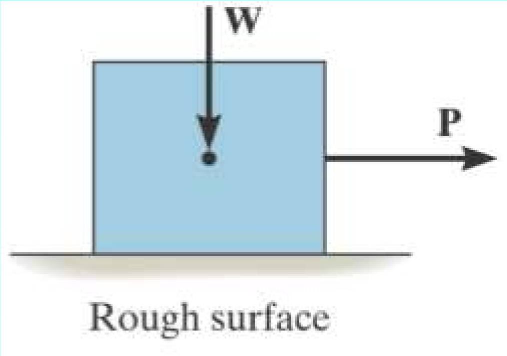
PHYS 170 Section 101
Lecture 15
October 10, 2017

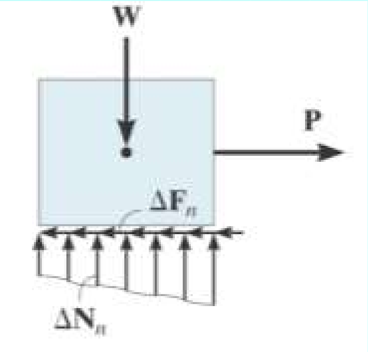
8.1 Characteristics of Dry Friction

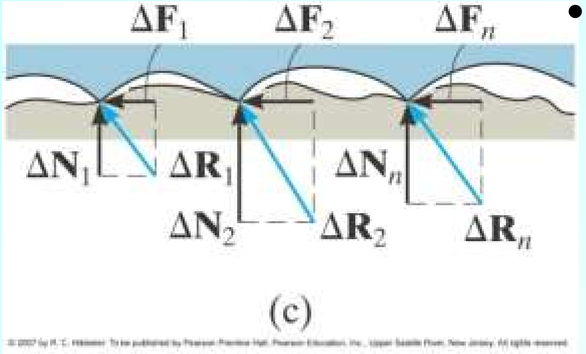
- **FRICTION** (definition): Force of resistance acting on a body that prevents/retards slipping of body relative to another body or surface with which it is in contact
  - Force always acts **tangent** to surface at point of contact
  - Direction of force opposes possible/actual motion of body relative to contact points
- Two general types of friction between surfaces
  - **Fluid friction**: Contacting surfaces separated by film of fluid (won't consider here)
  - **Dry friction**: No lubricating fluid between surfaces of body in contact

- **Important note**: As we will briefly see, a full understanding of dry friction requires a detailed examination of the microscopic interaction of rough surfaces. This is a complicated subject, and one which is still not completely understood. The approach adopted here, including the various equations for the magnitude of frictional forces, is largely empirical in nature (i.e. based on experiments).

Theory of Dry Friction



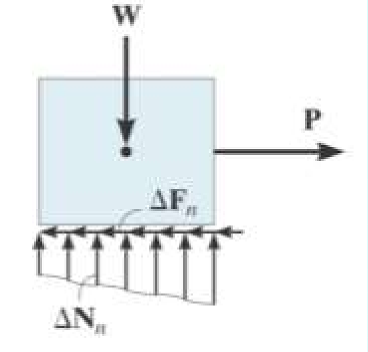




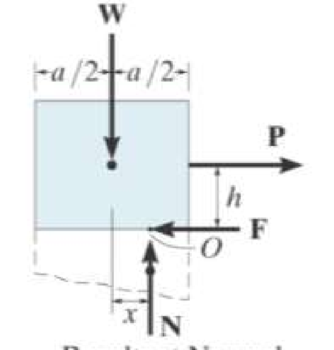
(c)

- Consider block of weight **W** being pulled horizontally by applied force **P** along rough horizontal surface
- To fully understand friction, must take into account deformations of surfaces of contact, i.e. surfaces cannot be considered rigid
- On microscopic scale, friction develops as a result of "interlocking" contacts between irregularities in the surfaces
- Each contact results in reactive force $\Delta \mathbf{R}_n$ which has a frictional component $\Delta \mathbf{F}_n$ and a normal component $\Delta \mathbf{N}_n$ (n labels the microscopic contact, Δ stresses that we are considering small-scale forces)

$$\Delta \mathbf{R}_n = \Delta \mathbf{F}_n + \Delta \mathbf{N}_n$$



- **Note**: We can define a normal force as a force of reaction due to the contact of 2 bodies, and which acts perpendicularly to the point/surface of contact (e.g. reactions on smooth surface support or roller in Table 5.2)



