

PHYS 170 Section 101  
Lecture 25  
November 5, 2018

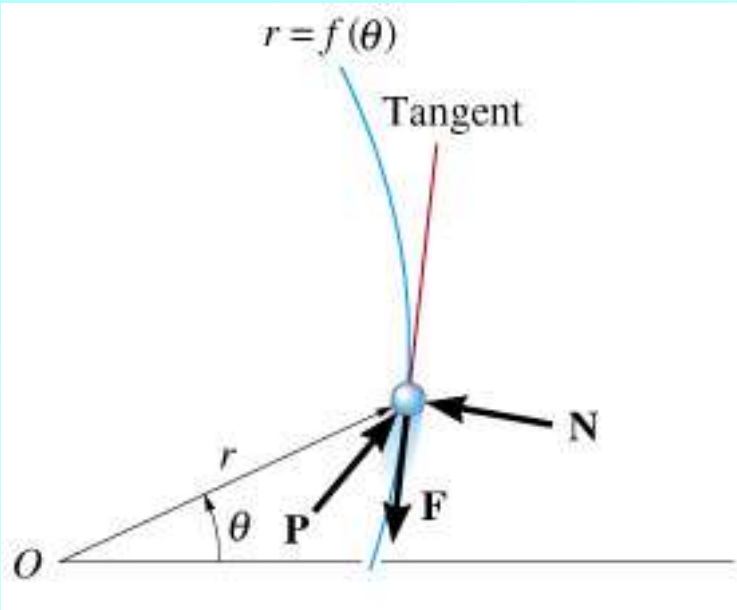
# Nov 5—Announcements

- Midterm review sessions in tutorial slots tomorrow
- Current homework due Tuesday, November 13, 11:59 PM
- Lecture 21 now includes a solution of problem 12.204 from the 8<sup>th</sup> homework assignment (the pulley question)

# Lecture Outline/Learning Goals

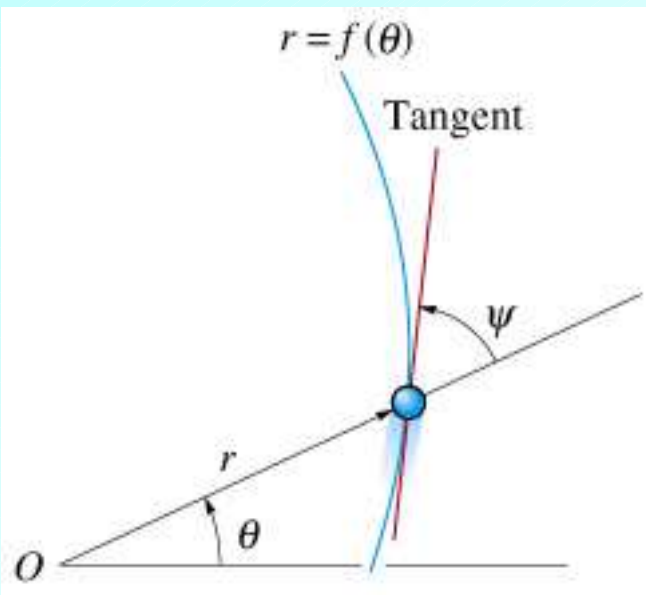
- Relationship between tangential-normal and polar coordinates
- Two worked examples of motion in polar coordinates
- One really bad engineering joke

# TANGENTIAL AND NORMAL FORCES



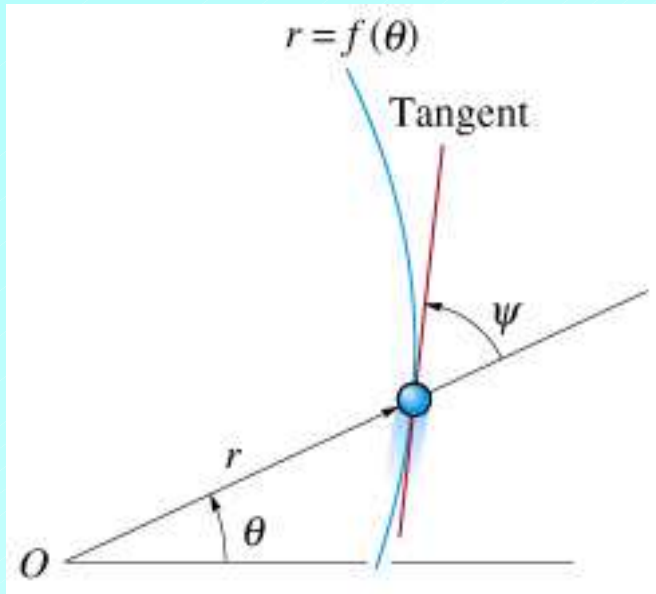
Many of the problems in this part of the course have the following features

