PHYS 170 Section 101
Lecture 23
October 31, 2017

October 31—Announcements

- Contrary to previous claims (somewhat), will need to switch calculator to radians for one question in current homework (12-177). See Canvas version of announcement for a bit more detail

- Have added to the online notes (Lecture 21) the solution to the pulley question I left as an exercise

Lecture Outline/Learning Goals

- Two worked examples solving equations of motion in rectangular coordinates

Recall: Relative Motion of Two Particles Using Translating Axes

Position

$$\vec{r}_{B/A} = \vec{r}_B - \vec{r}_A$$

$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$$

Velocity

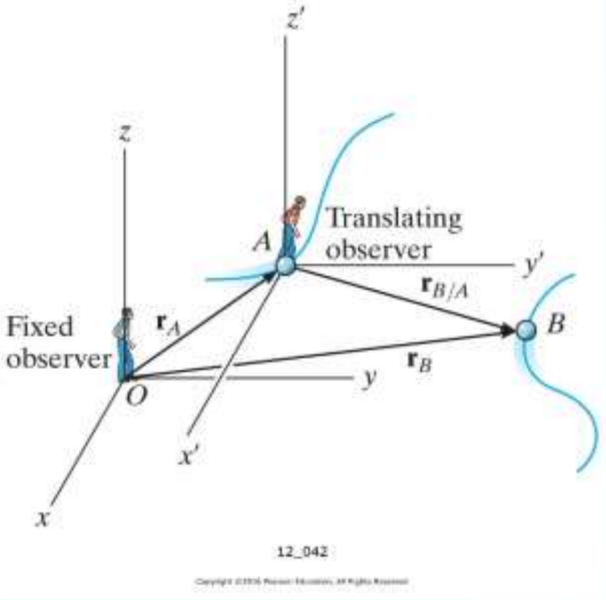
$$\vec{v}_{B/A} = \vec{v}_B - \vec{v}_A$$

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$

Acceleration

$$\vec{a}_{B/A} = \vec{a}_B - \vec{a}_A$$

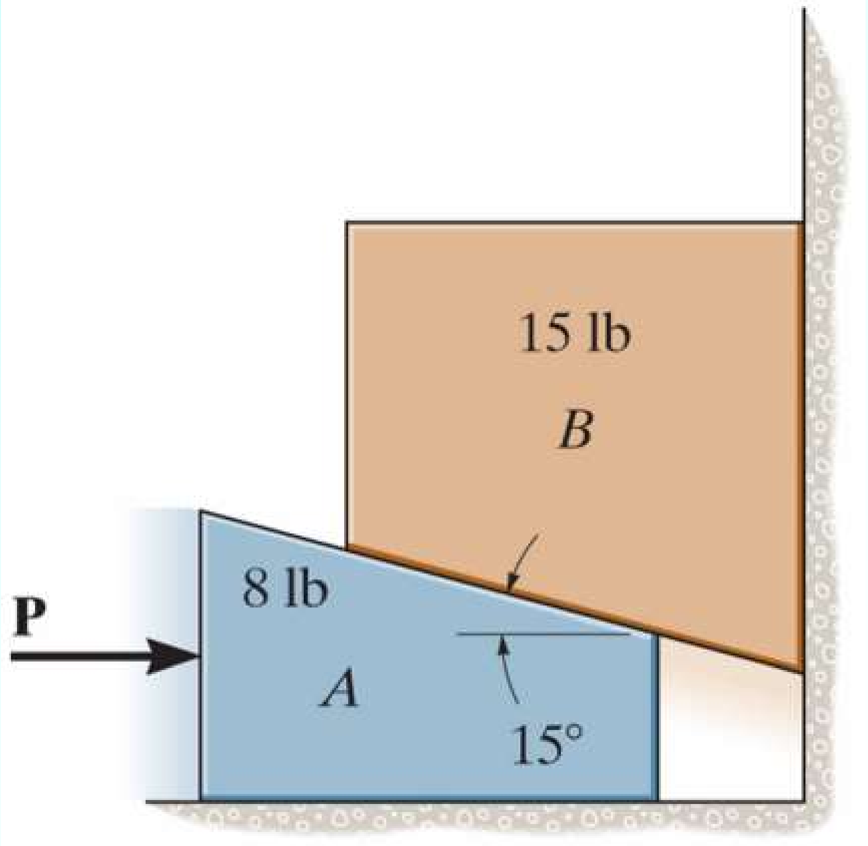
$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$