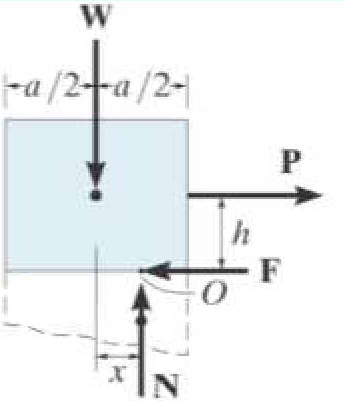
Resultant Normal and Frictional Forces

- Simplifying assumption: Can analyze friction effects by replacing loading due to all of the microscopic reactions with resultant vectors

$$\mathbf{F} = \sum_n \Delta \mathbf{F}_n \qquad \text{Resultant friction force}$$

$$\mathbf{N} = \sum_n \Delta \mathbf{N}_n \qquad \text{Resultant normal force}$$

- Now consider 3 cases separately, wherein the block is 1) in equilibrium (not moving, or about to move) 2) in equilibrium, but just about to move, and 3) moving



Resultant Normal and Frictional Forces

EQUILIBRIUM

- Assume block is in equilibrium and consider its FBD as in the figure to the left
- Observe
  1. **F** acts tangentially to surface and **opposes** **P**
  2. **N** acts normally (perpendicular) to surface, and is directed upwards to balance **W**

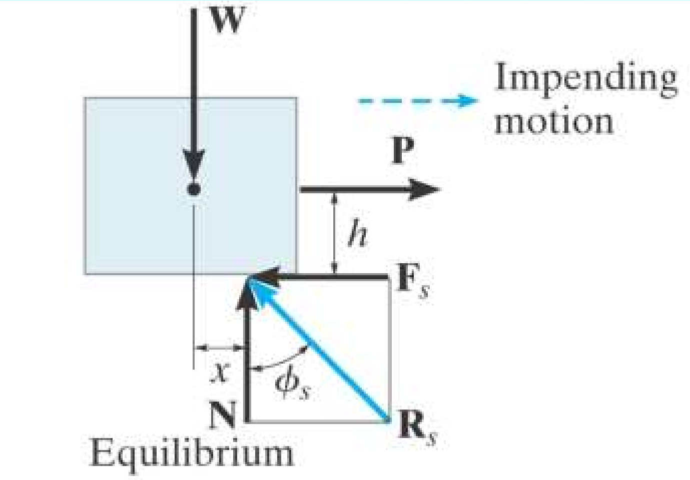
- We can determine *where* **N** acts by considering moment equilibrium about point O

$$Wx = Ph \qquad \text{(counterclockwise and clockwise moments balance)}$$

or, solving for x

$$x = \frac{Ph}{W}$$

- **Important special case:** When $x=a/2$, the normal forces acts at the right corner of the block, and block is on verge of tipping (i.e. on verge of not being in moment equilibrium)



IMPENDING MOTION

- Depending on circumstances (e.g. *h* small, contact surfaces "slippery") frictional force **F** may not be large enough to balance **P**

- In this case, the block *slips* (translates) before it tips

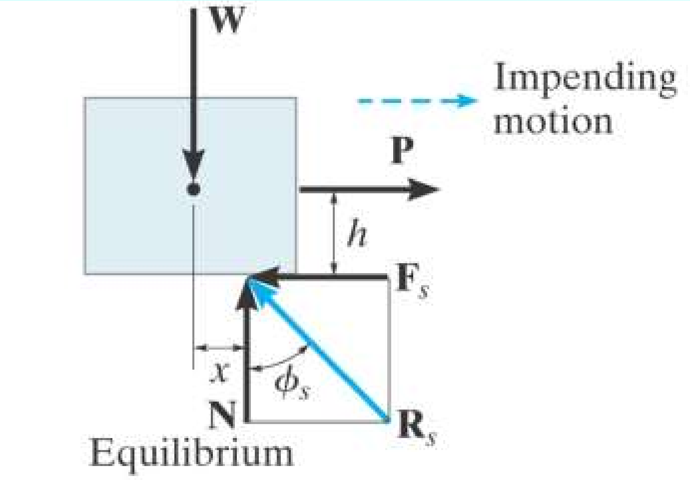
- Imagine slowly increasing *P* from 0 – then *F* also slowly increases, but only to some **maximum value**, F_s

- F_s is known as the **limiting static frictional force**; any further increase in *P* causes the block to slip

