PHYS 170 Section 101
Lecture 24
November 2, 2018

Midterm 2 (November exam)

- 2:00 PM to 2:50 PM, Friday, November 9, in our usual lecture hall

- The exam is worth 20 marks out of 100 marks for the course

- The exam will consist of 2 questions based on text Chapters 5, 8 and 12 (Lecture 12 [*after* the wrench problem] through first part of Lecture 22, inclusive)

- The two questions will not necessarily be worth the same number of marks each

Midterm 2 (November exam)

Graphing Calculator

You may bring a graphing calculator to any and all of the three exams.

Information Sheet

You may bring one handwritten (on both sides if you like) 8 ½ in by 11 in (216 mm by 279 mm) Information Sheet to any and all of the three exams.

You must prepare your own Information Sheet.

Your Information Sheet must not contain any reduced or printed material or any sample problems or solutions to sample problems.

In order for your exam to be marked, you must sign your Information Sheet and hand it in with your exam booklet.

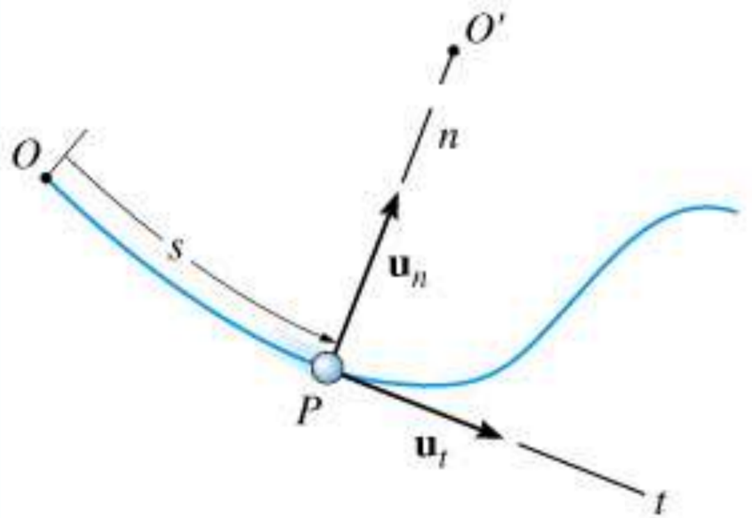
Review Sessions / Past Exams

- No tutorials next week
- Instead will again hold review sessions in usual tutorial locations at usual tutorial times
- Note that the second questions on both the March 2016 and March 2017 exams are from Chapter 13, and are therefore not relevant for the upcoming midterm

Lecture Outline/Learning Goals

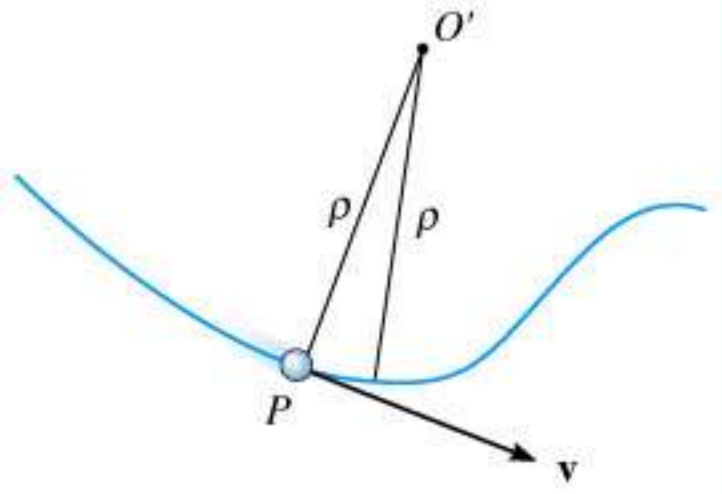
- 13.5 Equations of Motion: Normal and Tangential Coordinates
- Worked example of motion in tangential-normal coordinates
- 13.6 Equations of Motion: Cylindrical Coordinates

Recall—Curvilinear Motion: Normal and Tangential Components (Planar Motion)



POSITION

(a)

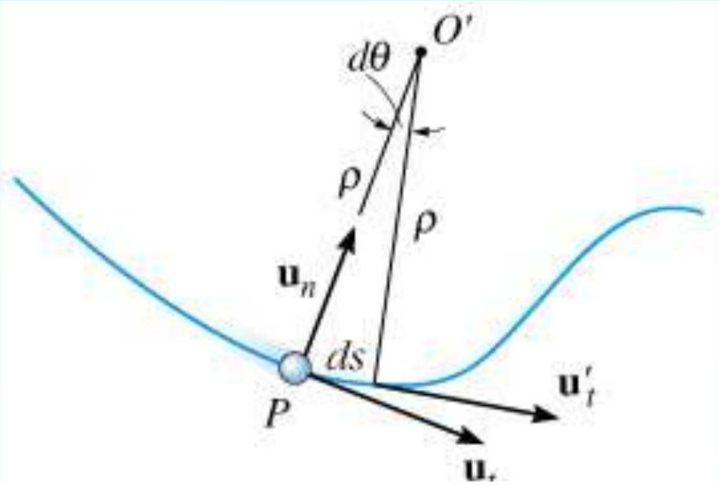


VELOCITY

(c)

