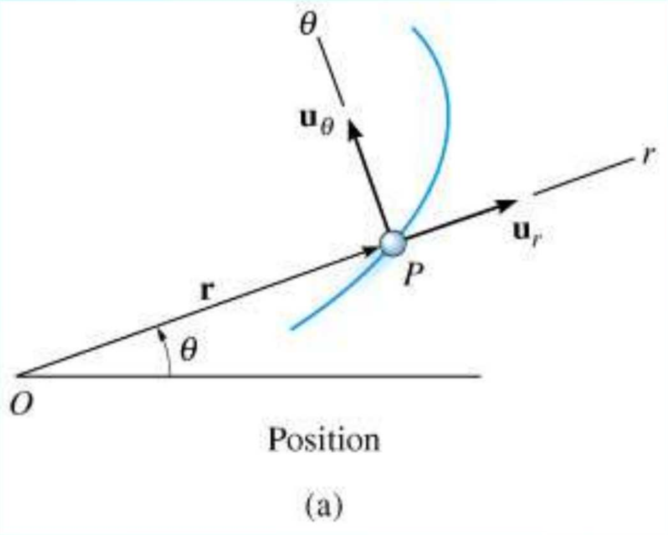
PHYS 170 Section 101
Lecture 20
October 24, 2018

Lecture Outline/Learning Goals

- Worked problem of curvilinear motion using polar components
- 12.9 Absolute Dependent Motion: Analysis of Two Particles
- Worked example(s) of absolute dependent motion (pulley and block systems)

Curvilinear Motion: Polar Coordinates—Summary



- **POSITION**

$$\mathbf{r} = r\,\mathbf{u}_r$$

- **VELOCITY**

$$\mathbf{v} = v_r\,\mathbf{u}_r + v_\theta\,\mathbf{u}_\theta$$

$$v_r = \dot{r}$$

$$v_\theta = r\,\dot{\theta}$$

- **ACCELERATION**

$$\mathbf{a} = a_r\,\mathbf{u}_r + a_\theta\,\mathbf{u}_\theta$$

$$a_r = \ddot{r} - r\dot{\theta}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

TIME DERIVATIVES

- Note that to determine **v** and **a** in polar coordinates, we need the four time derivatives $\dot{r},\ \ddot{r},\ \dot{\theta},\ \ddot{\theta}$

- Generally encounter 2 types of problems

1. Particle path is given in the form $r = r(t),\quad \theta = \theta(t)$ (parametric specification)

   – In this case, we can evaluate the derivatives directly

   – Example: Consider path given by $r = 4t^2$ and $\theta = 8t^3 + 6$. Then

$$\dot{r} = 8t \qquad \dot{\theta} = 24t^2$$

$$\ddot{r} = 8 \qquad \ddot{\theta} = 48t$$

TIME DERIVATIVES (continued)

2. Particle path is given in the form $r = f(\theta)$

– In this case we must use the **chain rule** to compute the relationship between the radial and angular (transverse) derivatives

– Example: Consider the path given by $r^2 = 6\theta^3$. Then

$$2r\dot{r} = 18\theta^2\dot{\theta}$$

and

$$2r\ddot{r} + 2\dot{r}^2 = 18\theta^2\ddot{\theta} + 36\theta\dot{\theta}^2$$

$$r\ddot{r} + \dot{r}^2 = 9(\theta^2\ddot{\theta} + 2\theta\dot{\theta}^2)$$

TIME DERIVATIVES (continued)

- Will usually know $\theta(t)$ so will be able to compute $\dot{\theta}$ and $\ddot{\theta}$ and then use equations like the above to calculate $\dot{r}$ and $\ddot{r}$.