- They are natural to treat in polar coordinates
- The path of the particle is specified (constrained motion)
- Some of the forces on the particle act in the **normal** or **tangential** directions



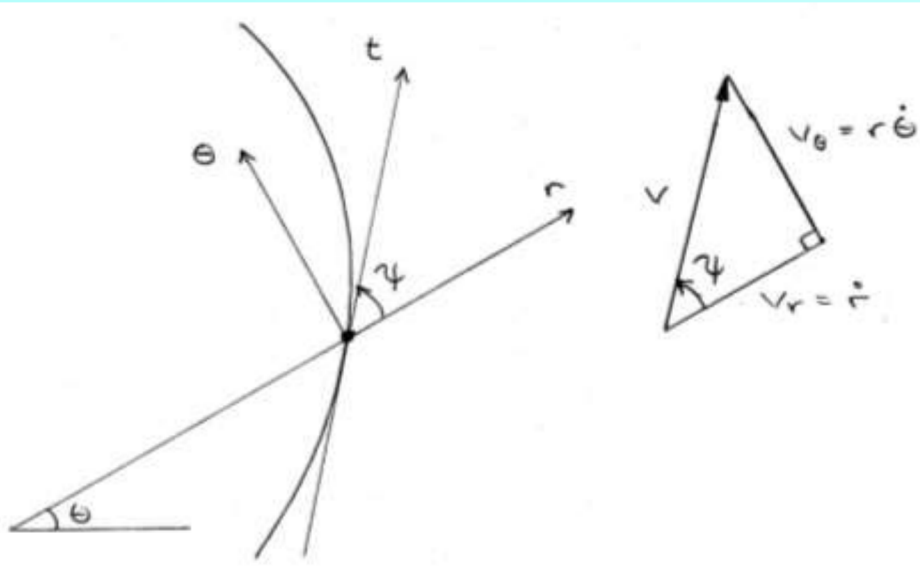
- We thus need to be able to determine the orientation of the  $(t, n)$  coordinate system with respect to the  $(r, \theta)$  system
- As shown in the figure, the tangent to the particle path will form an angle  $\psi$  with the **extended** radial line

## TANGENTIAL AND NORMAL FORCES (continued)



- Given the equation for the path in the form  $r = f(\theta)$ , the text derives an equation for  $\psi$ ; will not reproduce that derivation here
- Rather, we derive the formula using a different construction that is based on the observation that the velocity of the particle is **always tangent to the particle path**

## TANGENTIAL AND NORMAL FORCES (continued)



- We can then resolve the velocity vector into its  $r$  and  $\theta$  components as shown in the figure
- Clearly, we have

$$\tan \psi = \frac{v_{\theta}}{v_r} = \frac{r\dot{\theta}}{\dot{r}}$$

so that the **tangent of the angle between the  $r$  and  $t$  axes is given by the ratio of the  $\theta$  and  $r$  components of the velocity**

- We can manipulate this as follows

$$\tan \psi = \frac{r\dot{\theta}}{\dot{r}} = \frac{r}{\dot{r}/\dot{\theta}} = \frac{r}{(dr/dt) \times (dt/d\theta)}$$