$$\mathbf{v} = v\,\mathbf{u}_t$$

$$v = \frac{ds}{dt} = \dot{s}$$



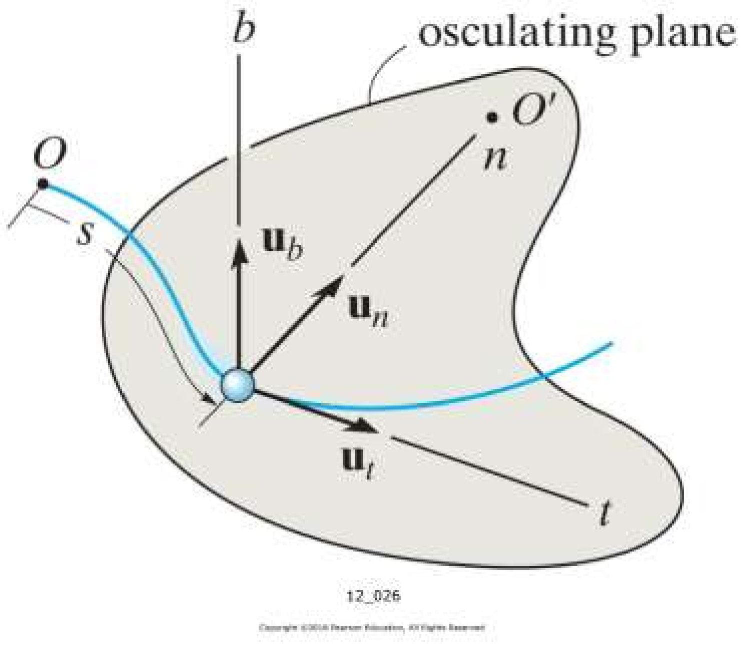
ACCELERATION

$$\mathbf{a} = a_t \mathbf{u}_t + a_n \mathbf{u}_n$$

$$a_t = \dot{v}$$

$$a_n = \frac{v^2}{\rho}$$

Curvilinear Motion: Normal and Tangential Components (Three Dimensional Motion)



Assume particle moves along space curve with no restriction to planar motion

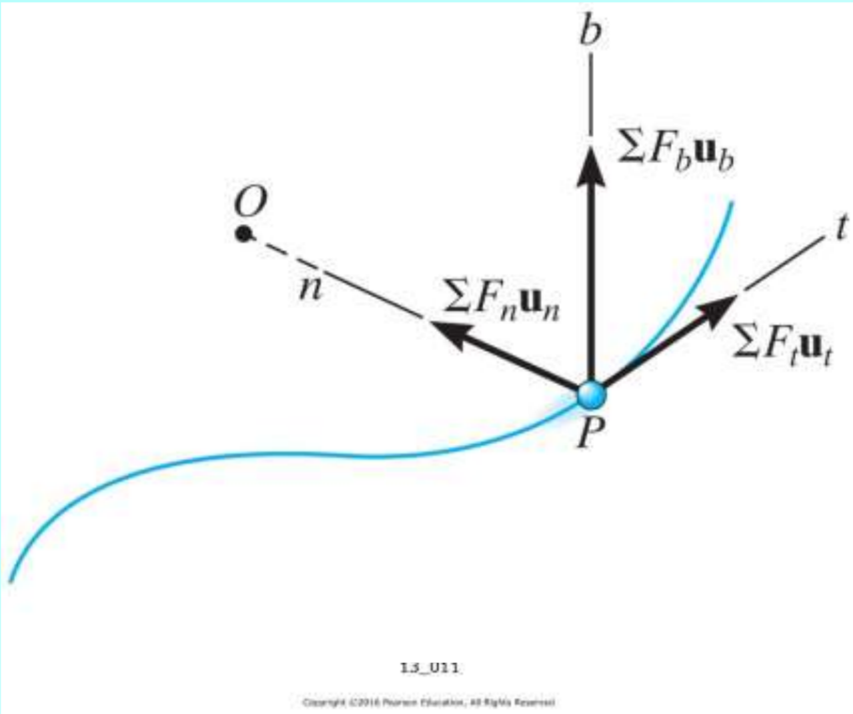
$\mathbf{u}_t$ is uniquely defined; choose $\mathbf{u}_n$ to point towards center of curvature of path, O'

$\mathbf{u}_t$ and $\mathbf{u}_n$ are always perpendicular; choose third axis, *b*, with unit normal defined by

$$\mathbf{u}_b = \mathbf{u}_t \times \mathbf{u}_n$$

Note that there will never be any *motion* in the *b* direction (the motion is restricted to the *t-n* plane, the so called osculating plane, which is not, in general, a single plane in 3D space).

