PHYS 170 Section 101

Lecture 12

October 1, 2018

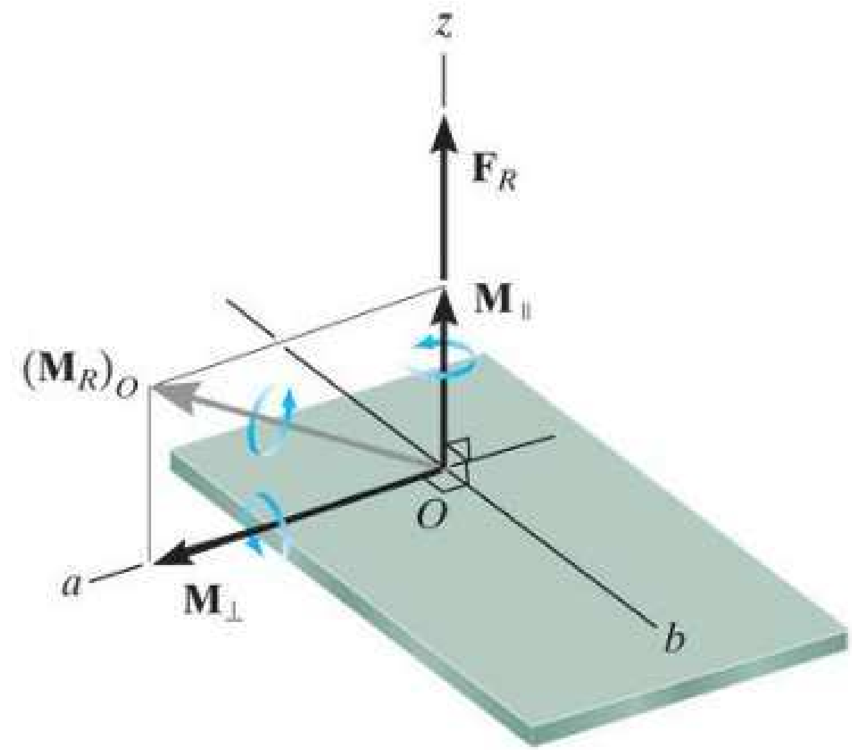
OCT 1—ANNOUNCEMENTS

- No tutorials next week
- Instead, will have review sessions for the midterm (Oct 9, usual tutorial locations)
- Attendance at these is strictly optional

Lecture Outline/Learning Goals

- Sample wrench problem
- END OF MATERIAL FOR FIRST MIDTERM (FRIDAY, OCT. 12)
- Begin Chapter 5—Equilibrium of a rigid body (three dimensional problems only)
- Conditions for rigid-body equilibrium
- Free-body diagrams