Problem 13-41 (page 129, 12th edition)

A horizontal force P = 20 lb is applied to block A. The coefficients of kinetic friction between block A and the horizontal surface, between the two blocks, and between block B and the vertical surface are 0.1, 0.2, and 0.3, respectively.

(1) Determine the acceleration of each block and all normal forces.



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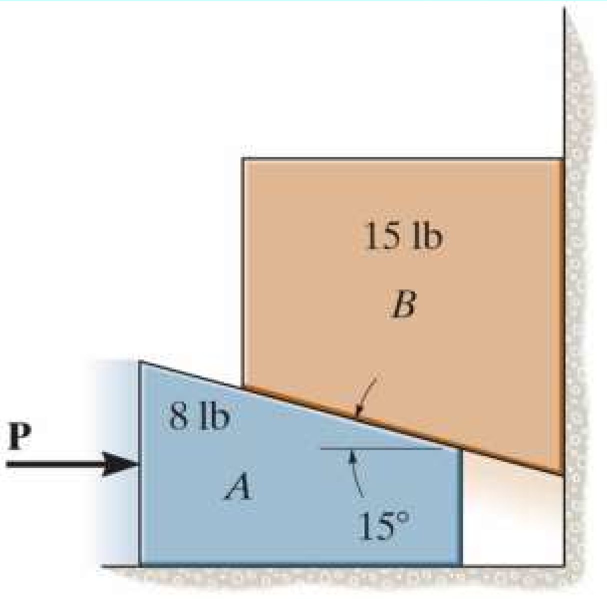