- The following experimentally determined formula relates the limiting frictional force to the normal force

$$F_s = \mu_s N$$

where μ_s is known as the **coefficient of static friction**



- When the block is on the verge of slipping, the normal force, **N**, and the friction force, $\mathbf{F}_s$, yield a resultant

$$\mathbf{R} = \mathbf{N} + \mathbf{F}_s$$

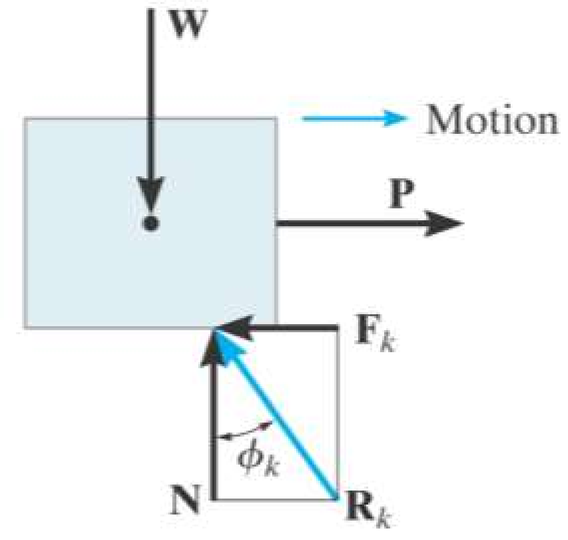
that makes an angle ϕ_s with the vertical as shown in the diagram

$$\phi_s = \tan^{-1}\left(\frac{F_s}{N}\right) = \tan^{-1}\left(\frac{\mu_s N}{N}\right) = \tan^{-1}\mu_s$$

- ϕ_s is thus known as the **angle of static friction**

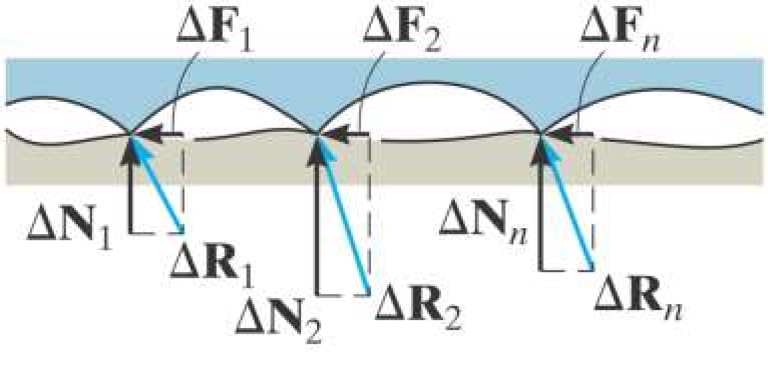
SOME REMARKS ON COEFFICIENT OF STATIC FRICTION, μ_s

- Has no dimension (no units)
- Tends to satisfy $\mu_s < 1$
- Depends on which two materials are in contact
- Typically given in tabular form from experiments (see, e.g., Table 8.1 of text), and for given materials, will frequently span some range of values

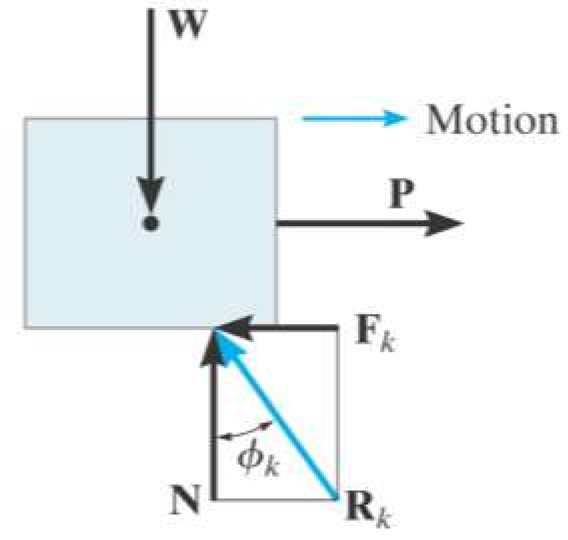


MOTION

- Increasing pulling force so that $P > F_s$ causes a decrease in the frictional force to a value F_k, known as the **kinetic frictional force** $(F_k < F_s)$
- When this occurs, the block will not be in equilibrium, so will begin to slide with some acceleration



- Drop in frictional force once block starts moving can be understood in terms of points of contact “riding over” one another – sticking-unsticking at high points is the primary contributor to kinetic friction
- Reaction forces are “more vertical” in this case, so tangential frictional components are reduced relative to static case



