PHYS 170 Section 101
Lecture 27
November 14, 2018

Nov 14—Announcements

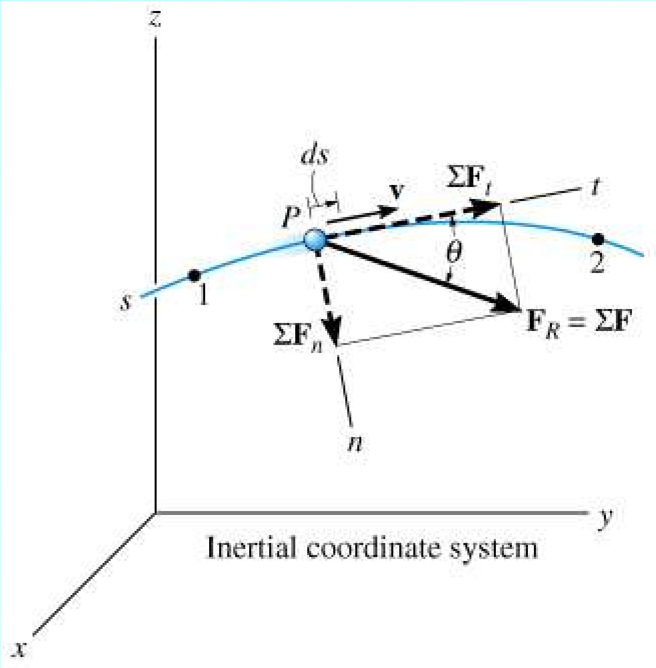
- Will hand midterm 2 back at the end of today’s class
- Average was just under 70%

Lecture Outline/Learning Goals

- Chapter 14.3: Principle of Work & Energy for a System of Particles
- Worked example of problem solved using principle of work and energy

The physics of extracting yourself from a hole

14.3 Principle of Work & Energy for a System of Particles



REVIEW from last lecture (single particle)

$$\Sigma \int_{s_1}^{s_2} F_t \, ds = \Sigma U_{1-2} = \int_{v_1}^{v_2} mv \, dv = \underbrace{\frac{1}{2} m v_2^{\,2}}_{T_2} - \underbrace{\frac{1}{2} m v_1^{\,2}}_{T_1} \qquad (1)$$

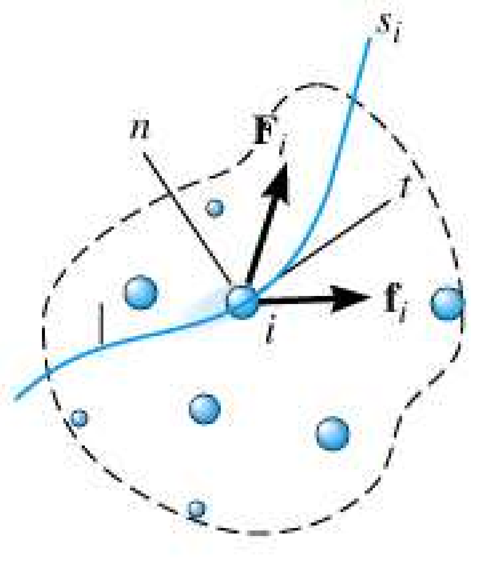
Principle of work & energy

$$\boxed{T_1 + \Sigma U_{1-2} = T_2}$$

$$a_t = \frac{dv}{dt} = \frac{dv}{ds}\frac{ds}{dt} = \frac{dv}{ds} v = v\frac{dv}{ds} \quad \Rightarrow \quad a_t ds = v dv$$

$$\Sigma F_t = m a_t$$

$$a_t \, ds = v \, dv \quad \Rightarrow \quad F_t \, ds = m a_t ds = mv \, dv$$



Inertial coordinate system

PWE for a system of particles (continued)

- We can generalize the principle to a system of particles as shown in the figure at left