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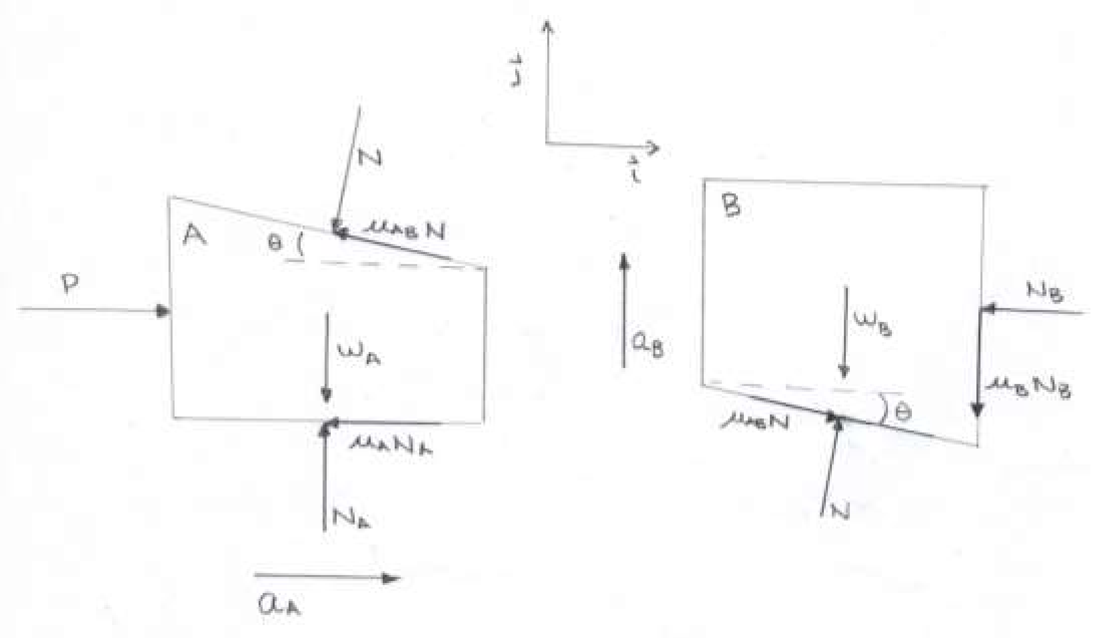
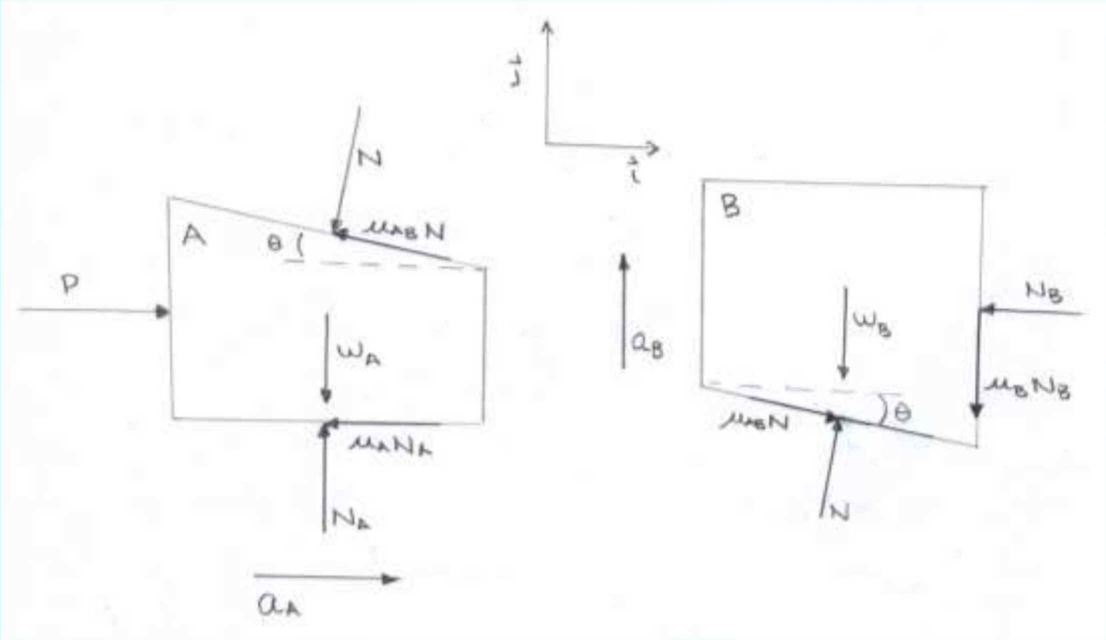