so we have our final result

$$\tan \psi = \frac{r}{dr/d\theta}$$

## TANGENTIAL AND NORMAL FORCES (continued)

- From

$$\tan \psi = \frac{r}{dr / d\theta}$$

it follows immediately that

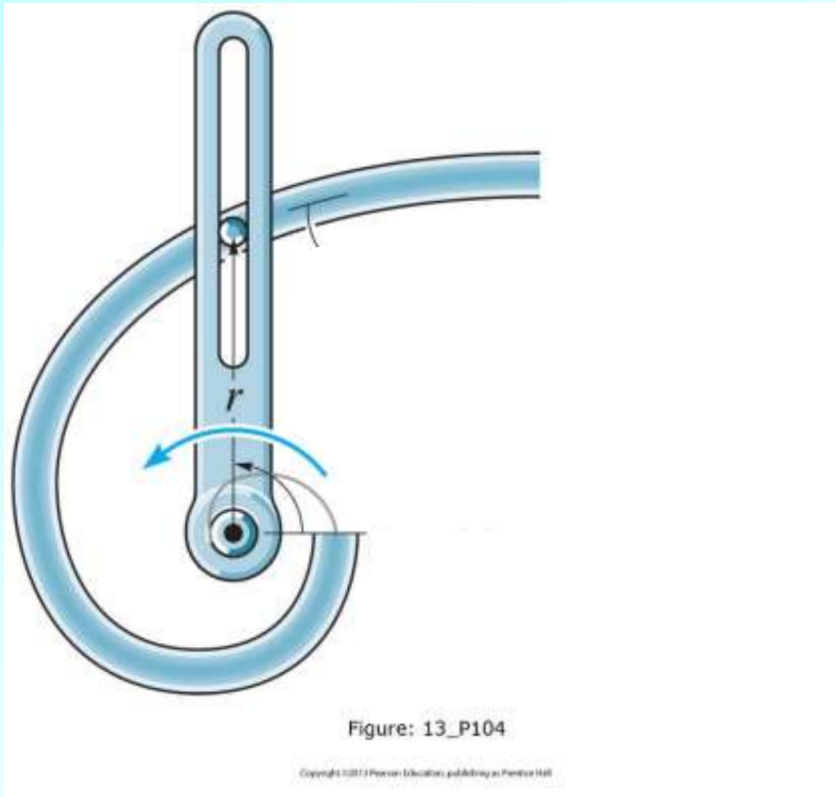
$$\psi = \tan^{-1} \left[ \frac{r}{dr / d\theta} \right]$$

- If the angle computed in this fashion is positive, then it is measured counterclockwise from the extended radial line to the tangent line; conversely, if the computed angle is negative, it is measured clockwise from the radial line
- That is, the sense of  $\psi$  is the same as the sense of  $\theta$  (the positive direction of  $\theta$  is counterclockwise)

## Problem 13-104 (Page 153, 13<sup>th</sup> edition)

The slotted rod moves the 4 kg particle around the curved slot whose shape is given by  $r = 0.6 / \theta$  m where  $\theta$  is in radians. **Motion is in the horizontal plane.**

(1) Determine the force that the rod exerts on the particle and the force of the slot on the particle when  $\theta = 90^\circ$ ,  $\dot{\theta} = 0.5$  rad/s and  $\ddot{\theta} = 0.6$  rad/s<sup>2</sup>. Friction may be neglected.