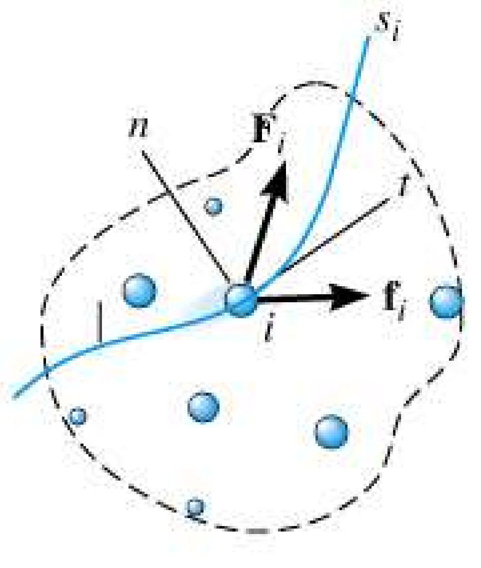
- Consider i-th particle, with mass m_i , and acted on by the following forces

$$\mathbf{F_i} \equiv \text{ resultant external force}$$

$$\mathbf{f}_i \equiv \text{ resultant internal force due to interactions with other particles}$$

- Applying equation (1) from above, which is valid in the tangential direction, we have

$$\frac{1}{2} m_i v_{i1}{}^2 + \int_{s_{i1}}^{s_{i2}} (F_i)_t \, ds + \int_{s_{i1}}^{s_{i2}} (f_i)_t \, ds = \frac{1}{2} m_i v_{i2}{}^2$$



PWE for a system of particles (continued)

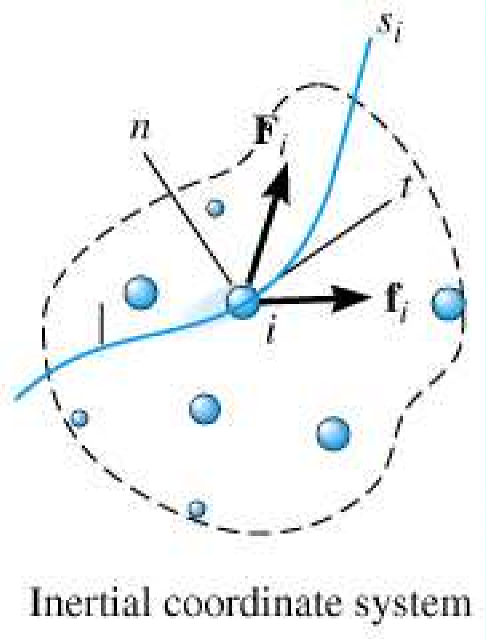
- We get analogous equations for all the other particles; since work and energy terms are all scalars, we can add them algebraically to get

$$\sum_i \frac{1}{2} m_i v_{i1}{}^2 + \sum_i \int_{s_{i1}}^{s_{i2}} (F_i)_t \, ds + \sum_i \int_{s_{i1}}^{s_{i2}} (f_i)_t \, ds = \sum_i \frac{1}{2} m_i v_{i2}{}^2$$

which we can write as

$$\sum_i T_1 + \sum_i U_{1-2} = \sum_i T_2$$

- In words: The initial kinetic energy of the system (ΣT_1), plus the work done by all the external and internal forces acting on the particles (ΣU_{1-2}), equals the final kinetic energy of the system (ΣT_2).



PWE for a system of particles (continued)

- Now, as we have noted before when discussing systems of particles, the internal forces come in action-reaction pairs

- Since each such pair is comprised of vectors of the form $\mathbf{f_i},\ -\mathbf{f_i}$ it is tempting to conclude that the total work done on the particle system due to these forces will vanish (recall that work can have either sign)

- However, this is **not** the case in general, since the pair of particles involved in the action-reaction pair of forces may travel **different paths**, in which case the total work done by the force pair is **not necessarily 0**

- Conversely, though, if all of the pairwise interacting particles in a system **do** travel equivalent paths (i.e. experience the same displacements), then we **can** safely ignore internal forces in our application of the principle of work and energy