Accelerations and relative motion



$$\vec{a}_A = a_A \vec{i}$$

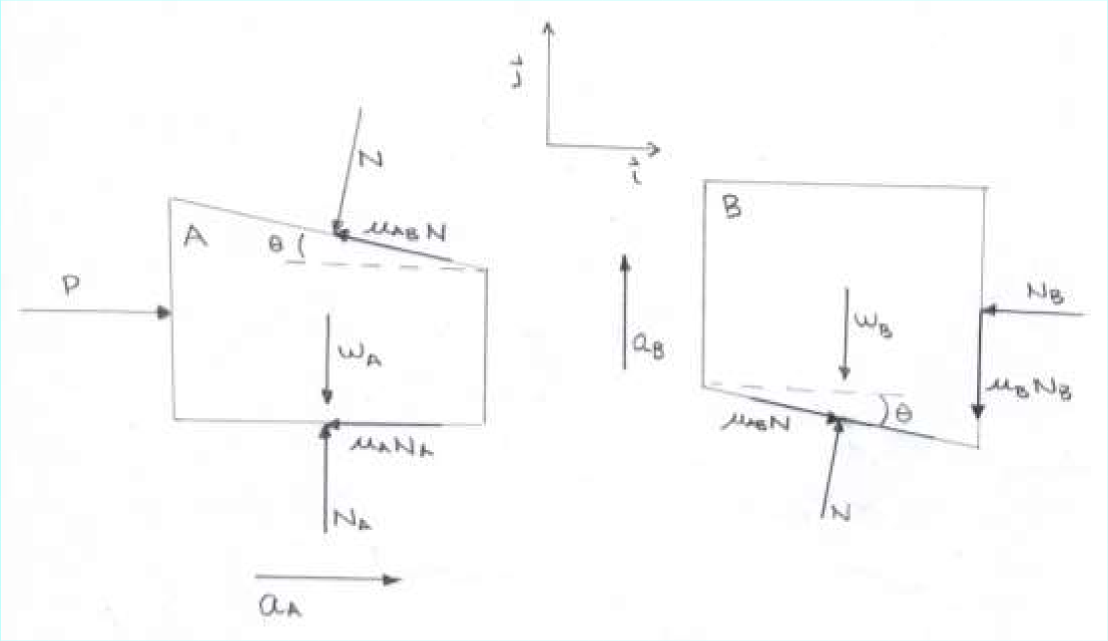
$$\vec{a}_B = a_B \vec{j}$$

Relative motion: Imagine riding on block *A*. What is *B*'s motion relative to you?

Answer: Up and to the left, at an angle θ from the horizontal.

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$

$$\vec{a}_{B/A} = a_{B/A}(-\cos\theta\,\vec{i} + \sin\theta\,\vec{j})$$



Solution strategy

Use equations for relative acceleration to express a_B in terms of a_A

Using FBDs, write down equations of motion for two blocks

Ensure that there are the same number of equations as unknowns

Solve the equations

Data

$$W_A = m_A g = 8 \text{ lb} \qquad W_B = m_B g = 15 \text{ lb} \qquad g = 32.2 \text{ ft/s}^2$$

$$P = 20 \text{ lb} \qquad \theta = 15^\circ$$

$$\mu_A = 0.1 \qquad \mu_{AB} = 0.2 \qquad \mu_B = 0.3$$

Accelerations

$$\vec{a}_A = a_A \vec{i}$$

$$\vec{a}_B = a_B \vec{j}$$

Relative motion

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$

$$\vec{a}_{B/A} = a_{B/A}(-\cos\theta\,\vec{i} + \sin\theta\,\vec{j})$$

Cartesian components of $\vec{a}_B$

x component: $0 = a_A - a_{B/A}\cos\theta \;\Rightarrow\; a_A = a_{B/A}\cos\theta$

y component: $a_B = a_{B/A}\sin\theta$

from which we have

$$a_B / a_A = (a_{B/A}\sin\theta)/(a_{B/A}\cos\theta) = \tan\theta$$

$$a_B = a_A \tan\theta$$

Equations of motion (refer to FBDs)

Equations of motion for *A*:

$\sum F_x = ma_x$: $\quad P - N\sin\theta - \mu_{AB} N\cos\theta - \mu_A N_A = m_A a_A \qquad (1)$

$\sum F_y = ma_y$: $\quad -N\cos\theta + \mu_{AB} N\sin\theta + N_A - W_A = 0 \qquad (2)$

Equations of motion for B :

$$\sum F_x = ma_x : \qquad N\sin\theta + \mu_{AB} N\cos\theta - N_B = 0 \qquad (3)$$

$$\sum F_y = ma_y : \qquad N\cos\theta - \mu_{AB} N\sin\theta - \mu_B N_B - W_B = m_B a_B$$

$$N\cos\theta - \mu_{AB} N\sin\theta - \mu_B N_B - W_B = m_B a_A \tan\theta \qquad (4)$$

Equations (1)-(4) contain 4 unknowns: a_A, N_A, N, N_B, so the system (probably) has a solution.

Data

$W_A = m_A g = 8 \text{ lb}$ $\qquad W_B = m_B g = 15 \text{ lb}$ $\qquad g = 32.2 \text{ ft/s}^2$

$P = 20 \text{ lb}$ $\qquad \theta = 15^\circ$

$\mu_A = 0.1$ $\qquad \mu_{AB} = 0.2$ $\qquad \mu_B = 0.3$

Equations to be solved

$$m_A a_A + (\sin\theta + \mu_{AB}\cos\theta)N + \mu_A N_A = P \qquad (1)$$

$$(-\cos\theta + \mu_{AB}\sin\theta)N + N_A = W_A \qquad (2)$$

$$(\sin\theta + \mu_{AB}\cos\theta)N - N_B = 0 \qquad (3)$$

$$-m_B \tan\theta\, a_A + (\cos\theta - \mu_{AB}\sin\theta)N - \mu_B N_B = W_B \qquad (4)$$

Numerical results (using rref([M]) where [M] is a 4×5 matrix.