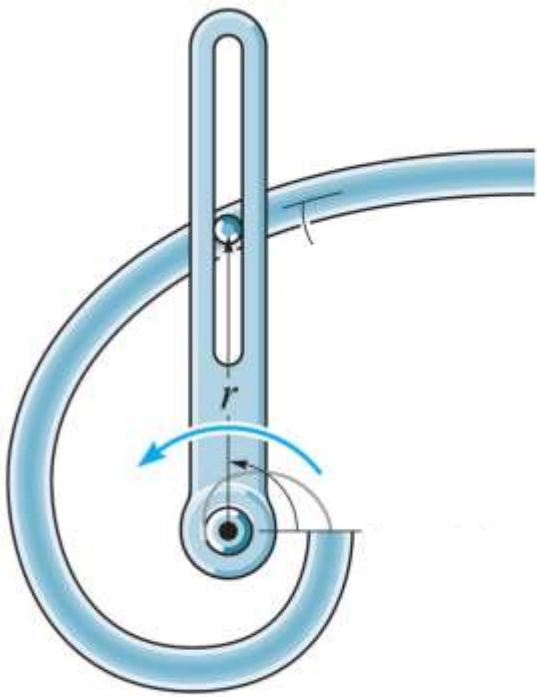
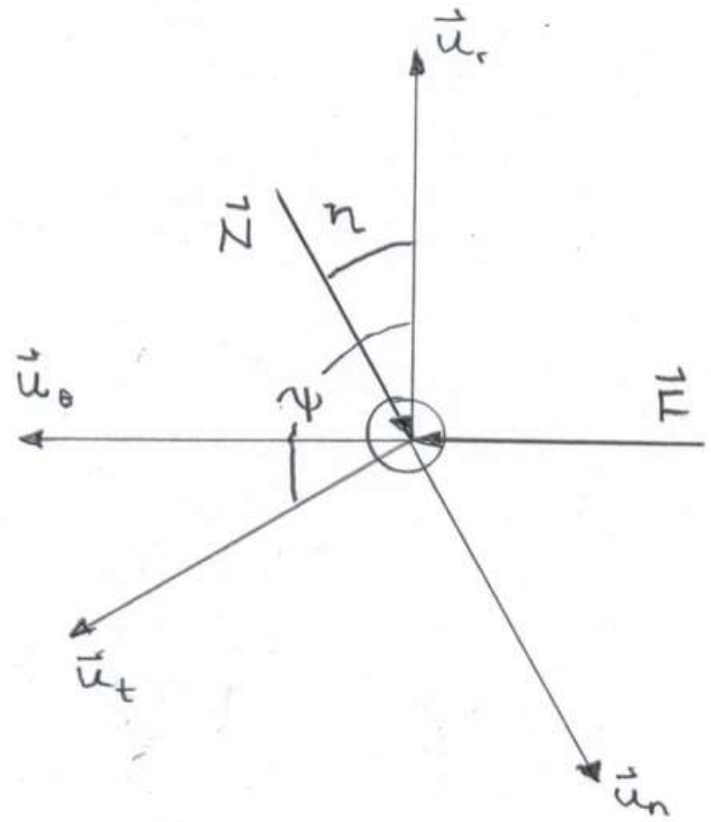


Figure: 13\_P104

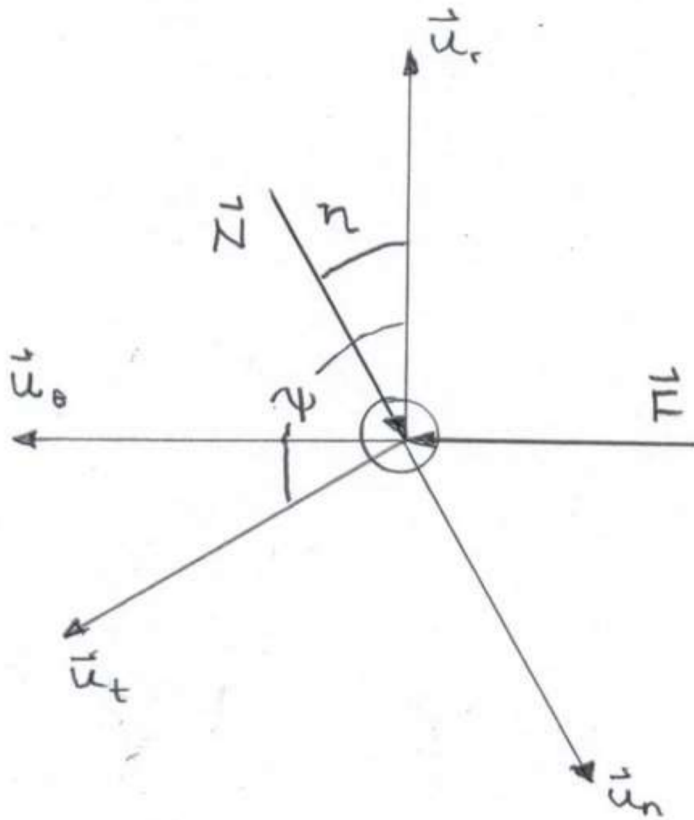
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## Solution strategy

Determine acceleration components in polar coordinates, then determine requested forces. Force that rod exerts on particle is in the  $\theta$  direction, force that the slot exerts on particle is in normal ( $n$ ) direction.