PWE for a system of particles (continued)

- For our purposes, there are two key situations where this is so

  1. When the particles comprise a **rigid** body that is **translating**

  2. When the particles are connected by ideal (i.e. inextensible) cords/cables (pulley-block systems)

- For notational simplicity, when working problems involving more than one particle, we will generally **assume** the summations over particles, i.e. we will write

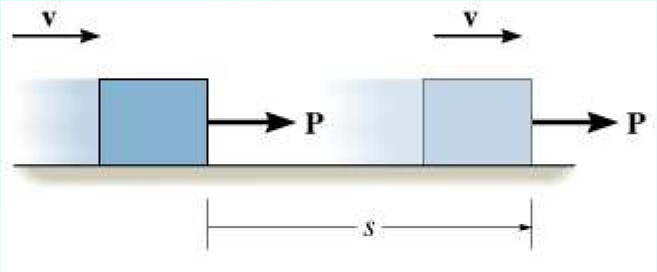
$$T_1 + \Sigma U_{1-2} = T_2 \qquad (\Sigma \rightarrow \text{ summation over forces doing work})$$

instead of

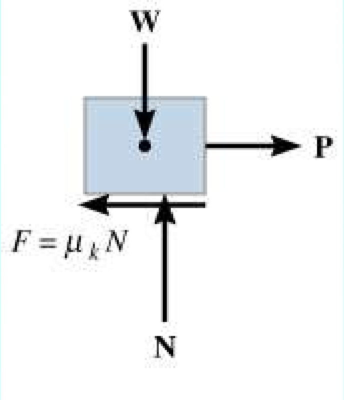
$$\sum_i T_1 + \sum_i U_{1-2} = \sum_i T_2$$

Work of Friction Caused by Sliding

- We can incorporate the work done by kinetic frictional forces into the principle of work and energy, but we have to be careful about the actual physical interpretation of this work



- Consider the situation shown in the figure at left where a force **P** is required to slide the block over a surface, with the interface characterized by a coefficient of kinetic friction, μ_k



- Then, considering the FBD for the block, we have $P = \mu_k N$ and we might expect that the PWE would give us

$$\frac{1}{2}mv^2 + Ps - \mu_k Ns = \frac{1}{2}mv^2$$

- However, we know that frictional forces generate heat, which is a form of energy **not** apparently accounted for in the above equation