- Alternatively, if we are given $r = f(\theta)$ and the velocity and acceleration **magnitudes**, then we can use the equations from $r = f(\theta)$, plus

$$v^2 = \dot{r}^2 + (r\dot{\theta})^2$$

$$a^2 = (\ddot{r} - r\dot{\theta}^2)^2 + (r\ddot{\theta} + 2\dot{r}\dot{\theta})^2$$

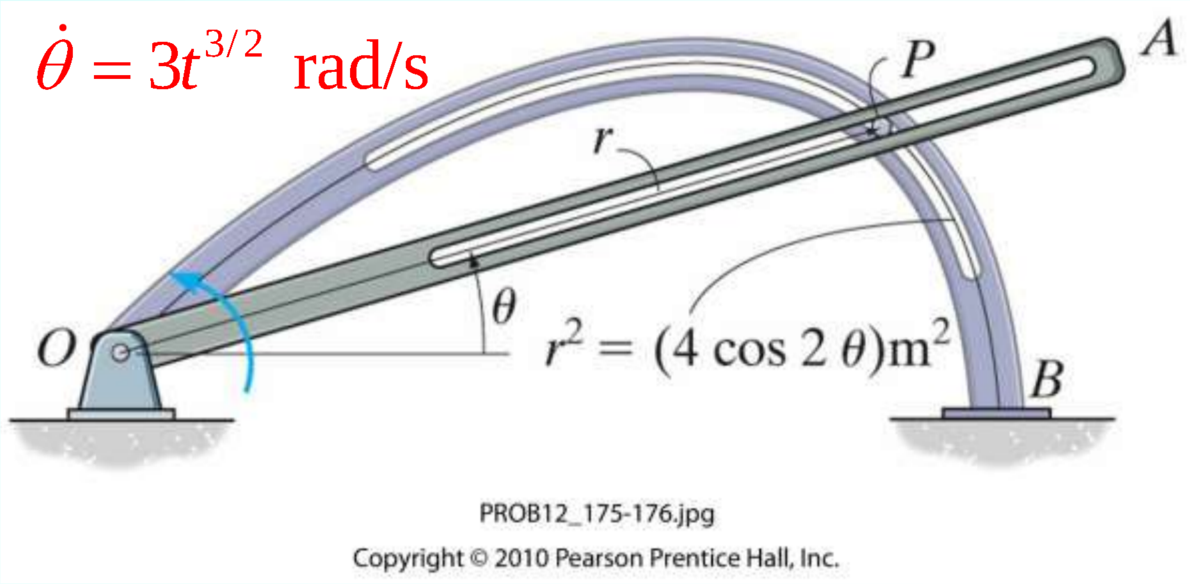
to determine $\dot{r}$, $\ddot{r}$, $\dot{\theta}$, $\ddot{\theta}$ (but only up to a sign, in general)

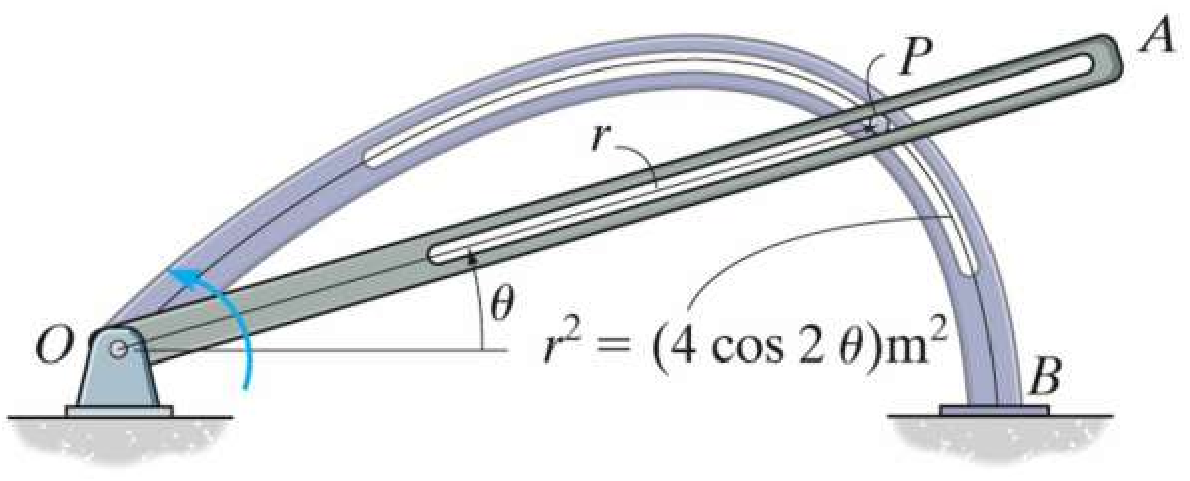
Problem 12-176 (page 78, 12th edition)

The motion of ball P is constrained by the curved slot in OB and by the slotted arm OA. OA rotates counterclockwise with angular speed $3t^{3/2}$ rad/s where t is in seconds and $\theta = 0$ when $t = 0$.