(Exercise: Verify the solution.)

$a_A = 26.0$ ft/s^2 $N_A = 29.4$ lb $N = 23.4$ lb $N_B = 10.6$ lb

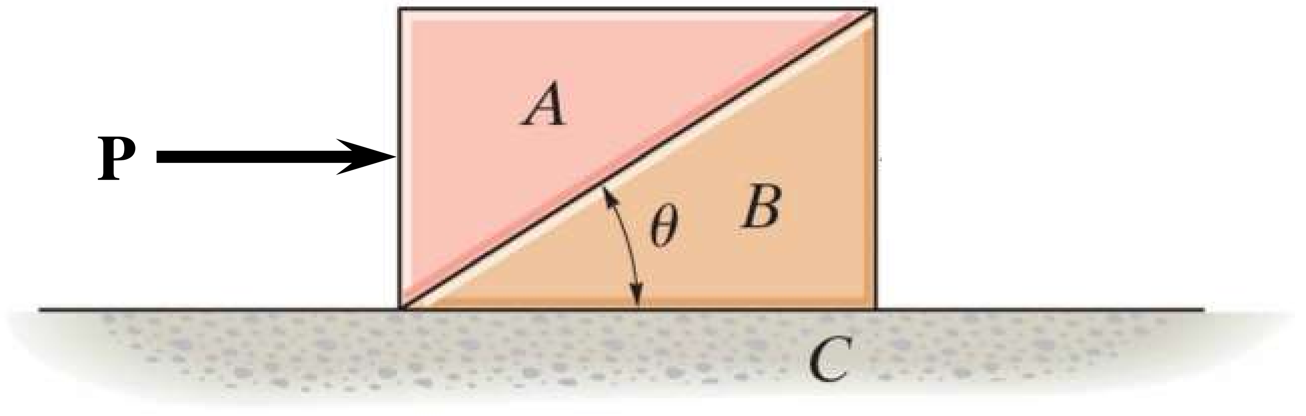
$a_B = 6.97$ ft/s^2 $a_{B/A} = 26.9$ ft/s^2

From the “Don’t do this at home folks!” department

Problem 13-46 (page 130, 13th edition)

The diagram shows two triangular blocks *A* and *B* each with mass 2 kg. *B* is on a horizontal surface. The sloped surface of *B* makes an angle $\theta = 40^\circ$ with the horizontal surface. The coefficient of kinetic friction between *B* and the horizontal surface is 0.5. The coefficient of kinetic friction between the two blocks is 0.2. A horizontal force $P = 50$ N acts to the right on *A*.

(1) Determine the acceleration of each block and the normal forces.



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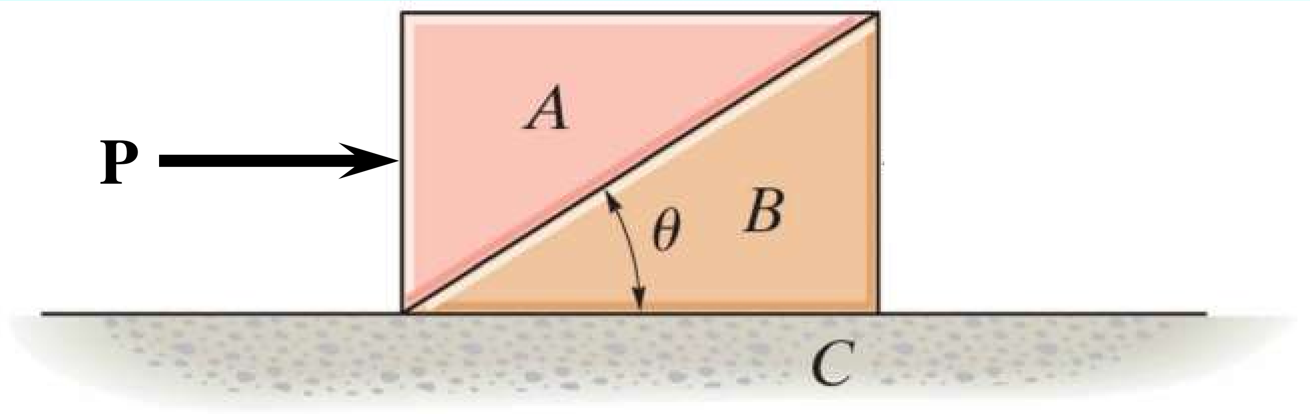