Determination of acceleration components requires computation of various derivatives ( $\dot{r}$ ,  $\ddot{r}$ )



Also need angle  $\psi$  between tangential and radial unit vectors, and angle  $\eta$  between normal and radial unit vectors; calculate from  $\tan \psi = r / (dr / d\theta)$  and  $\eta = \psi - 90^\circ$

## Data

$$m = 4 \text{ kg} \quad r(\theta) = a\theta^{-1} \quad a = 0.6 \text{ m} \quad \theta \text{ in radians}$$

$$\theta = 90^\circ \quad \dot{\theta} = 0.5 \text{ rad/s} \quad \ddot{\theta} = 0.6 \text{ rad/s}^2$$

Derivatives (Exercise: Verify all calculations here and below)

$$\dot{r} = -a\dot{\theta}\theta^{-2} \quad \ddot{r} = -a(\ddot{\theta}\theta^{-2} - 2\dot{\theta}^2\theta^{-3})$$

When  $\theta = 90^\circ$  ( $\pi / 2$  rad)

$$r = 0.3820 \text{ m} \quad \dot{r} = -0.1216 \text{ m/s} \quad \ddot{r} = -0.06850 \text{ m/s}^2$$

# Acceleration

$$\mathbf{a} = a_r \vec{u}_r + a_\theta \vec{u}_\theta$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -0.1640 \text{ m/s}^2 \quad \text{when } \theta = 90^\circ$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.1076 \text{ m/s}^2 \quad \text{when } \theta = 90^\circ$$

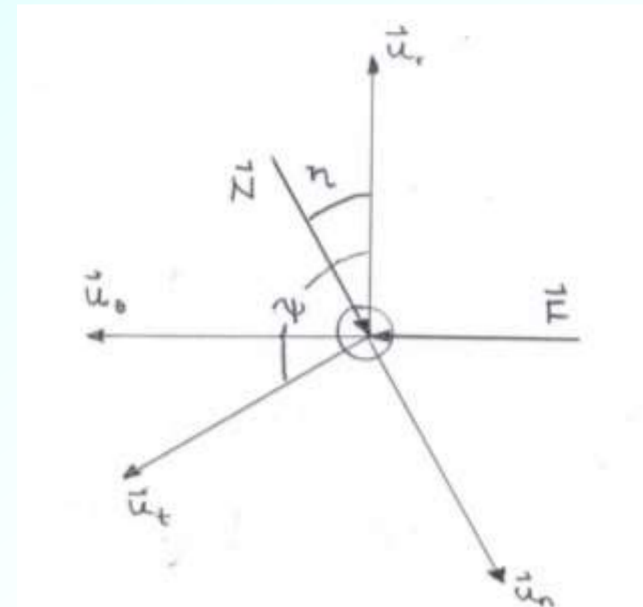
## Angle between tangential and radial unit vectors

$$\tan \psi = \frac{r}{dr/d\theta} = \frac{a\theta^{-1}}{-a\theta^{-2}} = -\theta = -\frac{\pi}{2}$$

$$\psi = \tan^{-1}(-\pi/2) = 122.5^\circ$$

$$\eta = \psi - 90^\circ = 32.48^\circ$$

Note: Calculator returns  $-57.518^\circ$ . Convert to  $122.5^\circ$  (refer to diagram) via  $\tan \theta = \tan(\theta + 180^\circ)$



## Forces

$$\vec{F} = F \vec{u}_\theta$$

$$\vec{N} = N \vec{u}_n = N(-\cos \eta \vec{u}_r - \sin \eta \vec{u}_\theta)$$

## Equations of motion

$$\sum F_r = ma_r : \quad -N \cos \eta = ma_r \quad (1)$$

$$\sum F_\theta = ma_\theta : \quad F - N \sin \eta = ma_\theta \quad (2)$$

Using givens and previously computed values determine  $N$  from (1), then compute  $F$  from (2)