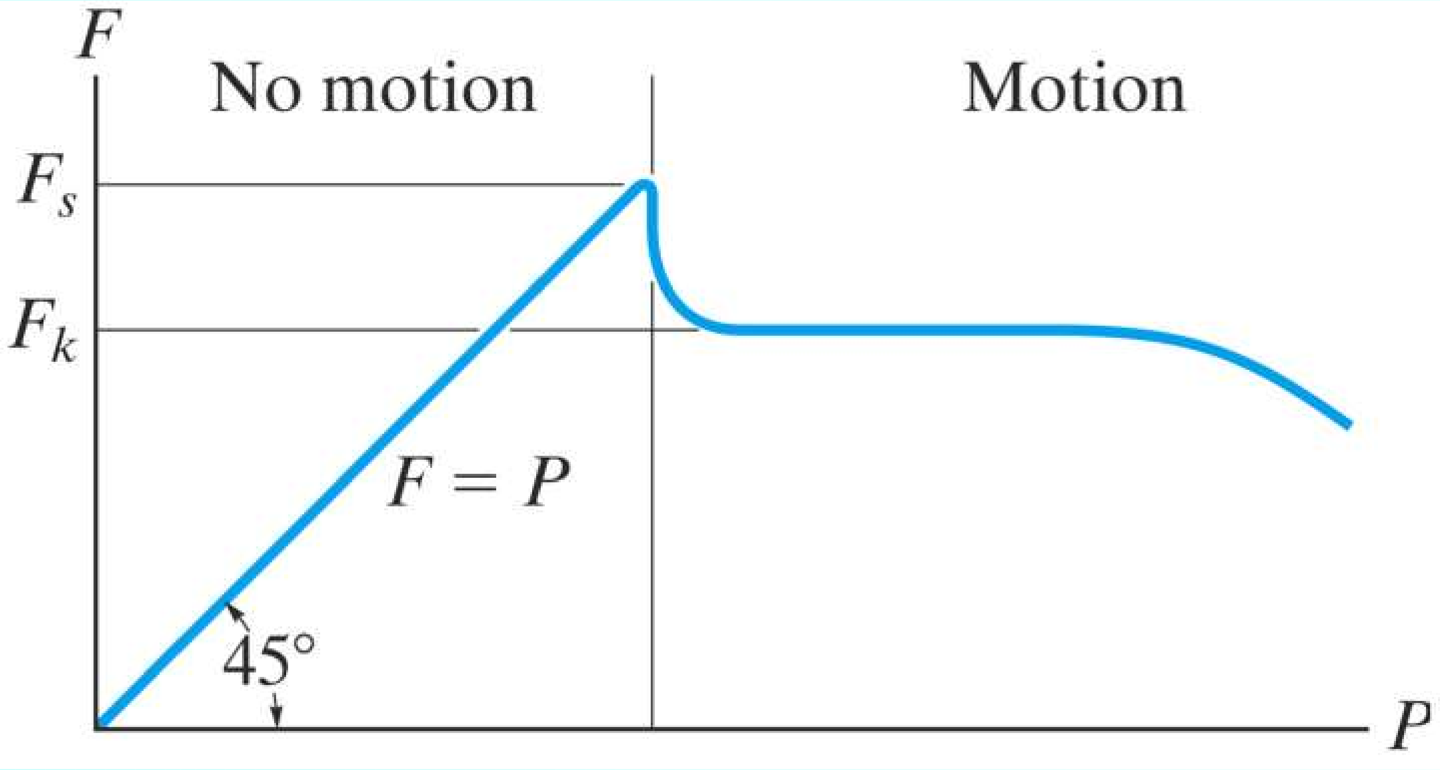
- As with the static friction case, we have an experimentally determined relation relating the normal and frictional forces

$$F_k = \mu_k N$$

where μ_k is known as the **coefficient of kinetic friction**, and tends to be about 25% smaller than μ_s

- Analogously to ϕ_s, we can define the **angle of kinetic friction**, ϕ_k, by

$$\phi_k = \tan^{-1}\left(\frac{F_k}{N}\right) = \tan^{-1}\left(\frac{\mu_k N}{N}\right) = \tan^{-1}\mu_k$$



- The variation of the magnitude of the frictional force, *F*, as a function of the magnitude of the applied force, *P*, is shown schematically in the above figure
- The decrease in *F* at large *P* is due to effects not considered here (such as aerodynamic effects since the body tends to be moving at high speed when *P* is very large)