(1) Determine the time when $\theta = 30^{\circ}$.

(2) Determine the radial and transverse components of the ball's velocity and acceleration when $\theta = 30^{\circ}$.





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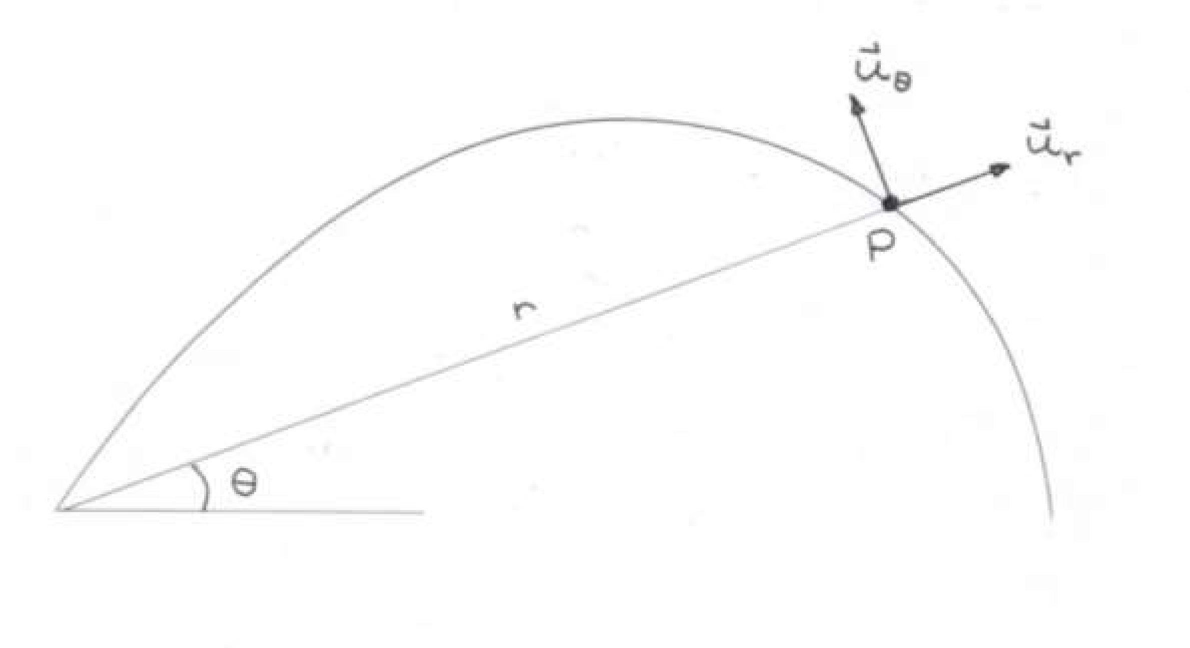
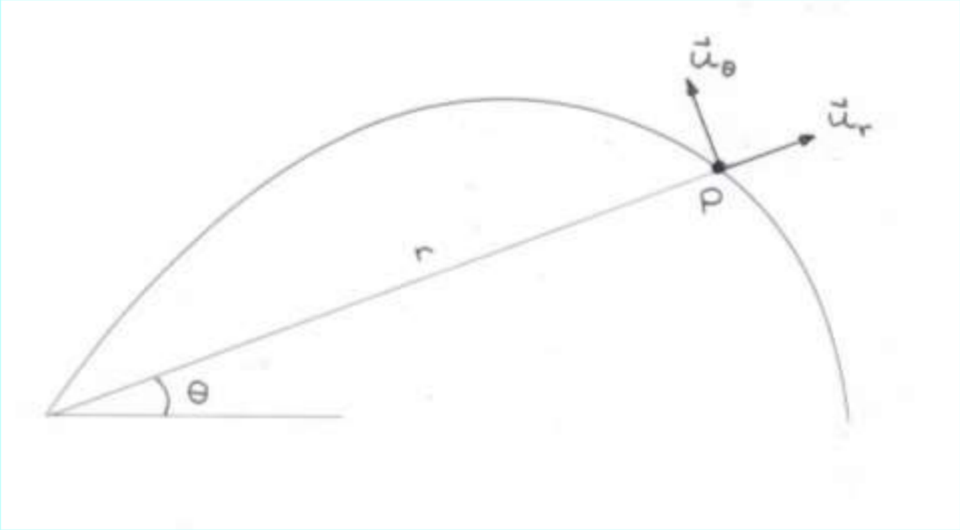