Work of Friction Caused by Sliding

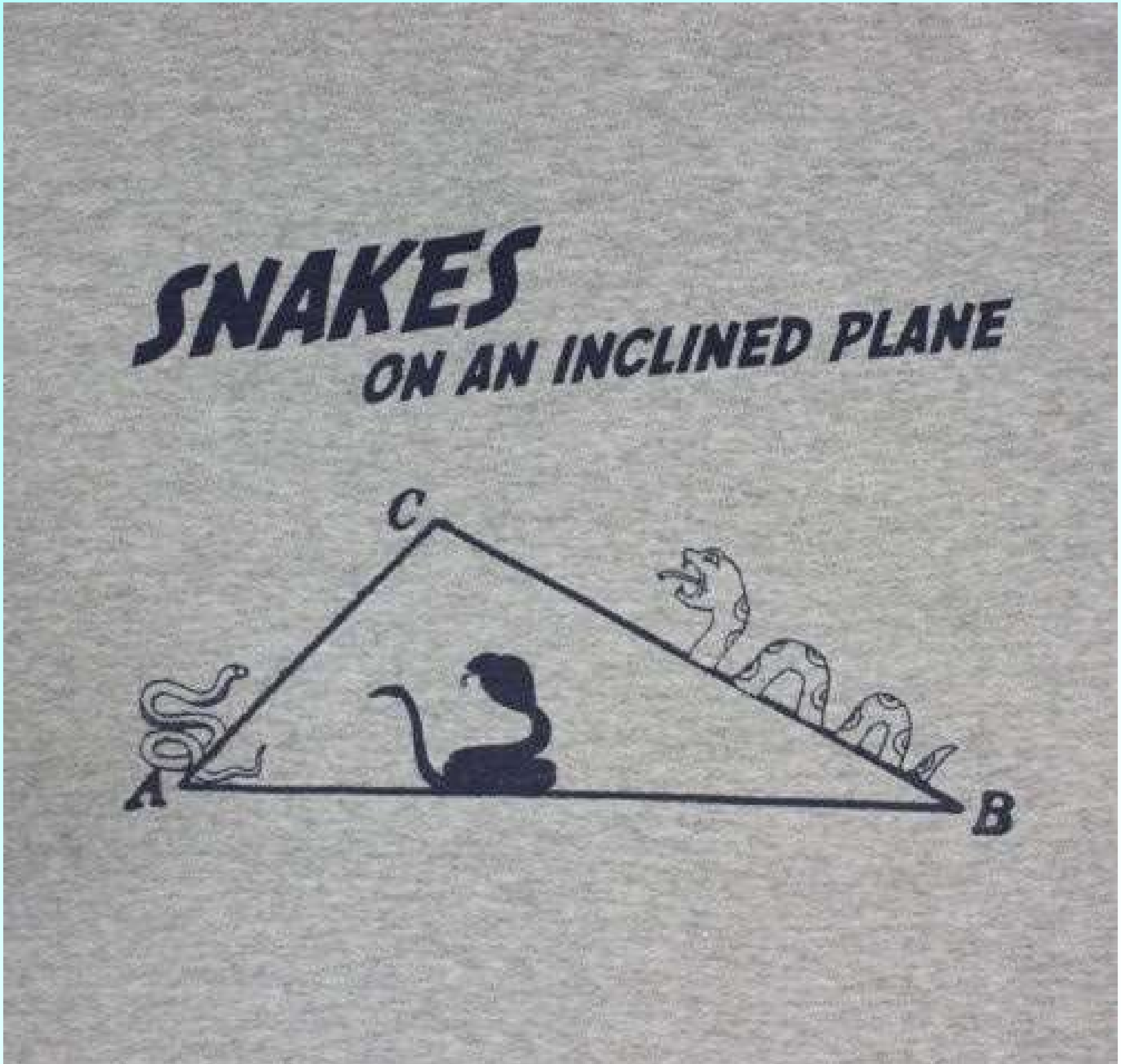
- As discussed in more detail in the text, a proper analysis of the microscopic origin of friction resolves this issue – in a nutshell, the displacement *s* above is **not** the true displacement over which $\mu_k N$ acts. The true displacement, s', is somewhat less that *s* so that we can write

$$\mu_k N s = \mu_k N s' + \mu_k N(s - s')$$

  where the first term represents the **external** work done by the frictional force, while the second term represents the **internal** work done by the force which primarily ends up as heat in the block

- However, having discussed this rather subtle point, we will proceed to essentially ignore it when we solve problems involving kinetic friction using work-energy methods

- I.e. given a kinetic frictional force, $\mu_k N$, and an (apparent) displacement *s*, we will use $\mu_k N s$ for the magnitude of the work done by the force (as in the example which follows)


