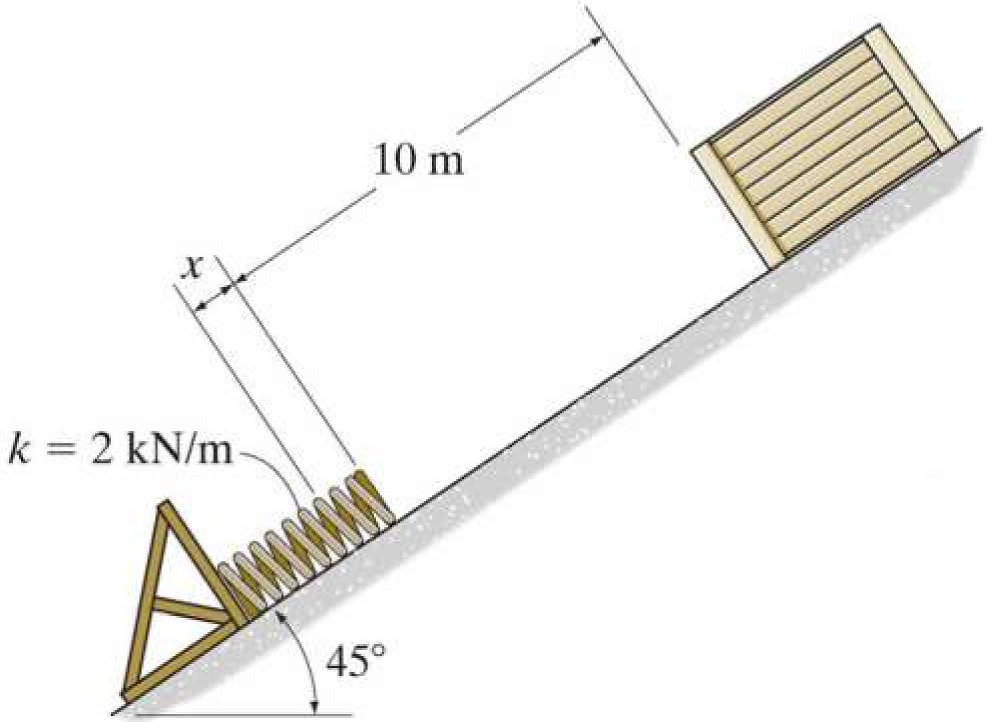
- See the text for the discussion of further implications, including the expression of a conservative force in terms of the negative of the gradient of the potential function

Problem 14-33 (Page 190, 12th edition)

The 100 kg crate slides down the plane. The coefficient of kinetic friction between the crate and the plane is 0.25. The spring is initially unstretched and the crate is initially at rest.

(1) Determine the compression of the spring required to bring the crate momentarily to rest.





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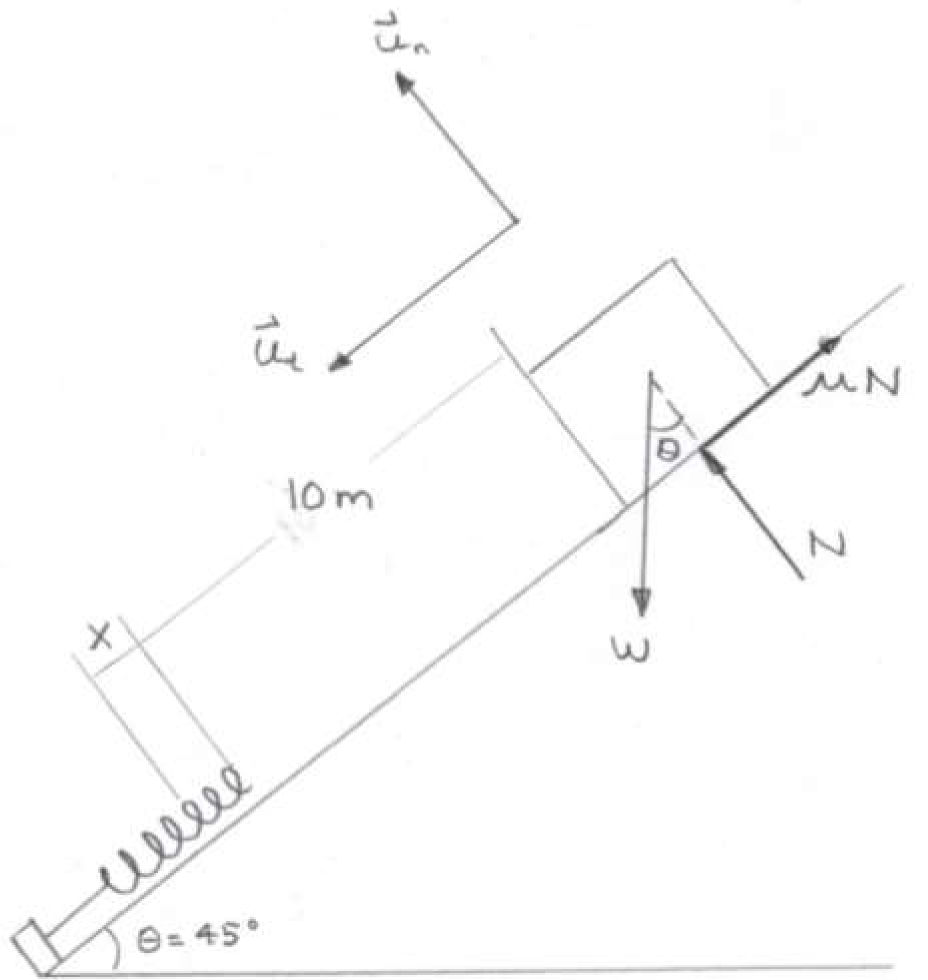