13.5 Equations of Motion: Normal and Tangential Coordinates



Can use equations of motion in normal and tangential coordinates when path of particle is known

Again, note that there is no motion in the *b* direction (i.e. no acceleration)

Equation of motion can be written as

$$\sum F_t \mathbf{u_t} + \sum F_n \mathbf{u_n} + \sum F_b \mathbf{u_b} = m\mathbf{a}_t + m\mathbf{a}_n$$

Above equation is satisfied provided

$$\sum F_t = ma_t = m\dot{v}$$

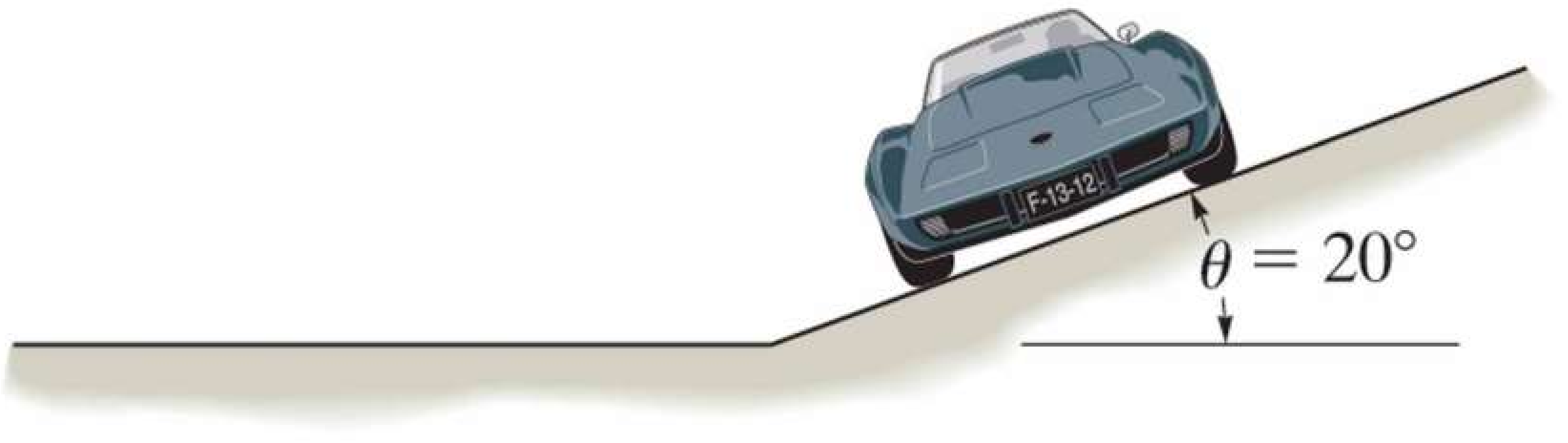
$$\sum F_n = ma_n = m\frac{v^2}{\rho}$$

$$\sum F_b = 0$$

Problem 13-53 (page 138, 12th edition)

A 1700 kg sports car travels around a 100 m radius horizontal circle on a 20° banked track. The coefficient of static friction between the tires and the road is 0.20.

(1) Determine the maximum constant speed the car can travel without slipping up the slope.