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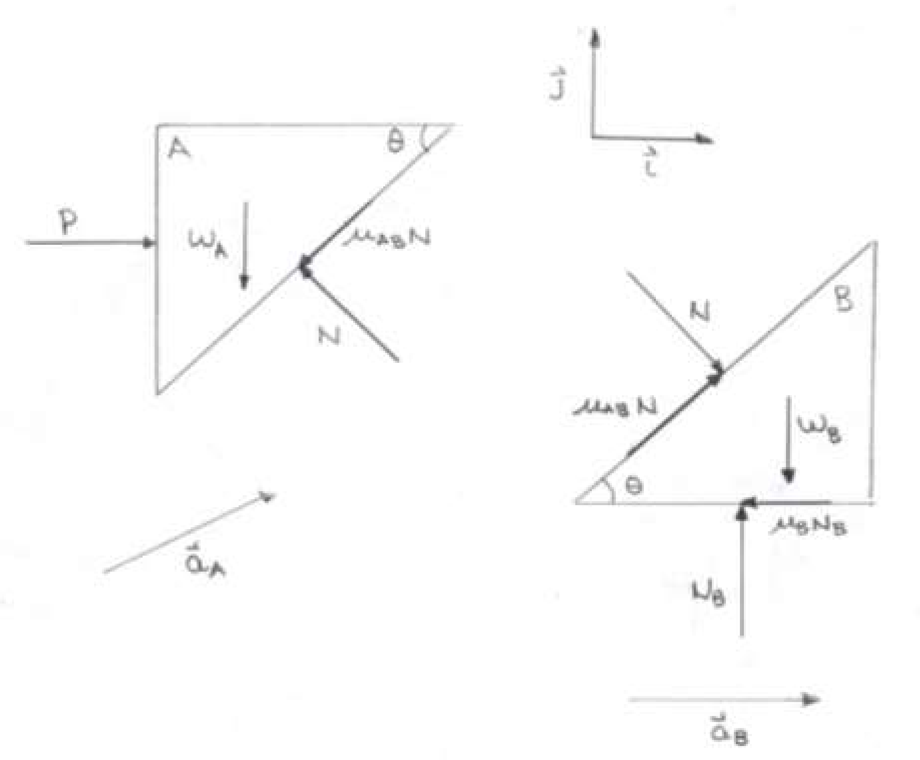
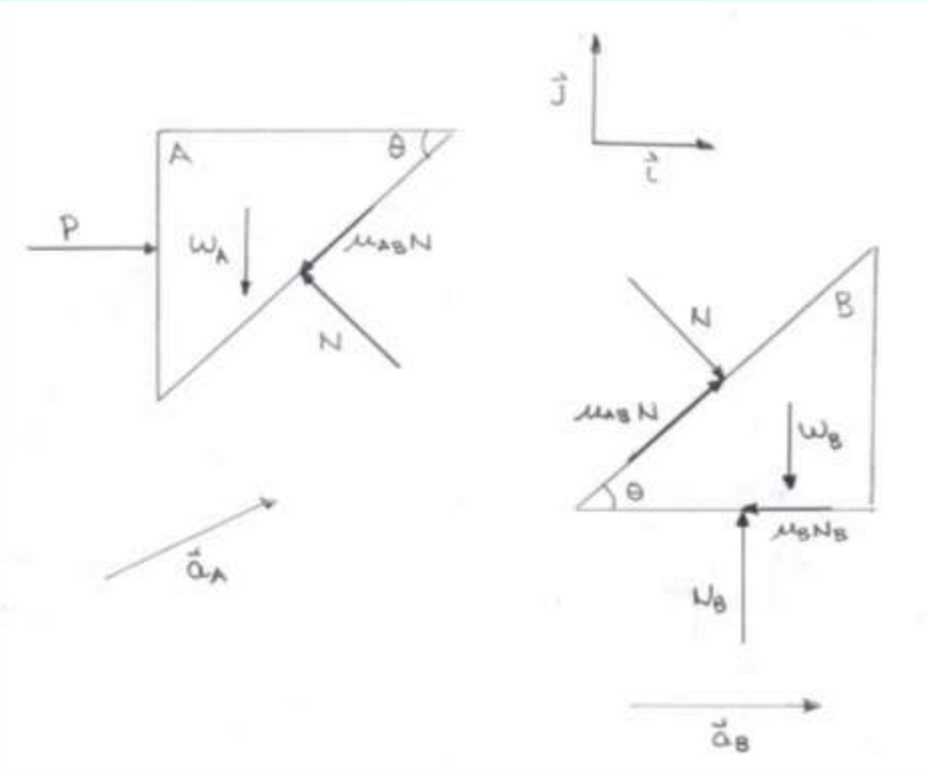