Solution strategy

Integrate angular speed $\dot{\theta}(t)$ to determine angle $\theta(t)$ arm travels through in time t.

Determine time t at which arm has travelled 30°.

Differentiate expressions for r^2 and $\dot{\theta}$ to get equations involving r, $\dot{r}$, $\ddot{r}$, θ, $\dot{\theta}$, $\ddot{\theta}$.

Use those equations to determine components of velocity and acceleration.

Data: $r^2 = 4\cos 2\theta \qquad \dot{\theta} = 3t^{3/2} \qquad \theta = 30^\circ = \pi / 6\,\text{rad}$

Integrate angular speed $\dot{\theta}$ to determine $\theta(t)$ (in radians!)

$$\dot{\theta} = \frac{d\theta}{dt}$$

$$\int_0^\theta d\theta = \int_0^t \dot{\theta}\,dt$$

$$\theta(t) = \int_0^t 3t^{3/2} dt = 6t^{5/2} / 5$$

Time t at which $\theta(t) = \pi / 6$ is given by the solution of

$$6t^{5/2} / 5 = \pi / 6$$

which yields $t = 0.718$ s

Data: $r^2 = 4\cos 2\theta$ (1) $\dot{\theta} = 3t^{3/2}$ (2) $\theta = 30^\circ = \pi/6\,\text{rad}$ (3)

Derivatives

Differentiate r with respect to t

$$2r\dot{r} = -8\dot{\theta}\sin 2\theta \qquad (4)$$

Differentiate again with respect to t

$$2(r\ddot{r} + \dot{r}^2) = -8(\ddot{\theta}\sin 2\theta + 2\dot{\theta}^2\cos 2\theta) \qquad (5)$$

Differentiate $\dot{\theta}$ with respect to t

$$\ddot{\theta} = \frac{9}{2}t^{1/2} \qquad (6)$$

Using $t = 0.7177$ s, we can now determine numerical values for r, $\dot{r}$, $\ddot{r}$, θ, $\dot{\theta}$, $\ddot{\theta}$ as follows:

θ: Given, eqn. (3) $= 30^\circ$

$\dot{\theta}$: Eqn. (2) $= 1.824$ rad/s

$\ddot{\theta}$: Eqn. (6) $= 3.812$ rad/s^2

r: Eqn. (1) $= 1.414$ m

$\dot{r}$: Eqn. (4) $= -4.468$ m/s

$\ddot{r}$: Eqn. (5) $= -32.862$ m/s^2

Using the above values we can now compute the radial and transverse components of the velocity and acceleration (Exercise: verify calculations)