$$N = 0.778 \text{ N} \quad F = 0.847 \text{ N}$$



VIA 9GAG.COM

**SCIENCE**

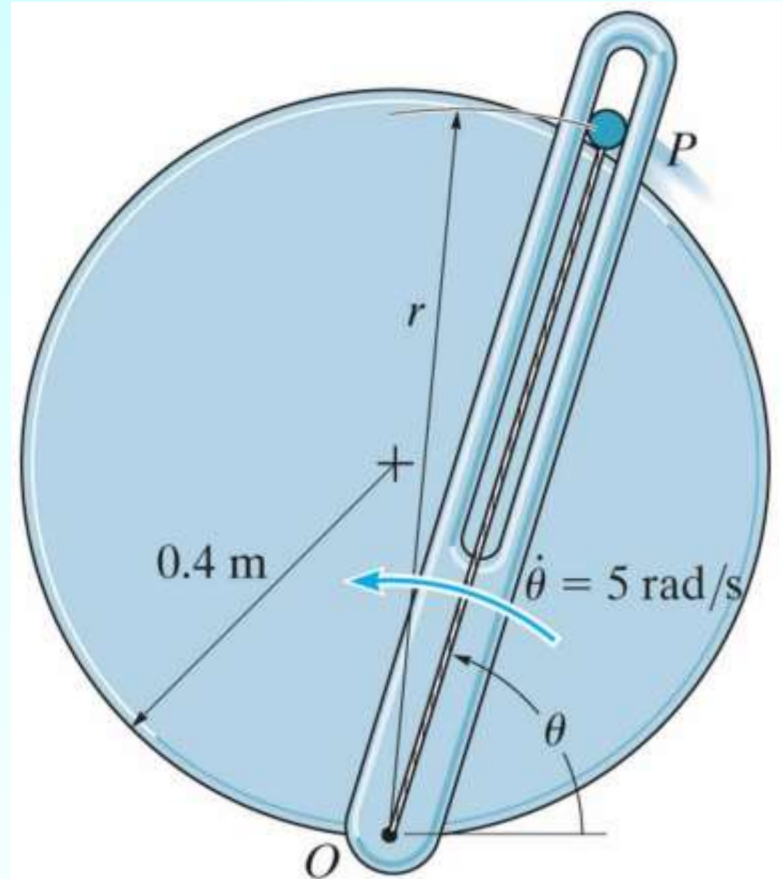


**ENGINEERING**

## Problem 13-110 (Page 153, 13<sup>th</sup> edition)

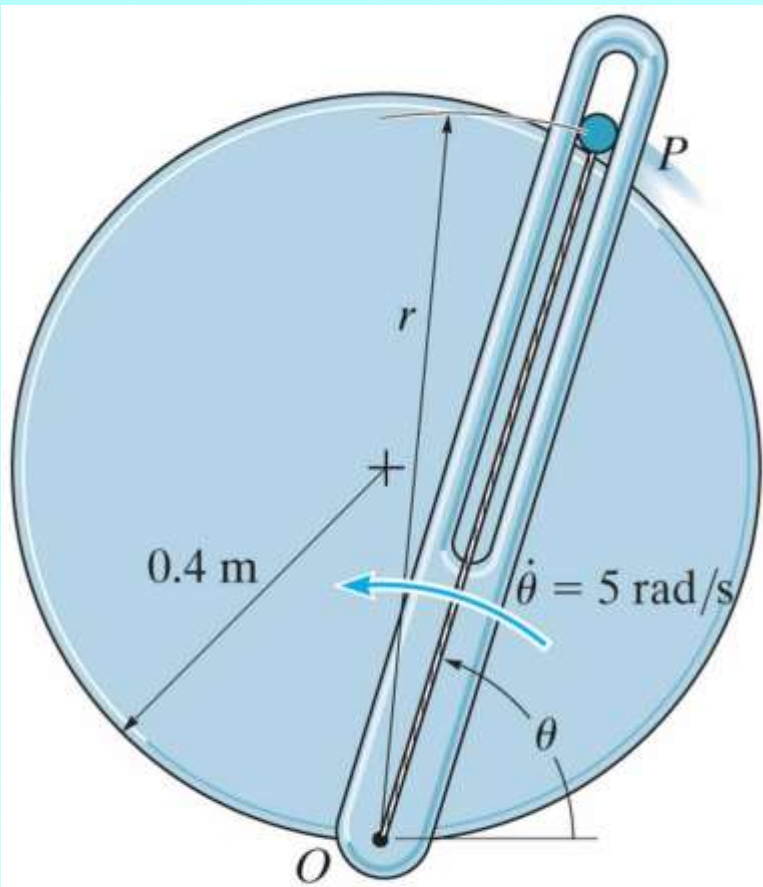
The slotted guide moves the 150 g particle  $P$  around the 0.4 m radius circular disk. Motion is in the vertical plane. Attached to  $P$  is an elastic cord extending from  $O$ . The cord has stiffness 30 N/m and unstretched length 0.25 m. Friction may be neglected.