Accelerations and relative motion



$$\vec{a}_A = a_{Ax}\,\vec{i} + a_{Ay}\,\vec{j}$$

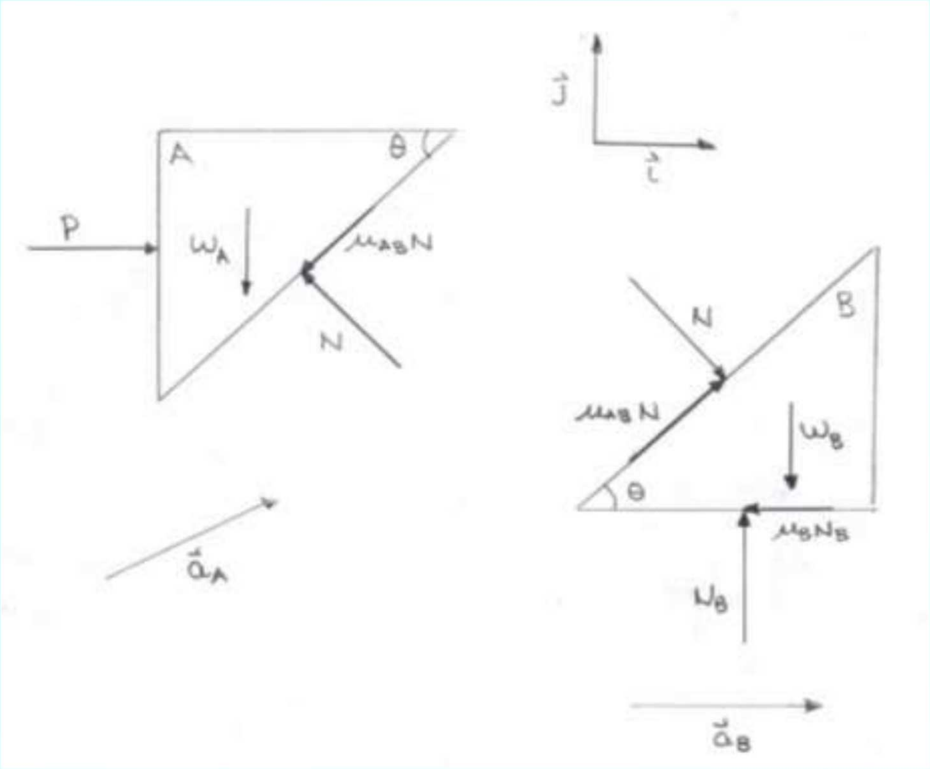
$$\vec{a}_B = a_B\,\vec{i}$$

Relative motion (similar to previous problem, except this time motion of A relative to B is up and to the right at angle θ to the horizontal)

$$\vec{a}_A = \vec{a}_B + \vec{a}_{A/B}$$

$$\vec{a}_{A/B} = a_{A/B}(\cos\theta\,\vec{i} + \sin\theta\,\vec{j})$$

Solution strategy



Use equations for relative acceleration to express components of $\vec{a}_A$ in terms of a_B and $a_{A/B}$