SNAKES
ON AN INCLINED PLANE
C
A
B

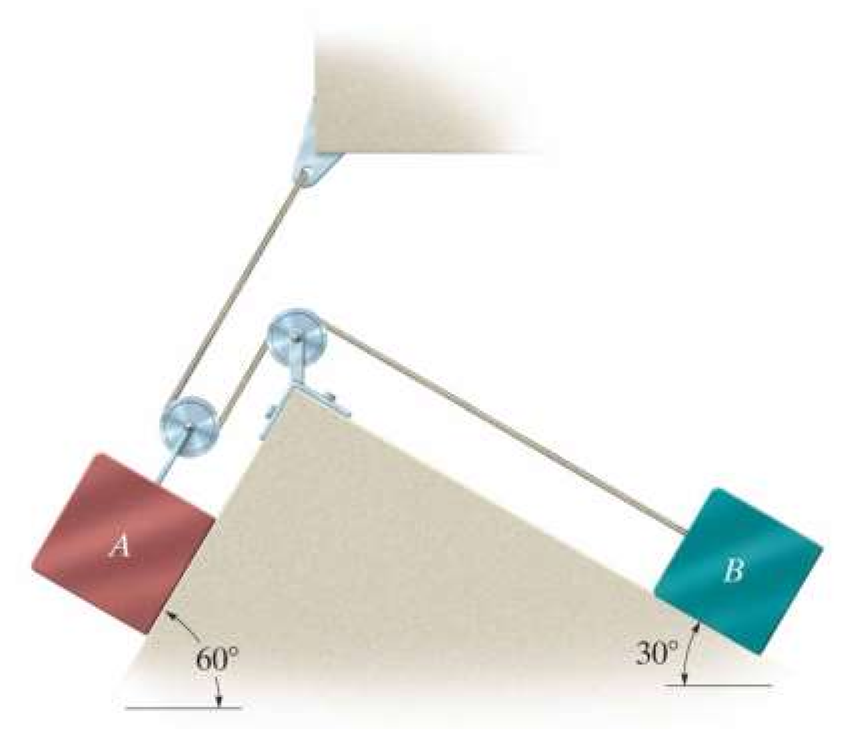
Problem 14-16 (Page 186, 13th edition)

Block B is given an initial speed down the plane. Block A moves up the plane. Both blocks eventually come to rest. The mass of A is 70 kg. The mass of B is 40 kg. The coefficient of kinetic friction between A and the inclined plane is 0.20. The coefficient of kinetic friction between B and the inclined plane is 0.05.

(1) Determine the initial speed of B in order that A travels 2 m up the plane before coming to rest.

(2) Determine the tension in the cord during the motion.

(3) Show that the tension force does not contribute to the total work done on the system.



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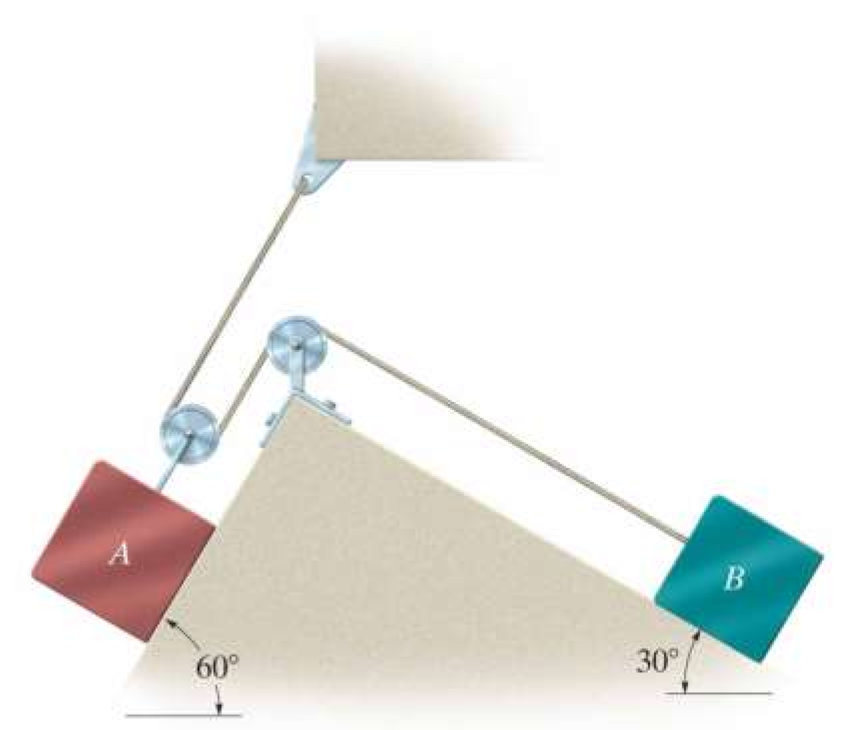