Velocity

$$\vec{v} = v_r \vec{u}_r + \mathrm{v}_\theta \vec{u}_\theta$$

$$v_r = \dot{r} = -4.47 \text{ m/s}$$

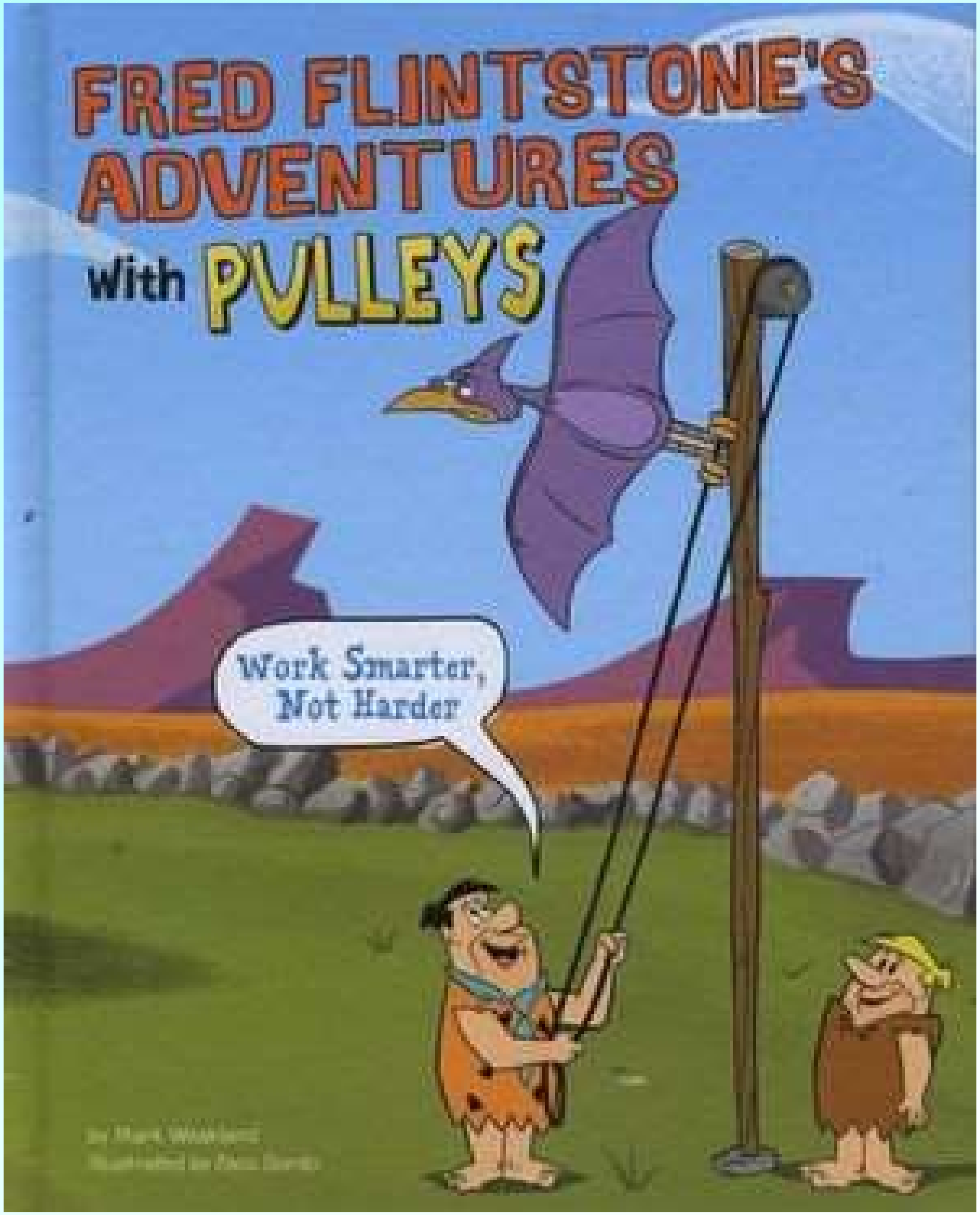
$$v_\theta = r\dot{\theta} = 2.58 \text{ m/s}$$