Recall: Reduction to a Wrench



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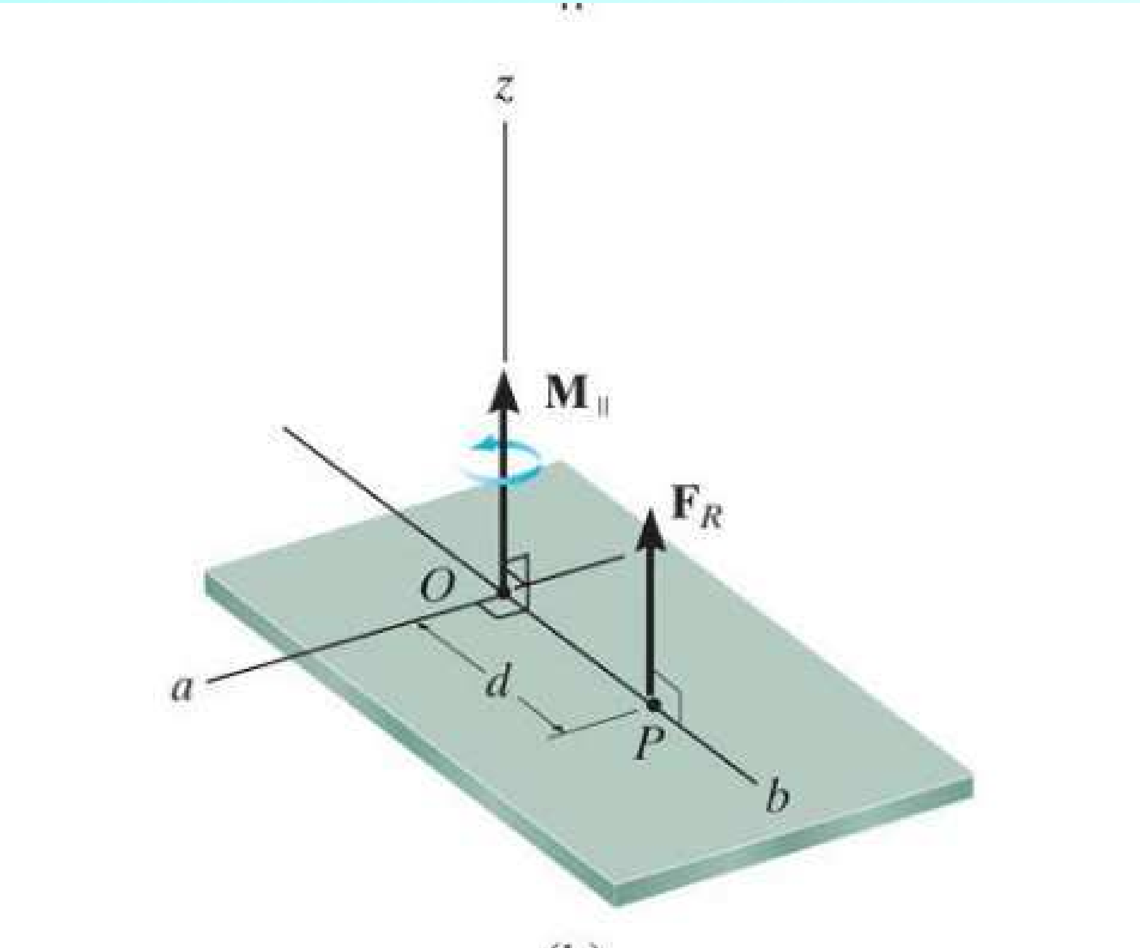
Reduction to a Wrench



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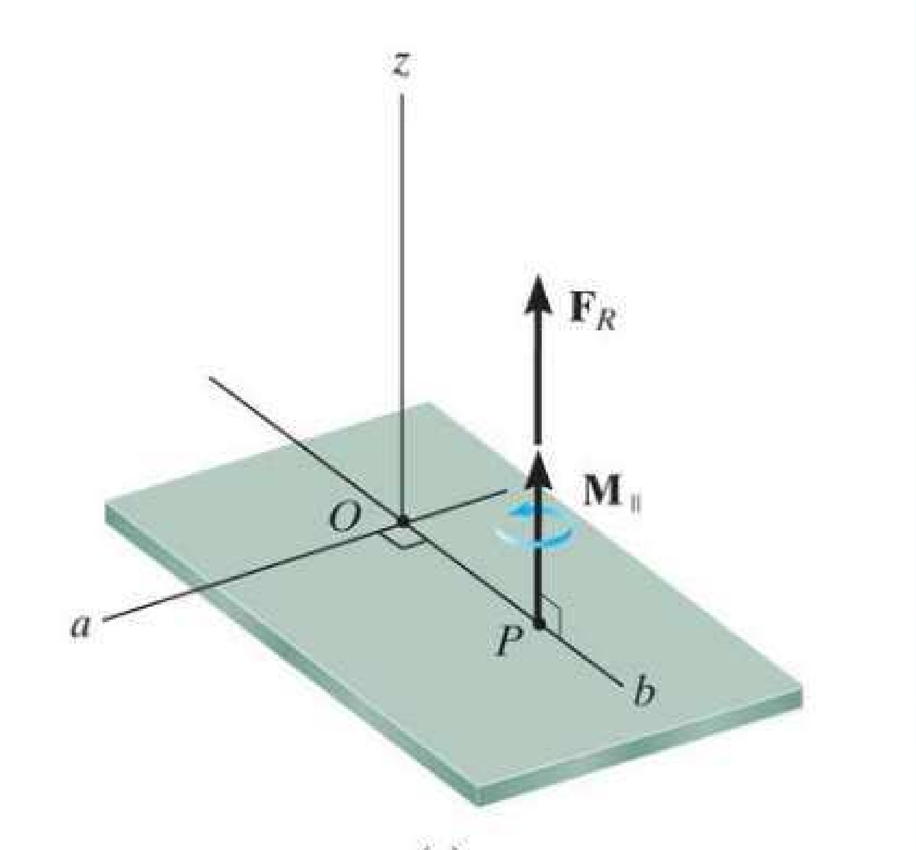
Reduction to a Wrench