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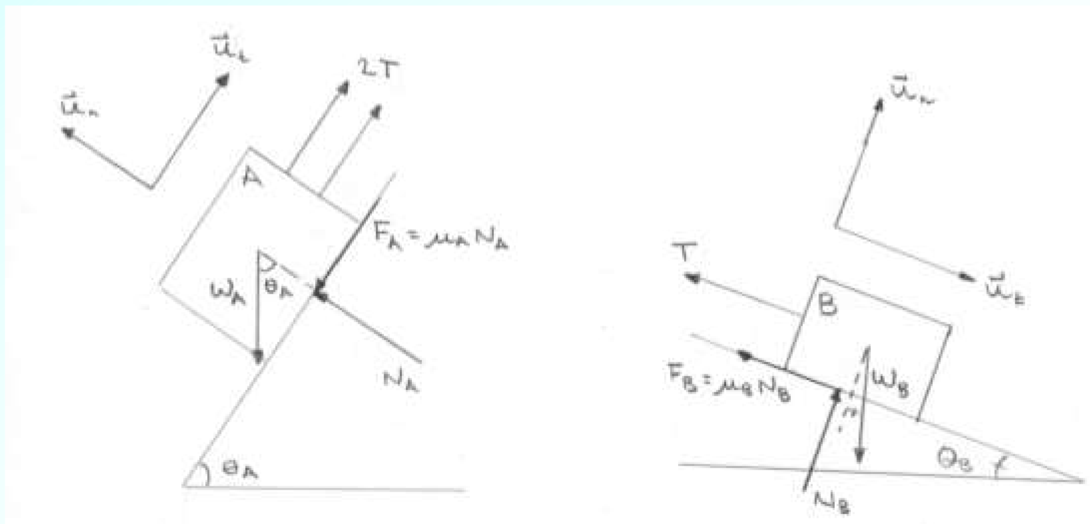