Acceleration

$$\vec{a} = a_r \vec{u}_r + a_\theta \vec{u}_\theta$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -37.6 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = -10.9 \text{ m/s}^2$$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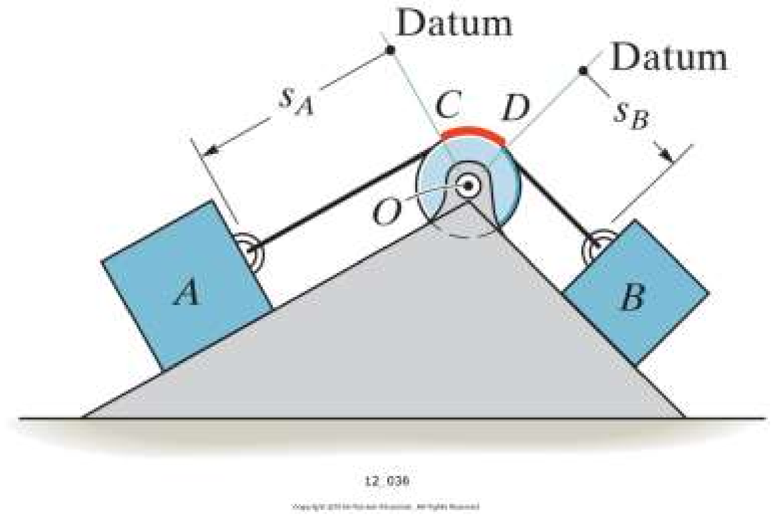
Absolute Dependent Motion: Analysis of Two Particles

- Consider situation where motion of one particle is dependent on motion of another

Datum: Convenient location from which a displacement is measured.

- Main application we will consider is a general system of pulleys and blocks: a very simple one is shown in the figure



- Two position coordinates s_A and s_B are related by

$$s_A + l_{CD} + s_B = l_T$$

- Now, l_{CD} = the length of rope from C to D and l_T = the total length of rope are *constant* lengths. Thus, differentiating the above with respect to time, we have

$$v_A + v_B = 0 \qquad \text{or} \qquad v_B = -v_A$$

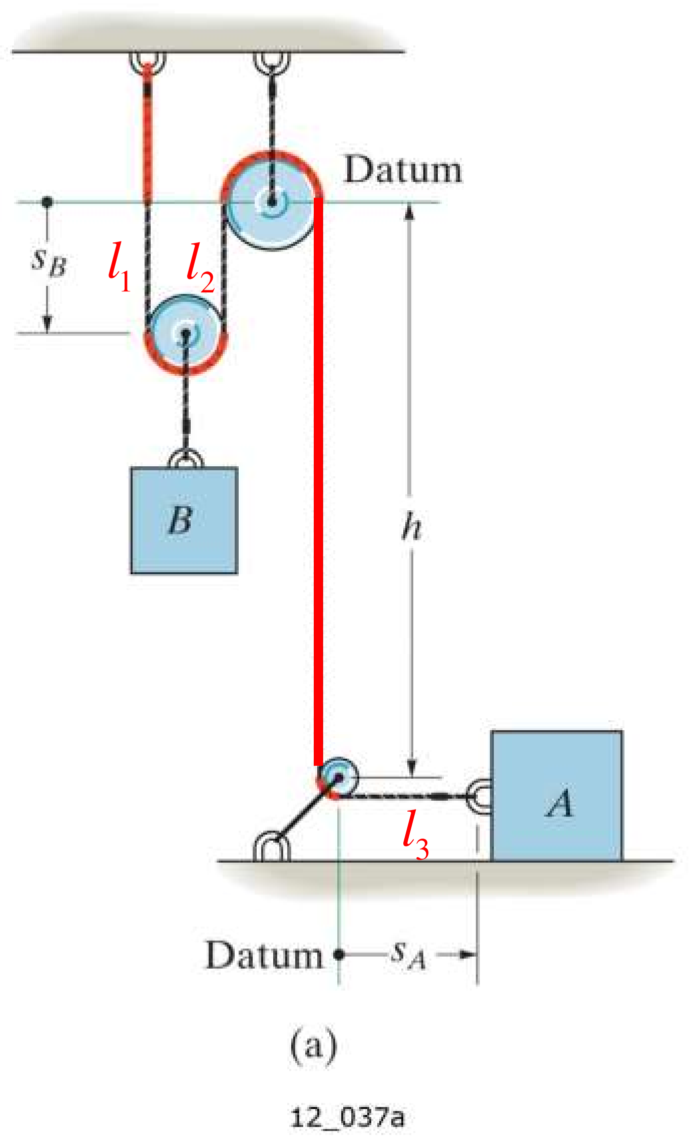
- The text outlines a general procedure for analyzing pulley-and-block systems, but we will adopt an even more systematic approach which is particularly useful when the systems get complicated. That said, there are many ways to solve this type of problem and if you find or invent one that suits you better, by all means use it.