PROB13_054.jpg

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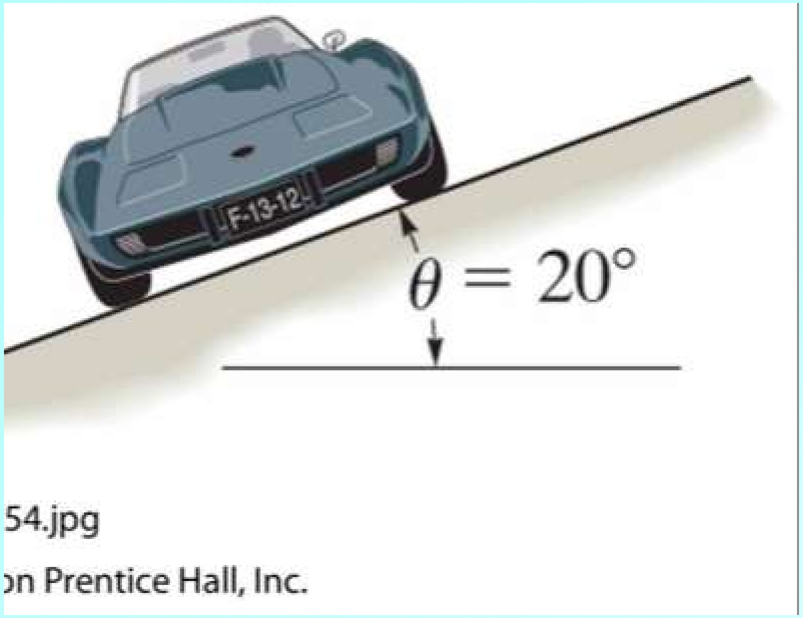
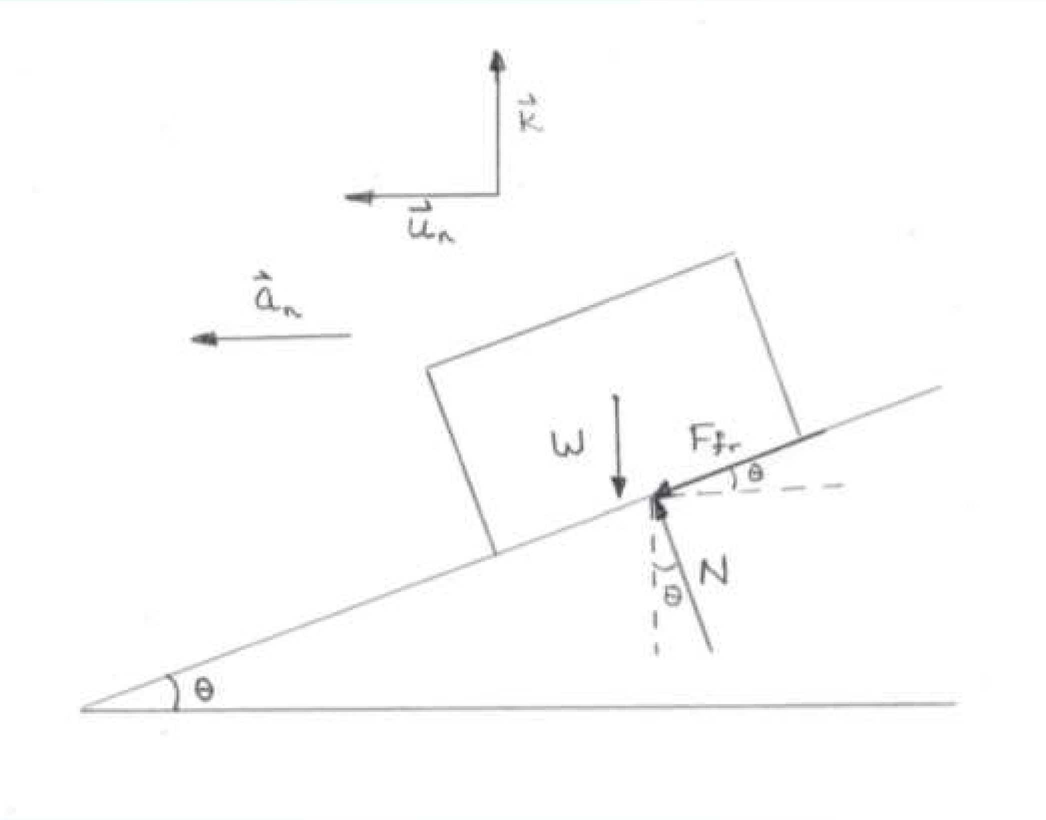
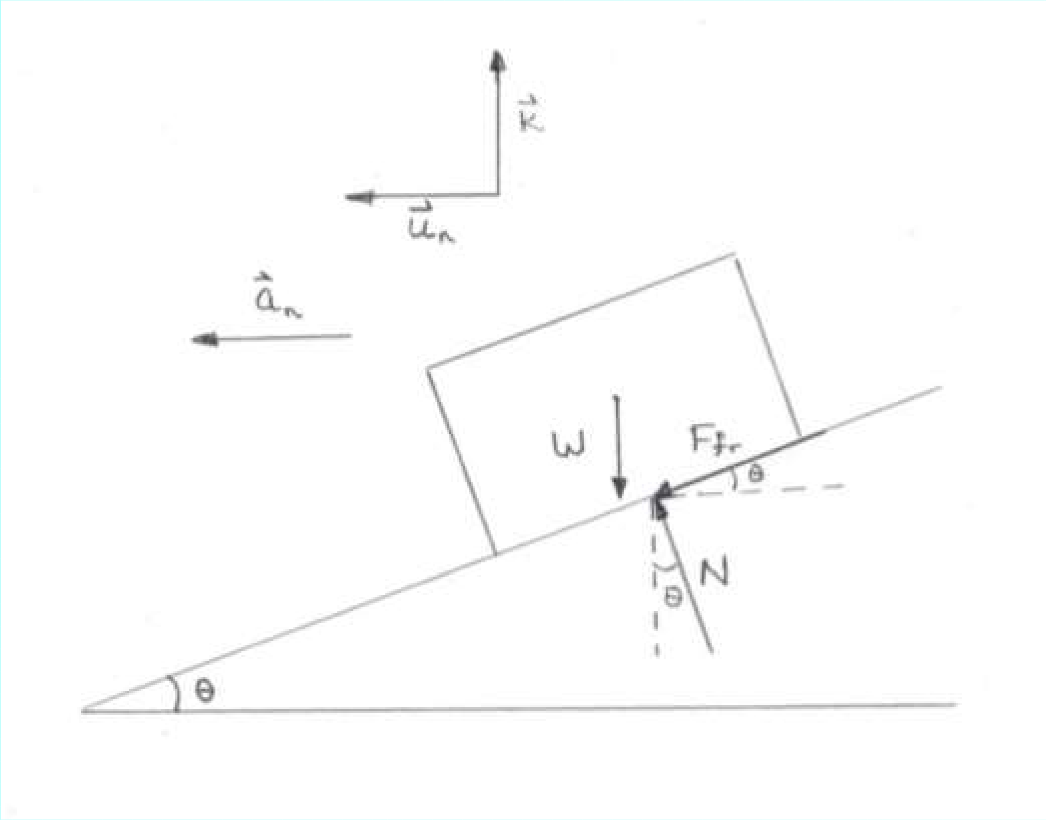
My physics teacher says my understanding of forces is the worst he's ever known.

I think he's pushing my leg.



Note that z is the b direction here. There is no acceleration in the t direction (constant speed), so no need to analyze motion in that direction.





Solution strategy

Using FBD, write down equations of motion in $(n,b)=(n,z)$ coordinates

Incorporate impending motion to treat frictional force

$$F_{fr} = \mu N$$

Solve equations to determine v

Impending motion is up the slope, so frictional force acts down the slope.