Summary and Key Characteristics of Dry Friction

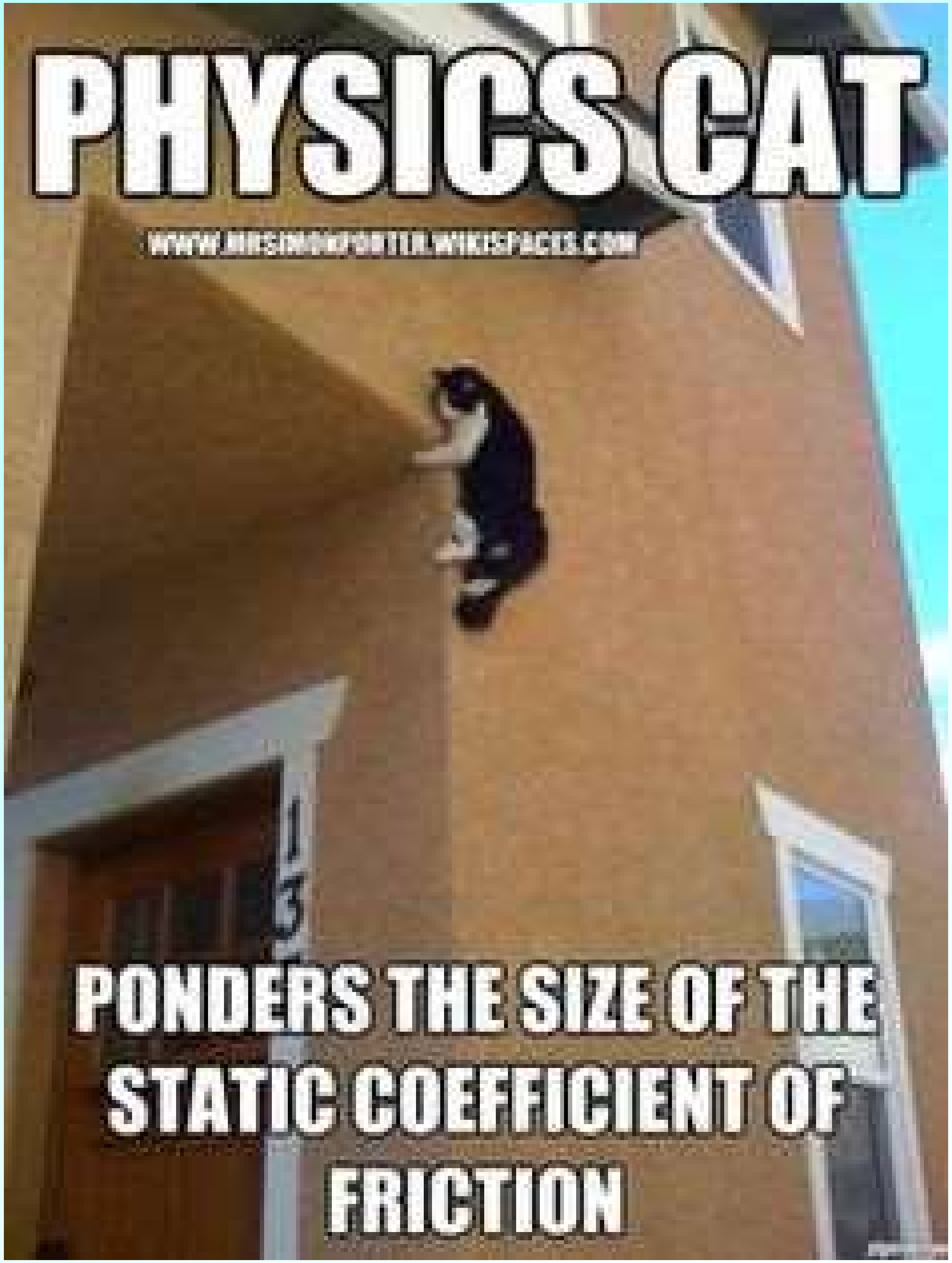
- Frictional force, **F**, acts tangentially to contact surfaces and opposes relative motion (or tendency for motion)

- Maximum static friction force, F_s , can usually be assumed to be **independent of contact area**