- The method is based on the use of two types of equations:

  - Rope equations: so called because they are based on the fact that the total length of any cord in a pulley system is a constant

  - Path equations: so called since they generally relate the displacement of one or more blocks to a sequence (path) of one or more segments of rope

- For both types of equations, we make use of the fact that certain segments of rope are of constant length (such as the segments that are wrapped around a pulley wheel)



Example

Determine the relationship between v_A and v_B .

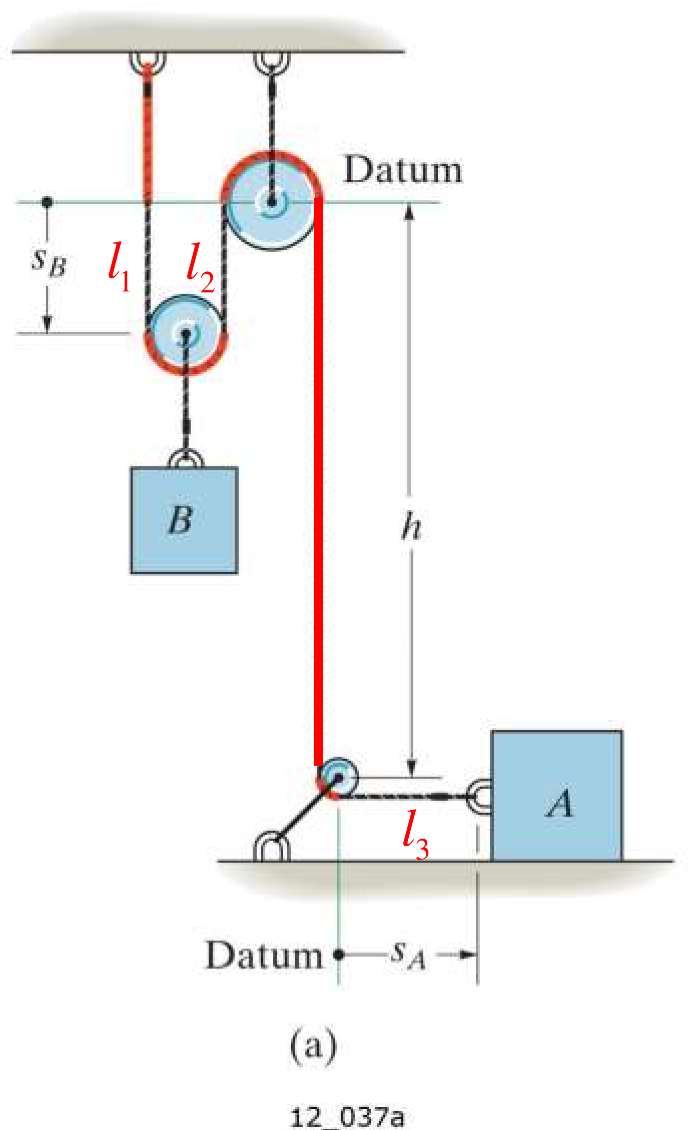
1. Label the non-constant length segments of the rope, l_1, l_2, l_3

2. Write down the rope equation

$$l_1 + l_2 + l_3 = \text{constant}$$

3. Write down the path equations (when summed these should contain all of the l_i's)

$$l_1 = s_B$$
$$l_2 = s_B$$
$$l_3 = s_A$$



4. Sum the path equations

$$l_1 + l_2 + l_3 = s_A + 2s_B$$

5. Use the rope equation to eliminate the l_i's

$$\text{constant} = s_A + 2s_B$$
$$s_A + 2s_B = \text{constant}$$

6. Differentiate with respect to time to get the relationship between the velocities

$$v_A + 2v_B = 0$$
$$v_B = -\frac{1}{2}v_A$$