Data

$m = 1700 \text{ kg} \qquad g = 9.81 \text{ m/s}^2$

$\theta = 20^\circ \qquad \rho = 100 \text{ m} \qquad \mu = 0.2$

Kinematics

$$\vec{a} = \frac{v^2}{\rho}\vec{u}_n$$

Forces

$$\vec{W} = -mg\,\vec{k}$$

$$\vec{N} = N(\sin\theta\,\vec{u}_n + \cos\theta\,\vec{k})$$

$$\vec{F}_{fr} = \mu N(\cos\theta\,\vec{u}_n - \sin\theta\,\vec{k})$$

Equations of motion

$$\sum F_n = ma_n : \qquad N\sin\theta + \mu N\cos\theta = \frac{mv^2}{\rho} \qquad (1)$$

$$\sum F_z = ma_z : \qquad N\cos\theta - \mu N\sin\theta - mg = 0$$

$$N\cos\theta - \mu N\sin\theta = mg \qquad (2)$$

Divide (1) by (2)

$$\frac{N\sin\theta + \mu N\cos\theta}{N\cos\theta - \mu N\sin\theta} = \frac{mv^2}{mg\rho} \quad \Rightarrow \quad v = \sqrt{g\rho\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}}$$

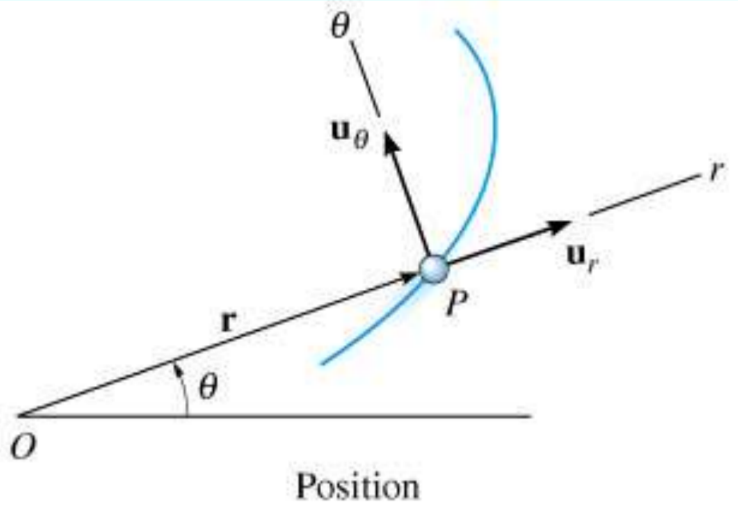
$$v = \sqrt{(9.81)(100)\frac{\sin 20^\circ + 0.2\cos 20^\circ}{\cos 20^\circ - 0.2\sin 20^\circ}} = 24.4\ \text{m/s} = 87.9\ \text{km/h}$$

Note that the result does not depend on the mass of the car. Does this surprise you? Can you provide an explanation for why this is the case?

EXERCISE: Determine the minimum constant speed the car can travel without sliding down the slope.

13.5 Equations of Motion: Cylindrical Coordinates

RECALL: Ch 12 – Polar components: Planar Motion (2D)

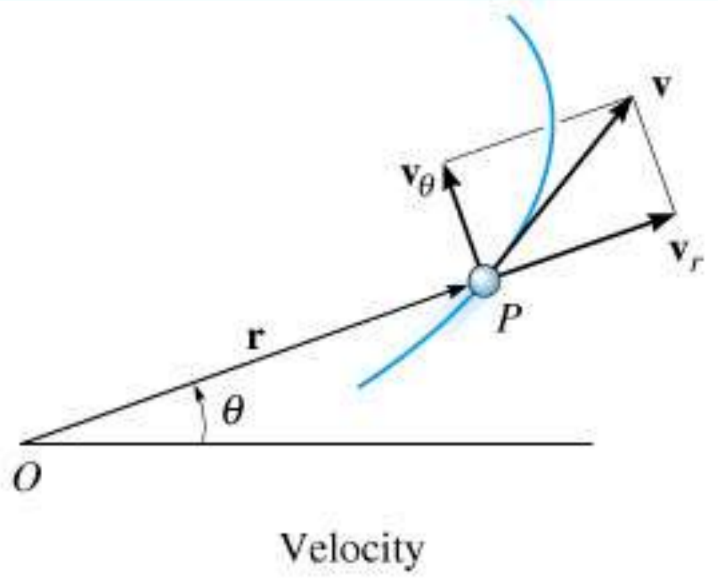


POSITION

$$\mathbf{r} = r\,\mathbf{u}_r$$

Note that, in general, r, θ coordinates of particle will be functions of time, i.e.

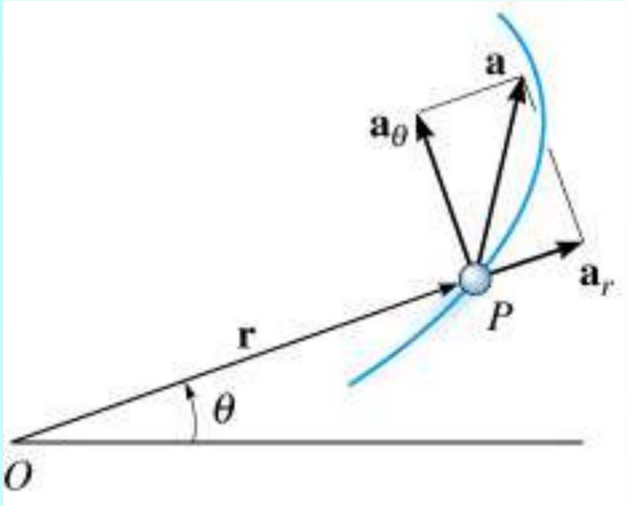
$$r = r(t)$$
$$\theta = \theta(t)$$



VELOCITY

$$\mathbf{v} = \dot{r}\,\mathbf{u}_r + r\dot{\theta}\,\mathbf{u}_\theta$$

Polar components: Planar Motion (2D) [continued]