- Generally, $F_s > F_k$ for any two surfaces in contact

- When slipping/sliding is **about to occur**, we have $F_s = \mu_s N$

- When slipping/sliding **is occurring**, we have $F_k = \mu_k N$


PHYSICS CAT
PONDERS THE SIZE OF THE STATIC COEFFICIENT OF FRICTION

8.2 Problems Involving Dry Friction

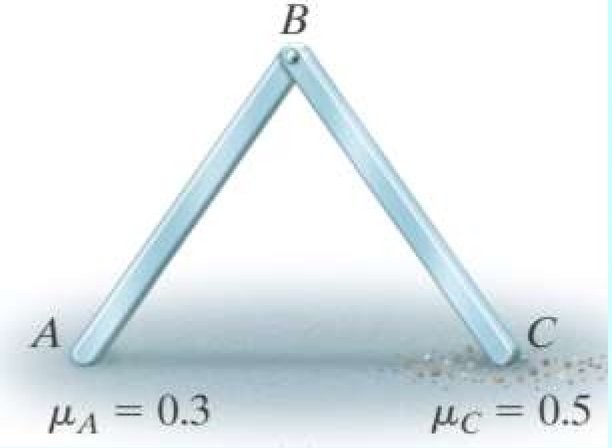
- In solving rigid body equilibrium problems involving dry friction, force system will have to satisfy laws of frictional forces in addition to equations of equilibrium

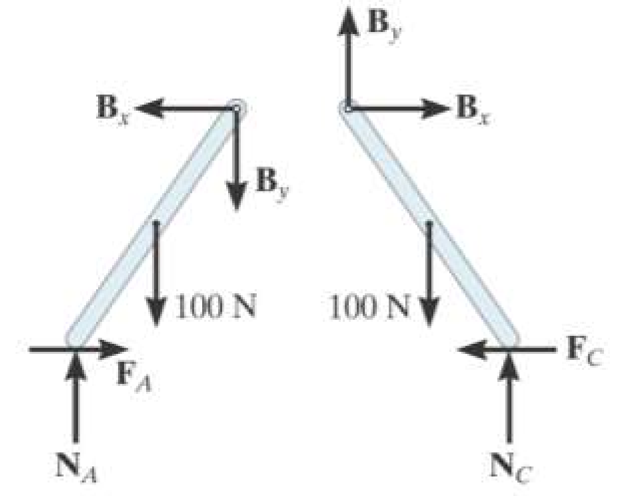
- Problems generally fall into 1 of 3 types; type can be identified from FBDs, identification and enumeration of unknowns, comparison of number of unknowns with the total number of available equilibrium equations and statements about state of free bodies (e.g. motion, impending motion)

- Proceed to briefly discuss each type in turn

1. EQUILIBRIUM

- This type of problem involves strict equilibrium, no motion or impending motion
- Total number of unknowns must equal total number of available equilibrium equations





- In example at left need to check that frictional forces at *A* and *C* are sufficient to maintain equilibrium

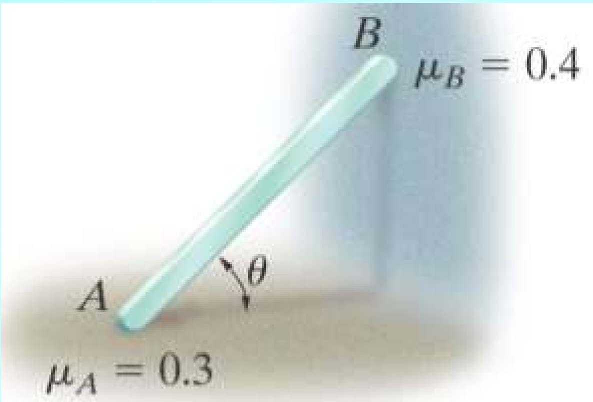
- From FBDs for two members, we see that we have a total of 6 unknowns, which **does** equal the total number of available equilibrium equations ($\Sigma F_x = \Sigma F_y = \Sigma M = 0$ for each member)

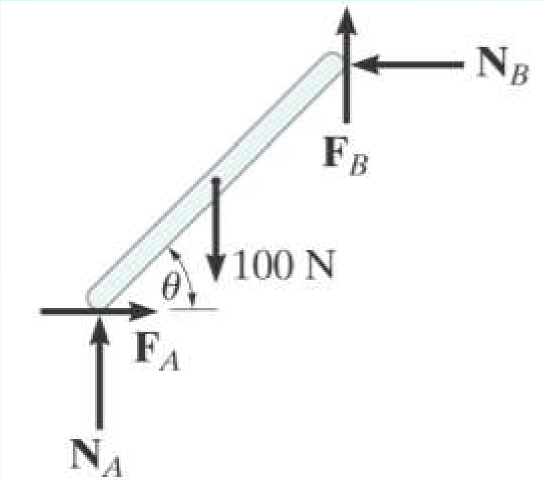
- Once frictional forces $\mathbf{F}_A$ and $\mathbf{F}_C$ are determined, must check to ensure that they satisfy $F \leq \mu_s N$ or slippage occurs and the members **cannot** be in equilibrium