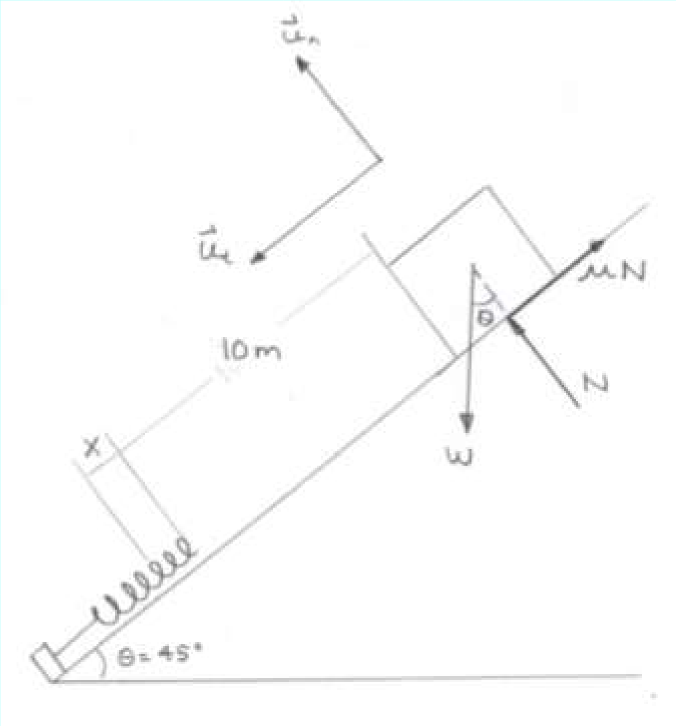

Solution strategy

Use energy balance equation (principle of work energy)

Set 0 of gravitational potential energy at height corresponding to point at which crate is instantaneously at rest

Use equilibrium in normal direction to relate normal force to weight of crate

Data $\vec{u}_t$ points down the plane

$m = 100$ kg $\quad \theta = 45^\circ \quad L = 10$ m

$\mu = 0.25 \quad k = 2000$ N/m

$h_1 = (L + x)\sin\theta \quad h_2 = 0$

$v_1 = 0 \quad v_2 = 0$

Energy Balance Equation

$$\frac{1}{2}mv_1^2 + mgh_1 + (U_{\text{other}})_{1-2} = \frac{1}{2}mv_2^2 + mgh_2 + \frac{1}{2}kx^2$$

$(U_{\text{other}})_{1-2}$ is the work done by friction

Using data values we have

$$mg(L + x)\sin\theta - \mu N(L + x) = \frac{1}{2}kx^2 \qquad (1)$$

Equilibrium in Normal Direction

$$N = mg\cos\theta$$

Substituting in (1) and evaulating numerical values we have

$$100(9.81)(\sin 45^\circ - 0.25\cos 45^\circ)(10 + x) = \frac{1}{2}(2000)x^2$$

This is a quadratic equation in *x*. Either use quadratic formula or "solver" on calculator to find

$$x = 2.56\text{ m}$$

14.6 Conservation of Energy

- Consider a particle which has both conservative and non-conservative forces acting on it

- Then we can write the Principle of Work and Energy in the form

$$T_1 + (\Sigma U_{1-2})_{\text{conservative}} + (\Sigma U_{1-2})_{\text{non-conservative}} = T_2$$

- But we have seen that

$$(\Sigma U_{1-2})_{\text{conservative}} = V_1 - V_2$$

so we have

$$T_1 + (V_1 - V_2) + (\Sigma U_{1-2})_{\text{non-conservative}} = T_2$$

or

$$T_1 + V_1 + (\Sigma U_{1-2})_{\text{non-conservative}} = T_2 + V_2$$

CONSERVATION OF ENERGY (continued)

- Our last formula was

$$T_1 + V_1 + (\Sigma U_{1-2})_{\text{non-conservative}} = T_2 + V_2$$

- We now consider the special but important case when **only** conservative forces act on the particle

- We then have **conservation of mechanical energy** or simply **conservation of energy**, stated as

$$T_1 + V_1 = T_2 + V_2$$

- In words: For a particle acted on solely by conservative forces, the sum of the particle's kinetic and potential energy is **constant** during the motion of the particle

CONSERVATION OF ENERGY (continued)

- An immediate implication of this important principle – which may be familiar to you from previous physics courses – is that if a particle's kinetic energy increases by a certain amount as it moves, then its potential energy must decrease by that precise amount and vice versa

SYSTEMS OF PARTICLES

- The extension of conservation of energy to systems of particles is straightforward – provided that the particles are acted on by **only conservative forces**, then we have

$$\Sigma T_1 + \Sigma V_1 = \Sigma T_2 + \Sigma V_2$$

  where the summations are over all the particles

- Again, it is crucial to bear in mind that conservation of energy can **only** be applied when all forces are conservative – for our purposes, gravitational and elastic – and thus problems involving friction e.g. are specifically excluded!

PROBLEM SOLVING USING CONSERVATION OF ENERGY

- Use conservation of energy to solve problems involving velocity, displacement and conservative forces

- Generally easier to use than PWE since involves quantities at only two specific points (typically initial and final points of a particle’s path), rather than requiring the computation of work done by forces through the displacement along the path

- Be careful with signs when dealing with gravitational potential energy, and keep in mind that elastic (spring) potential energy is always positive

- May still be beneficial to draw free body diagrams, as we will see in the example that follows