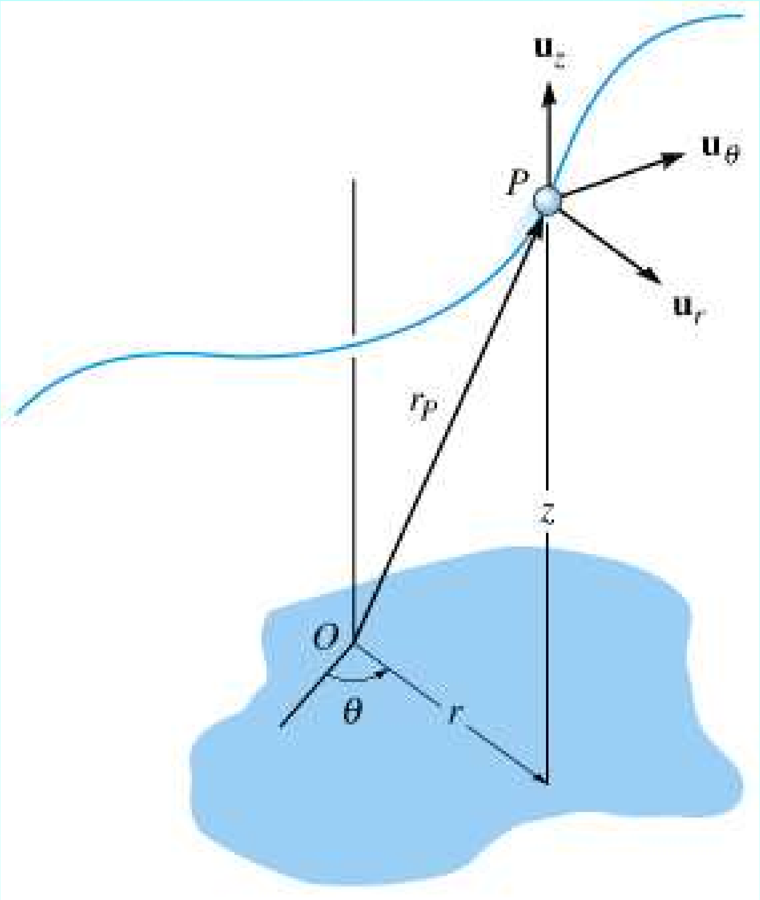


Acceleration

ACCELERATION

$$\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\mathbf{u}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{u}_\theta$$

THREE DIMENSIONAL MOTION (Cylindrical coordinates)



All 3 cylindrical coordinates, r, θ and z will generally be functions of time

$$r = r(t)$$

$$\theta = \theta(t)$$

$$z = z(t)$$

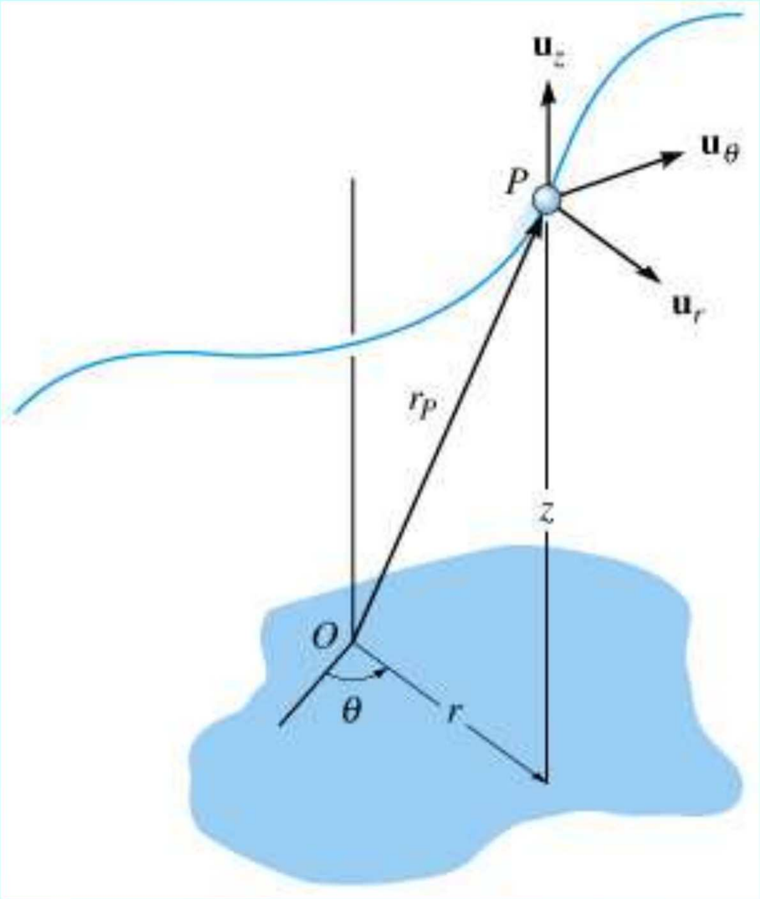
- As earlier (and largely for completeness) we now want to extend our discussion to 3D motion