(1) Determine the force of the guide on  $P$  and the normal force of the disk on  $P$  when  $\theta = 70^\circ$ ,  $\dot{\theta} = 5 \text{ rad/s}$  and  $\ddot{\theta} = 2 \text{ rad/s}^2$ .



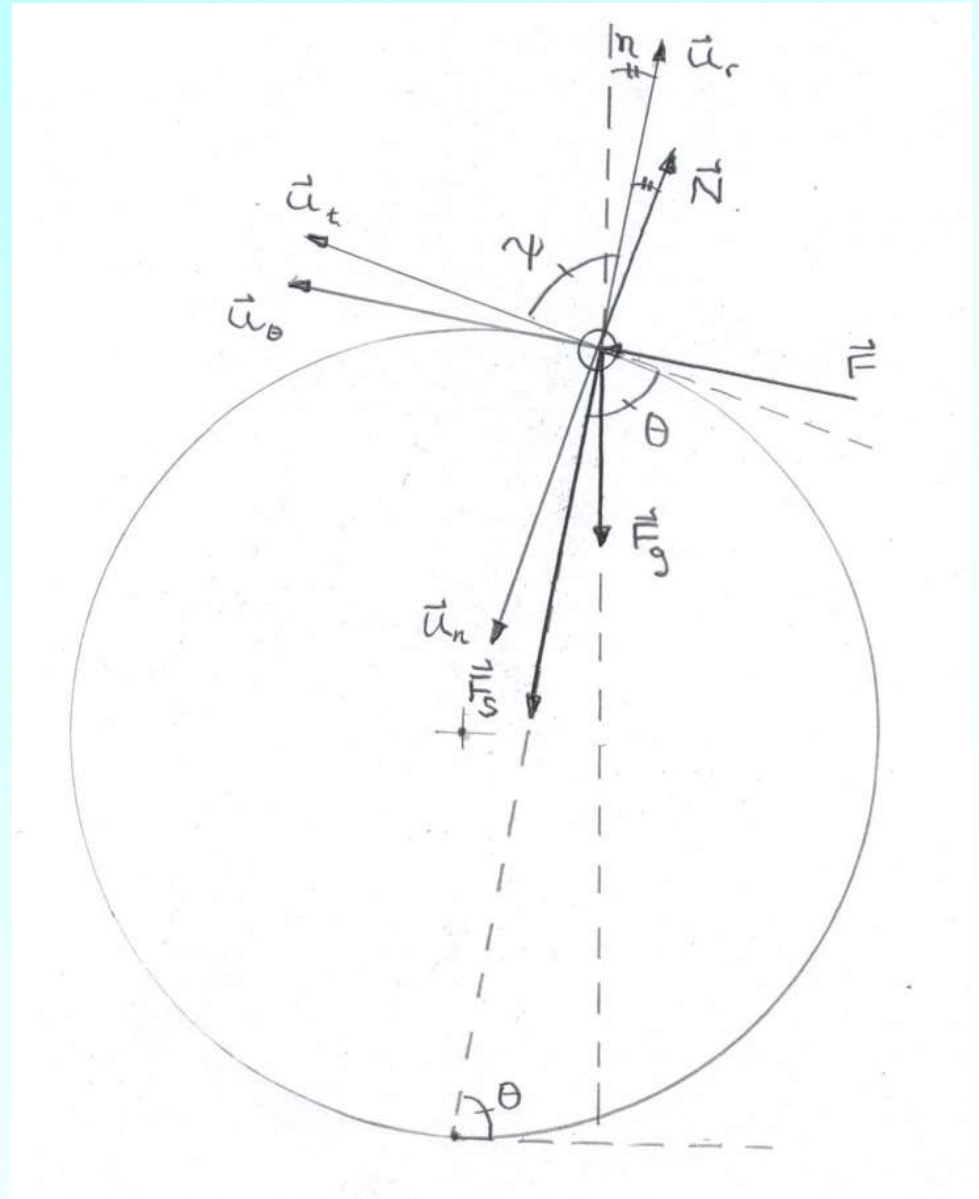
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PROB13\_105-106.jpg