2. IMPENDING MOTION AT ALL POINTS (ALL INTERFACES)

- Here, the total number of unknowns will equal the total number of available equilibrium equations, plus the total number of available frictional equations

Motion impending:	$F_s = \mu_s N$
Body slipping:	$F_k = \mu_k N$





- In example at right, want to find smallest angle θ such that bar does not slip
- Have 5 unknowns in the FBD, N_A, F_A, N_B, F_B and θ
- Have 3 equations for static equilibrium
- Also have 2 frictional equations (for a total of 5 equations) since motion is impending at both *A* and *B*

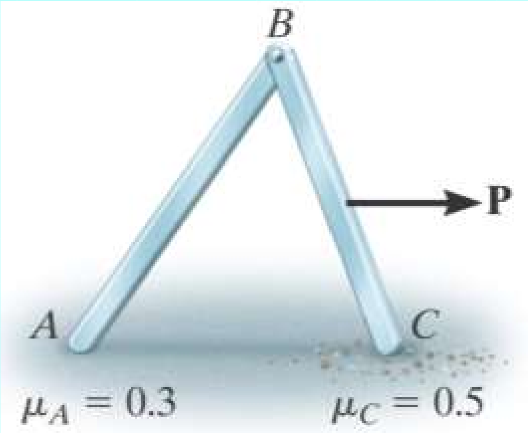
$$F_A = \mu_A N_A = 0.3\,N_A$$
$$F_B = \mu_B N_B = 0.4\,N_B$$

3. IMPENDING MOTION AT SOME POINTS

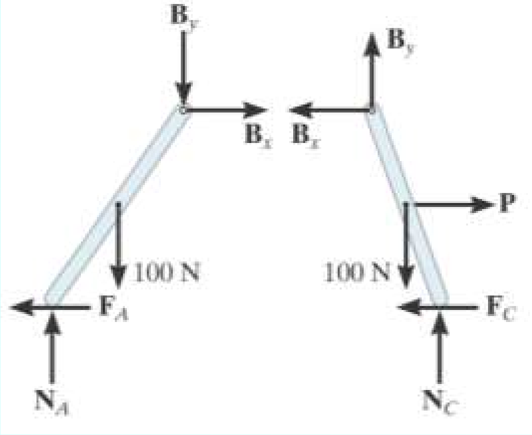
- Here, the total number of unknowns will be less than the total number of available equilibrium equations, plus the total number of available frictional equations or conditional equations for tipping

- For this type of problem, there will be more than 1 possibility for motion/impending motion and part of the solution of the problem will involve figuring out which motion occurs, or is about to occur

3. IMPENDING MOTION AT SOME POINTS



- In example at right, want to determine the minimum applied force, P, that will cause movement
- Have 7 unknowns, N_A, F_A, N_C, F_C, B_x, B_y and P
- Have 6 equations for static equilibrium and 2 possible frictional equations



- Thus need to decide (and ultimately verify) which of the 2 frictional equations will be satisfied
- Two possibilities

Slipping at A, no slipping at C: $F_A = \mu_A N_A$ and $F_C \le 0.5 N_C$

Slipping at C, no slipping at A: $F_C = \mu_C N_C$ and $F_A \le 0.3 N_A$

- Calculate P for both possibilities, choose the one that gives smaller value of P
- More generally, also possible than only one case will be consistent with assumptions

Equilibrium vs Frictional Equations

- Recall that direction of frictional force, **F**, always opposes motion of body over surface

- In strict equilibrium problems, we can **assume** the direction of a force in the FBD; if the assumption is incorrect, the solution of the equations will yield a negative value for the "magnitude" of the force

- However, whenever $F = \mu N$ is used, we **cannot** assume the direction, so **F** must always be shown with the correct sense in the FBD in such cases (Imagine using $F = \mu N$ with an assumed direction for **F** and finding $F < 0$. Then the frictional equation would imply $N < 0$ as well.)