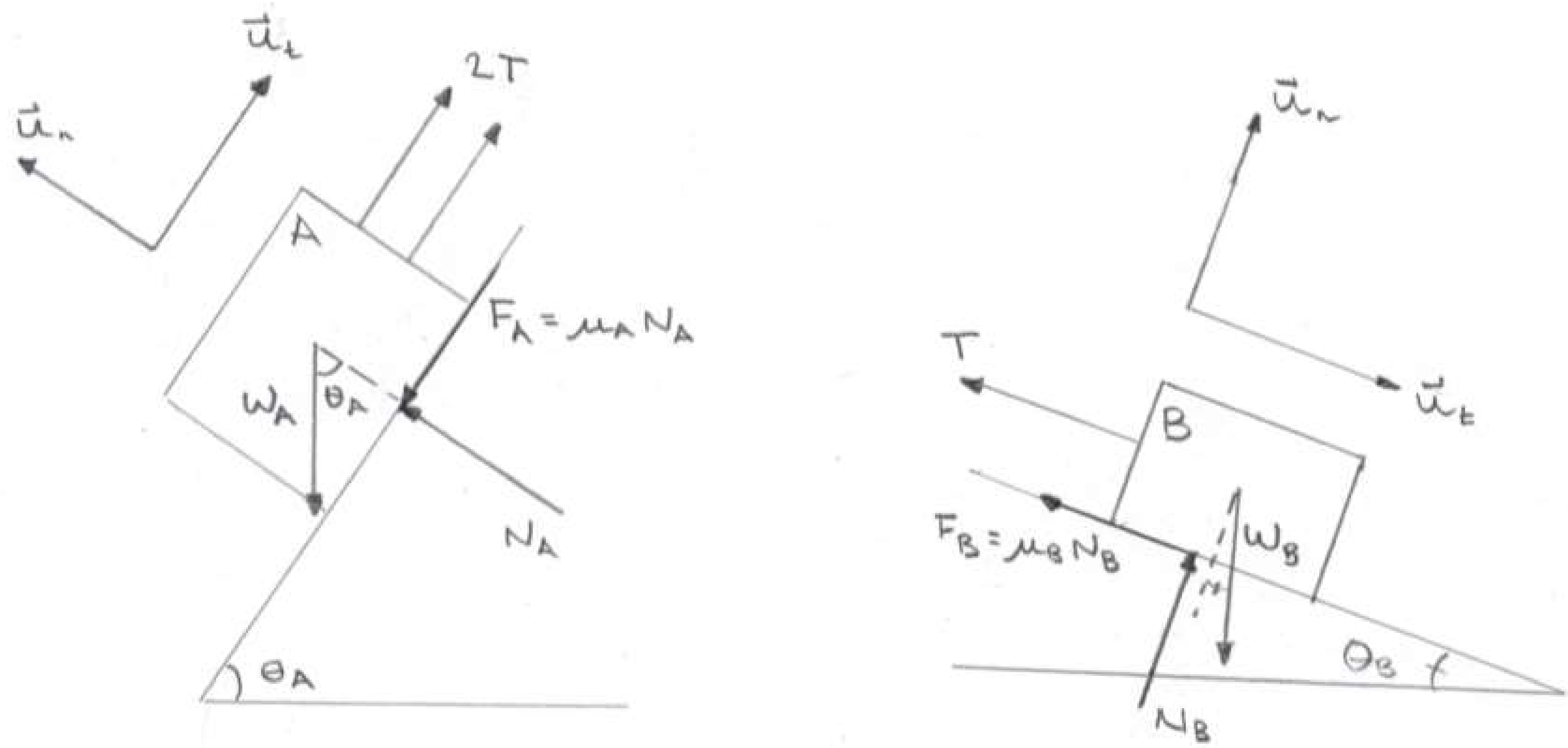
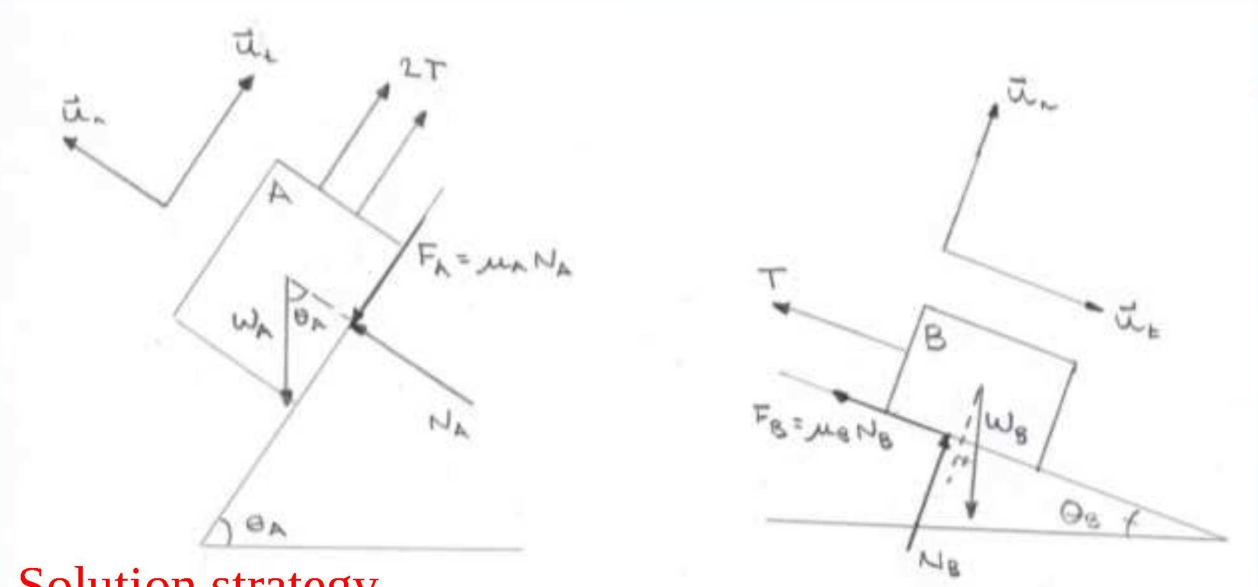