Using FBDs, write down equations of motion for two blocks

Ensure that there are the same number of equations as unknowns

Solve the equations

Data

$m_A = 2 \text{ kg}$ $m_B = 2 \text{ kg}$

$W_A = m_A g$ $W_B = m_B g$ $g = 9.81 \text{ m/s}^2$

$P = 50 \text{ N}$ $\theta = 40^\circ$

$\mu_B = 0.5$ $\mu_{AB} = 0.2$

Accelerations

$$\vec{a}_A = a_{Ax}\,\vec{i} + a_{Ay}\,\vec{j}$$

$$\vec{a}_B = a_B\,\vec{i}$$

Relative motion

$$\vec{a}_A = \vec{a}_B + \vec{a}_{A/B}$$

$$\vec{a}_{A/B} = a_{A/B}(\cos\theta\,\vec{i} + \sin\theta\,\vec{j})$$

Cartesian components of $\vec{a}_A$

x component: $a_{Ax} = a_B + a_{A/B}\cos\theta$

y component: $a_{Ay} = a_{A/B}\sin\theta$

Equations of motion (refer to FBDs)

Equations of motion for A:

$\sum F_x = ma_x$: $P - N\sin\theta - \mu_{AB}N\cos\theta = m_A a_{Ax}$

$$P - N\sin\theta - \mu_{AB}N\cos\theta = m_A(a_B + a_{A/B}\cos\theta) \qquad (1)$$

$\sum F_y = ma_y$: $N\cos\theta - \mu_{AB}N\sin\theta - W_A = m_A a_{Ay}$

$$N\cos\theta - \mu_{AB}N\sin\theta - W_A = m_A a_{A/B}\sin\theta \qquad (2)$$

Equations of motion for B :

$$\sum F_x = ma_x : \qquad N\sin\theta + \mu_{AB} N\cos\theta - \mu_B N_B = m_B a_B \qquad (3)$$

$$\sum F_y = ma_y : \qquad -N\cos\theta + \mu_{AB} N\sin\theta + N_B - W_B = 0 \qquad (4)$$

The above 4 equations contain 4 unknowns: a_B, $a_{A/B}$, N, N_B, so the problem likely has a solution.

Data

$m_A = 2$ kg $\qquad m_B = 2$ kg

$W_A = m_A g \qquad W_B = m_B g \qquad g = 9.81$ m/s^2

$P = 50$ N $\qquad \theta = 40^\circ$

$\mu_B = 0.5 \qquad \mu_{AB} = 0.2$

Equations to be solved

$$m_A a_B + m_A \cos\theta\, a_{A/B} + (\sin\theta + \mu_{AB}\cos\theta)N = P \qquad (1)$$

$$-m_A \sin\theta\, a_{A/B} + (\cos\theta - \mu_{AB}\sin\theta)N = W_A \qquad (2)$$

$$-m_B a_B + (\sin\theta + \mu_{AB}\cos\theta)N - \mu_B N_B = 0 \qquad (3)$$

$$(-\cos\theta + \mu_{AB}\sin\theta)N + N_B = W_B \qquad (4)$$

Numerical results (using rref([M]) where [M] is a 4×5 matrix)

(Exercise: Verify the solution)

$a_B = 4.86 \text{ m/s}^2$ $\quad a_{A/B} = 5.03 \text{ m/s}^2$ $\quad N = 40.9 \text{ N}$ $\quad N_B = 45.7 \text{ N}$

$a_{Ax} = 8.71 \text{ m/s}^2$ $\quad a_{Ay} = 3.23 \text{ m/s}^2$

$$\vec{a}_A = 9.29(\cos 20.4^\circ \, \vec{i} + \sin 20.4 \, \vec{j}) \text{ m/s}^2$$