- Again, this is easy to do – we simply introduce a z coordinate which is **identical** to the z coordinate used in the description of 3D motion using rectangular (Cartesian) coordinates

- Thus, as shown in the figure, we label a particle at point P with **cylindrical** coordinates (r, θ, z) and note that associated with the z-axis is a unit vector $\mathbf{u}_z$ that can be defined in terms of $\mathbf{u}_r$ and $\mathbf{u}_\theta$ via

$$\mathbf{u}_z = \mathbf{u}_r \times \mathbf{u}_\theta$$

THREE DIMENSIONAL MOTION (Cylindrical coordinates) [continued]



All 3 cylindrical coordinates, r, θ and z will generally be functions of time

$$r = r(t)$$

$$\theta = \theta(t)$$

$$z = z(t)$$

- The modification of our formulae for position, velocity and acceleration in polar coordinates to corresponding expressions in cylindrical coordinates is straightforward

- Specifically, we have

$$\mathbf{r}_P = r\,\mathbf{u}_r + z\,\mathbf{u}_z$$

$$\mathbf{v} = \dot{r}\,\mathbf{u}_r + r\dot{\theta}\,\mathbf{u}_\theta + \dot{z}\,\mathbf{u}_z$$

$$\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\,\mathbf{u}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\mathbf{u}_\theta + \ddot{z}\,\mathbf{u}_z$$

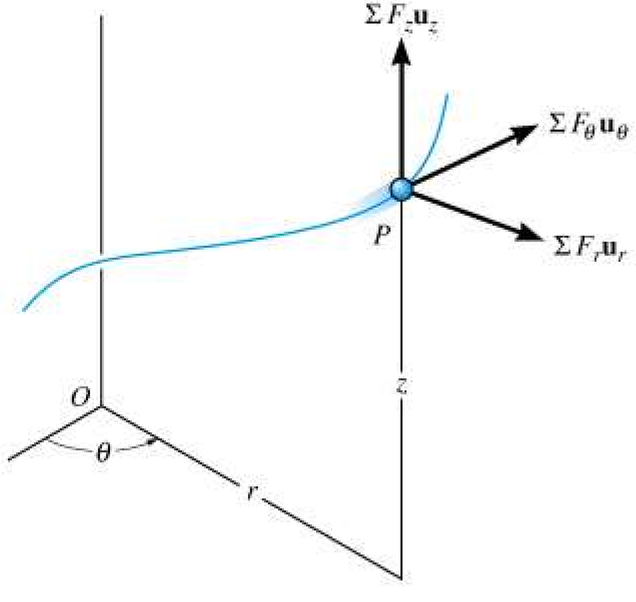
- Note that these are **identical** to the expressions for polar coordinates, except for the addition of the terms

Position: $z\,\mathbf{u}_z$

Velocity: $\dot{z}\,\mathbf{u}_z$

Acceleration: $\ddot{z}\,\mathbf{u}_z$

EQUATIONS OF MOTION



Inertial coordinate system

- Once we have established the three dimensional cylindrical coordinate system (r, θ, z), it is again straightforward to write down the corresponding equations of motion

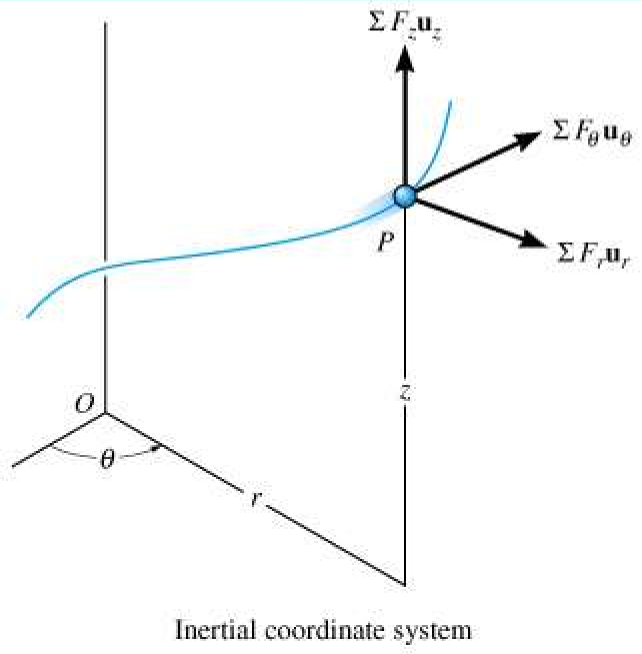
- Specifically (and referring to the diagram at left), Newton's second law

$$\Sigma \mathbf{F} = m\mathbf{a}$$

can be written in cylindrical coordinates as

$$(\Sigma F_r)\mathbf{u}_r + (\Sigma F_\theta)\mathbf{u}_\theta + (\Sigma F_z)\mathbf{u}_z = ma_r\mathbf{u}_r + ma_\theta\mathbf{u}_\theta + ma_z\mathbf{u}_z$$

EQUATIONS OF MOTION (continued)



Inertial coordinate system

- The previous vector equation is satisfied if and only if its r, θ and z components are independently satisfied