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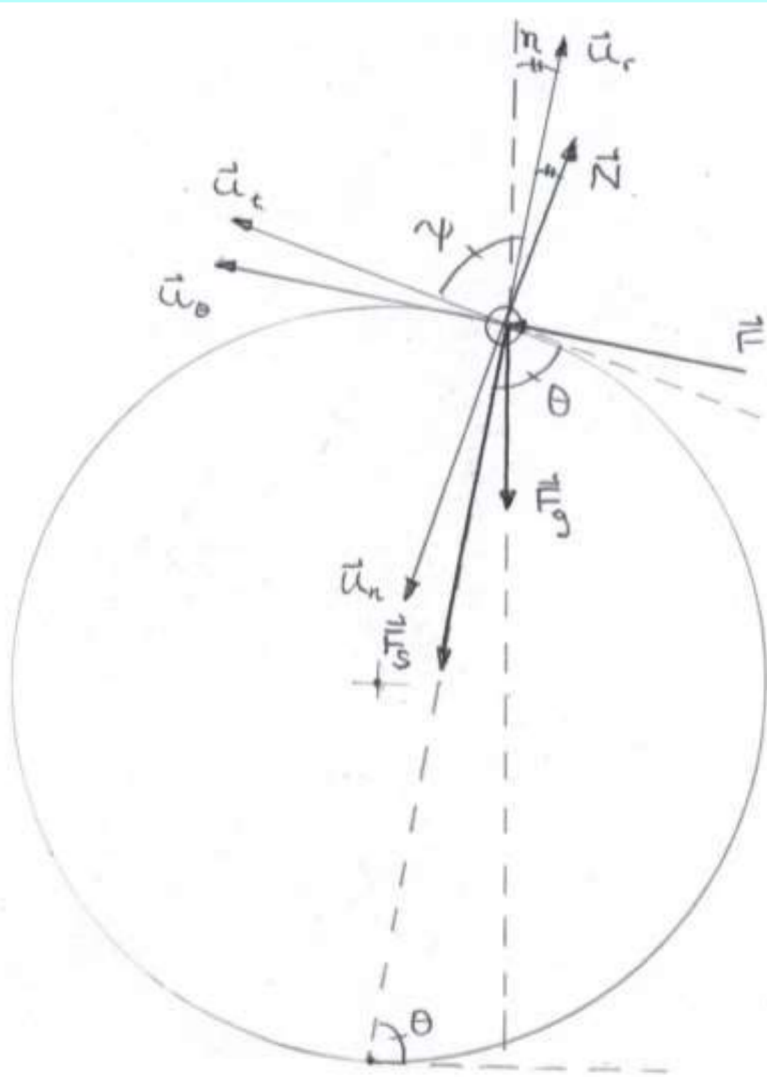
From geometry

$$\psi = \theta$$

$$\eta = 90 - \psi = 90 - \theta$$

$n$  and  $y$  axes both make an angle  $\eta$  with  $r$  axis





## Solution strategy

Determine acceleration components in polar coordinates, then determine requested forces from equations of motion using acceleration components and other forces (elastic cord, weight)

As before, determination of acceleration components requires computation of various derivatives ( $\dot{r}$ ,  $\ddot{r}$ ) and we must also determine angle,  $\psi$ , between radial and tangential unit vectors

Exercise: Refer to the original diagram. Show that  $r(\theta)$  is given by  $r(\theta) = 2R \sin \theta$ , where  $R$  is the radius of the disk.

**Solution continues in Lecture 26**