Additional Remarks on Solution of Friction Problems

- Will consider only **2D problems** in worked examples and homeworks

- Will generally solve using **scalar analysis** (for computing moments in particular)

- **FBDs**
  - Problems typically involve 2 (or more) free bodies, some unknowns (e.g. specific force magnitudes) will be common between diagrams (action/reaction pairs)
  - Will label force vectors with magnitudes; direction of vector in FBD gives actual/assumed direction of force

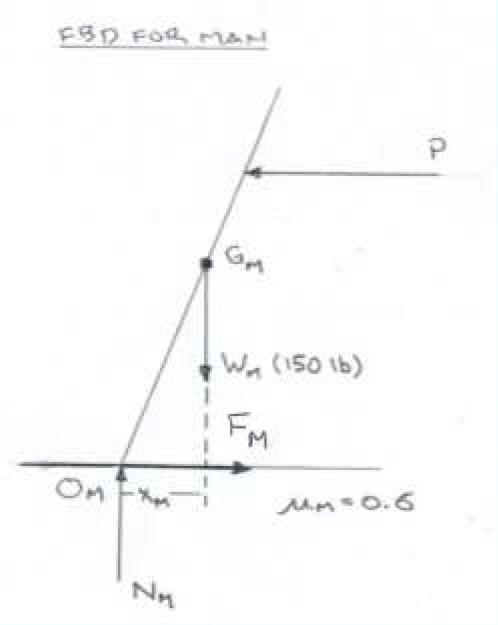
- In worked examples, will focus on problems of third type (impending motion at some points), since they tend to be the most challenging – if you master them, the other two types should be straightforward!

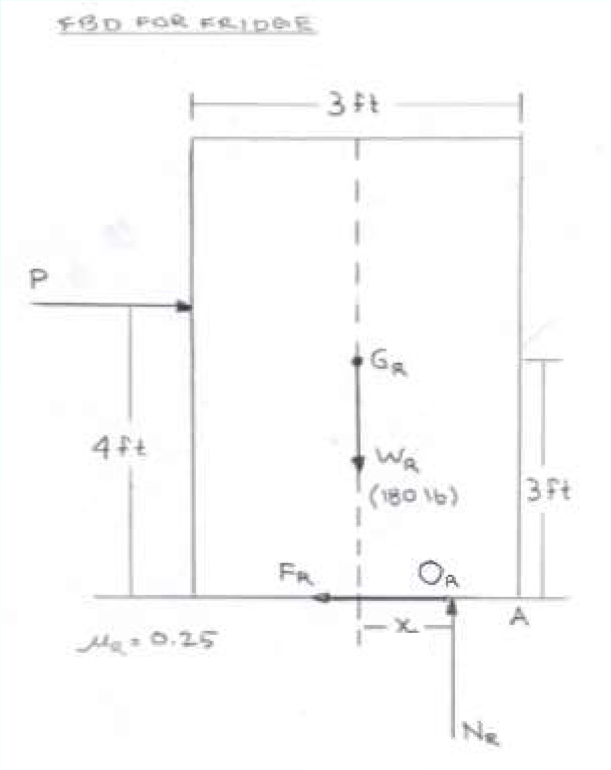
Problem 8-27 (page 405, 12th edition)

The refrigerator weighs 180 lb and rests on a tile floor. The coefficient of static friction μ_R between the refrigerator and the floor is 0.25.
The man weighs 150 lb. The coefficient of static friction μ_M between his shoes and the floor is 0.6. The man pushes horizontally on the refrigerator.

(1) Determine whether the man can move the refrigerator. If so, does the refrigerator slip or tip?


3 ft
G
1.5 ft
4 ft
3 ft
A
PROB08_026-027.jpg
Copyright © 2010 Pearson Prentice Hall, Inc.


FBD FOR MAN
P
G_M
W_M (150 lb)
F_M
O_M
x_M
$\mu_M = 0.6$
N_M


FBD FOR FRIDGE
3 ft
P
G_R
4 ft
W_R
(180 lb)
3 ft
F_R
O_A
A
x
$\mu_R = 0.25$
N_R

Data

$W_R = 180 \text{ lb} \qquad \mu_R = 0.25 \qquad W_M = 150 \text{ lb} \qquad \mu_M = 0.6$

Cartesian component equations of equilibrium

Refrigerator (take moments about O_R):

$$\sum F_x = 0: \qquad F_R = P \qquad (1)$$
$$\sum F_y = 0: \qquad N_R = 180$$
$$\sum (M_z)_R = 0: \qquad 180 x_R = 4P \qquad (2)$$

Man (take moments about O_M) :

$$\sum F_x = 0: \qquad F_M = P \qquad (3)$$
$$\sum F_y = 0: \qquad N_M = 150$$
$$\sum (M_z)_M = 0: \qquad 150 x_M = 4P \qquad (4)$$

Eqns. (1) to (4) contain 5 unknowns:

$F_R,\ P,\ F_M,\ x_R,\ x_M$

Need another equation to solve the problem.

ASSUME impending sliding for the refrigerator

Solution continues in Lecture 16