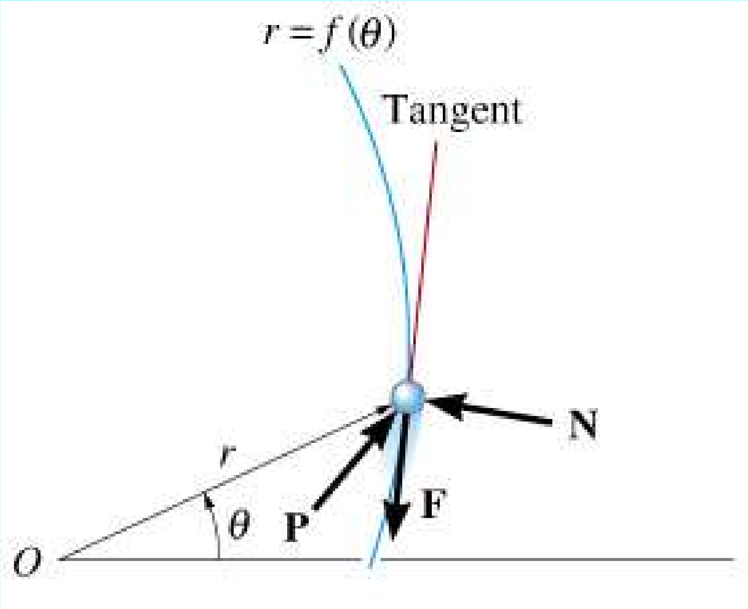
- We thus have the following scalar equations of motion in cylindrical components

$$\Sigma F_r = m a_r$$
$$\Sigma F_\theta = m a_\theta$$
$$\Sigma F_z = m a_z$$

- Note that in contrast to the case of (t,n,b) coordinates, there **can** be motion in the third (z) dimension in this case, and that **all** of the scalar components of the acceleration, a_r, a_θ and a_z, can have either sign in general

- Will usually be dealing with problems in which the motion is restricted to 2D, i.e. to the $r-\theta$ plane. In this case, only the first two of the above equations apply

TANGENTIAL AND NORMAL FORCES

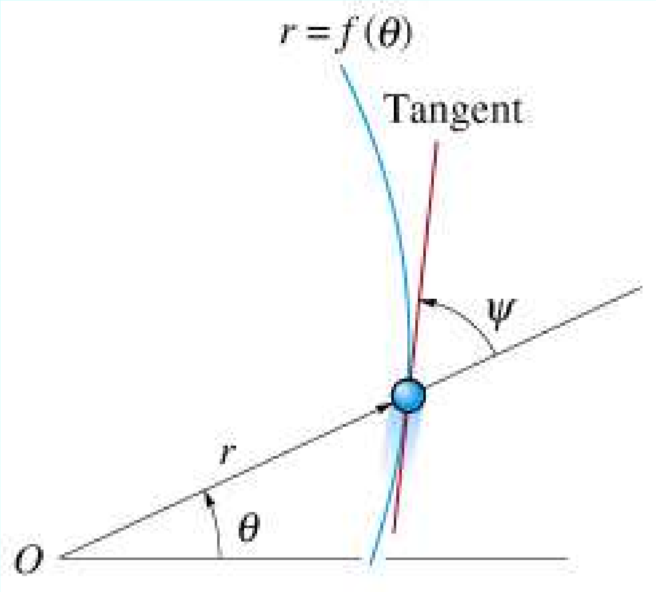


Many of the problems in this part of the course have the following features

- They are natural to treat in polar coordinates

- The path of the particle is specified (constrained motion)

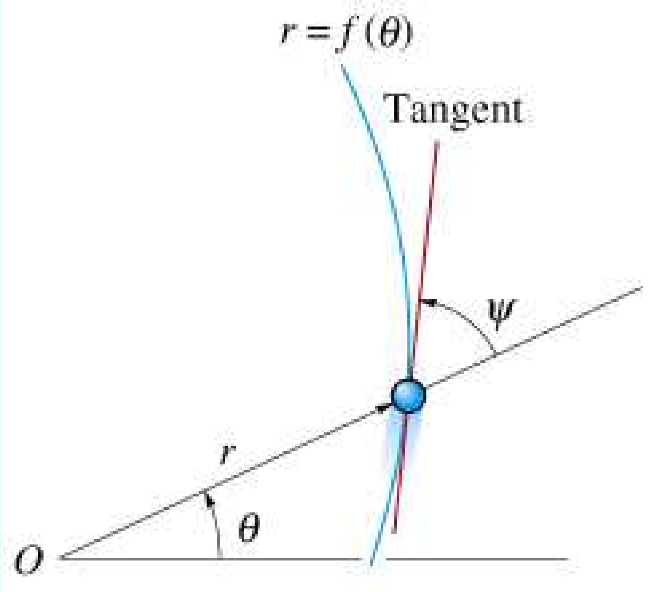
- Some of the forces on the particle act in the **normal** or **tangential** directions



- We thus need to be able to determine the orientation of the (t,n) coordinate system with respect to the (r,θ) system

- As shown in the figure, the tangent to the particle path will form an angle ψ with the **extended** radial line

TANGENTIAL AND NORMAL FORCES (continued)



- Given the equation for the path in the form $r = f(\theta)$, the text derives an equation for ψ; will not reproduce that derivation here

- Rather, we derive the formula using a different construction that is based on the observation that the velocity of the particle is **always tangent to the particle path**