$\vec{u}_t$
$\vec{u}_n$
2T
A
$F_A = \mu_A N_A$
W_A
θ_A
N_A
θ_A
$\vec{u}_n$
T
$\vec{u}_t$
B
$F_B = \mu_B N_B$
W_B
θ_B
N_B



Solution strategy

Determine dependent motion of two blocks via pulley system

For each block write down (a) energy balance equation for motion along block (tangential direction) and (b) equilibrium eqn in normal direction (no motion along $\vec{u}_n$)

Manipulate equations and solve for initial speed of block B, v_{B1}

Further manipulate equations and solve for tension, T. Show that T does not contribute to total work done on system

Data

Block A: $\vec{u}_t$ points up the plane

$m_A = 70\text{ kg}$ $\theta_A = 60^\circ$ $\mu_A = 0.2$

$h_{A1} = 0$ $h_{A2} = \Delta s_A \sin\theta_A$ $\Delta s_A = s_{A2} - s_{A1} = 2\text{ m}$

$v_{A2} = 0$

Block B: $\vec{u}_t$ points down the plane

$m_B = 60\text{ kg}$ $\theta_B = 30^\circ$ $\mu_B = 0.05$

$h_{B1} = \Delta s_B \sin\theta_B$ $h_{B2} = 0$ $\Delta s_B = s_{B2} - s_{B1}$

$v_{B2} = 0$

Dependent motion (Exercise: Derive if the equations are not clear to you)

$$2s_A = s_B + \text{constant}$$

$$2\Delta s_A = \Delta s_B \qquad (1)$$

$$2v_{A1} = v_{B1} \qquad (2)$$

Energy Balance Equation

$$\frac{1}{2}mv_1^2 + mgh_1 + (U_{\text{other}})_{1-2} = \frac{1}{2}mv_2^2 + mgh_2$$

where $(U_{\text{other}})_{1-2}$ is the work done by tension and friction forces

Block A: $\vec{u}_t$ points up the plane

Energy Balance

$$\frac{1}{2}m_A v_{A1}^2 + (2T - \mu_A N_A)\Delta s_A = m_A g h_{A2} \tag{3}$$

Equilibrium in n direction (perpendicular to plane)

$$N_A = m_A g \cos\theta_A \tag{4}$$

Block B : $\vec{u}_t$ points down the plane

Energy Balance

$$\frac{1}{2} m_B v_{B1}^2 + m_B g h_{B1} - (T + \mu_B N_B)\Delta s_B = 0 \qquad (5)$$

Equilibrium in n direction (perpendicular to plane)

$$N_B = m_B g \cos\theta_B \qquad (6)$$

Initial speed of B

Add equations (3) and (5). Use equations (1), (2), (4) and (6) as well as $h_{A2} = \Delta s_A \sin\theta_A$ and $h_{B1} = \Delta s_B \sin\theta_B$.

Solve for v_{B1} (Exercise: Do the algebra!)

$$v_{B1} = \left\{ \frac{8\left[m_A(\sin\theta_A + \mu_A\cos\theta_A) + 2m_B(\mu_B\cos\theta_B - \sin\theta_B)\right] g\Delta s_A}{m_A + 4m_B} \right\}^{1/2}$$

$$= 2.55 \text{ m/s}$$

Tension in the cord

Solve equation (5) for T. Use (1), (6), $h_{B1} = \Delta s_B \sin\theta_B$ (Exercise: Do the algebra)

$$T = m_B g\left(\sin\theta_B - \mu_B\cos\theta_B\right) + \frac{m_B v_{B1}^2}{4\Delta s_A} = 317 \text{ N}$$

Work done by the tension force

Block A: $2T\Delta s_A$

Block B: $-T\Delta s_B$

Since we have $2\,\Delta s_A = \Delta s_B$ (equation (1)), it follows that the tension force does not contribute to the total work done on the system.