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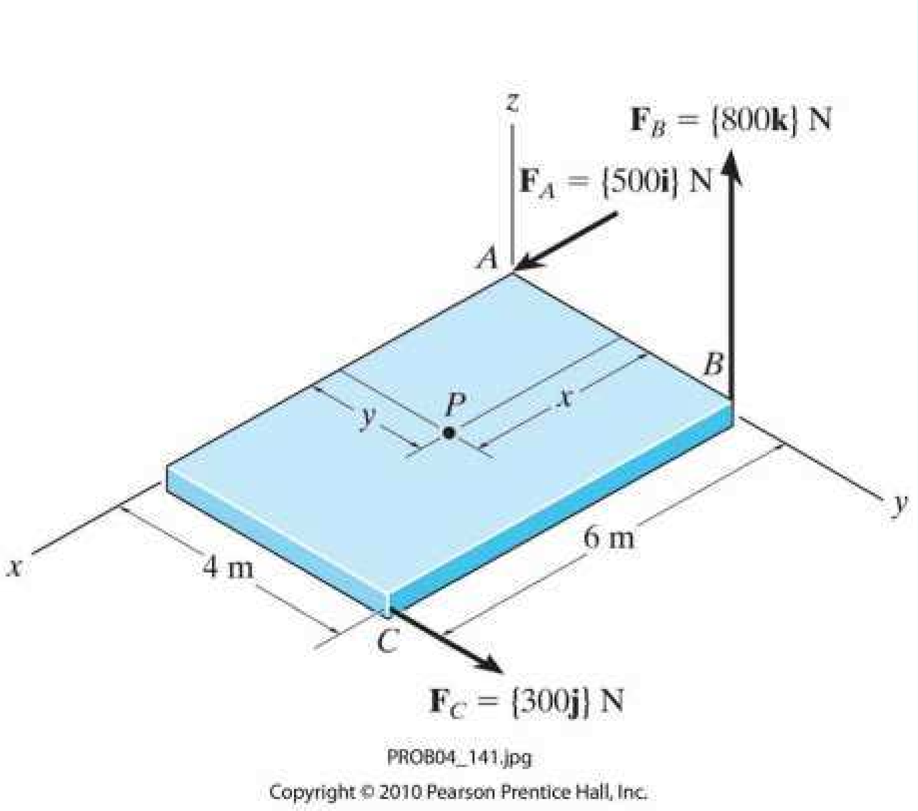
Problem 4-137 (page 182, 12th edition)

(1) Replace the three forces acting on the plate by a wrench.

(2) Determine the magnitude of the force and couple moment of the wrench and the point $P(x, y, 0)$ where its line of action intersects the plate.



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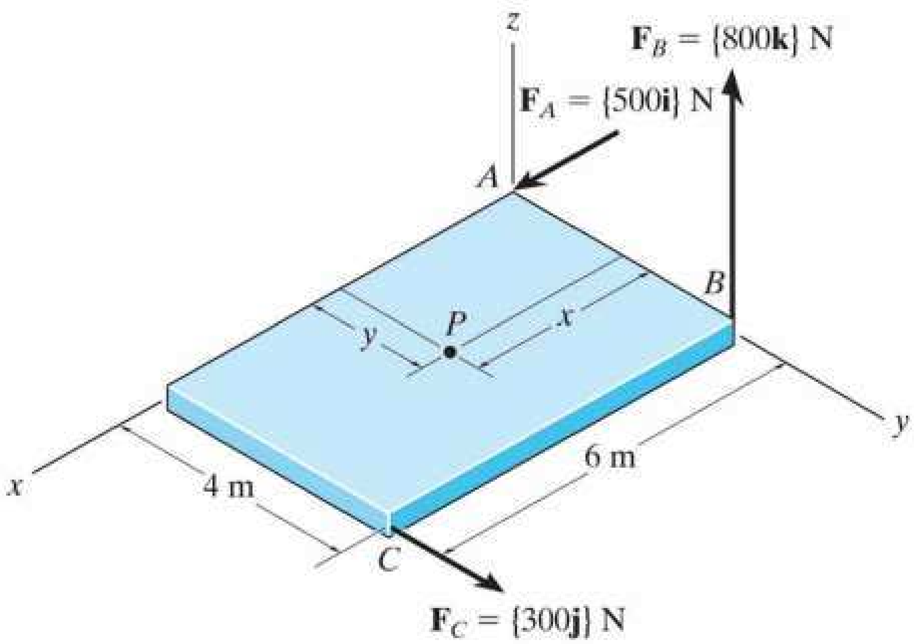

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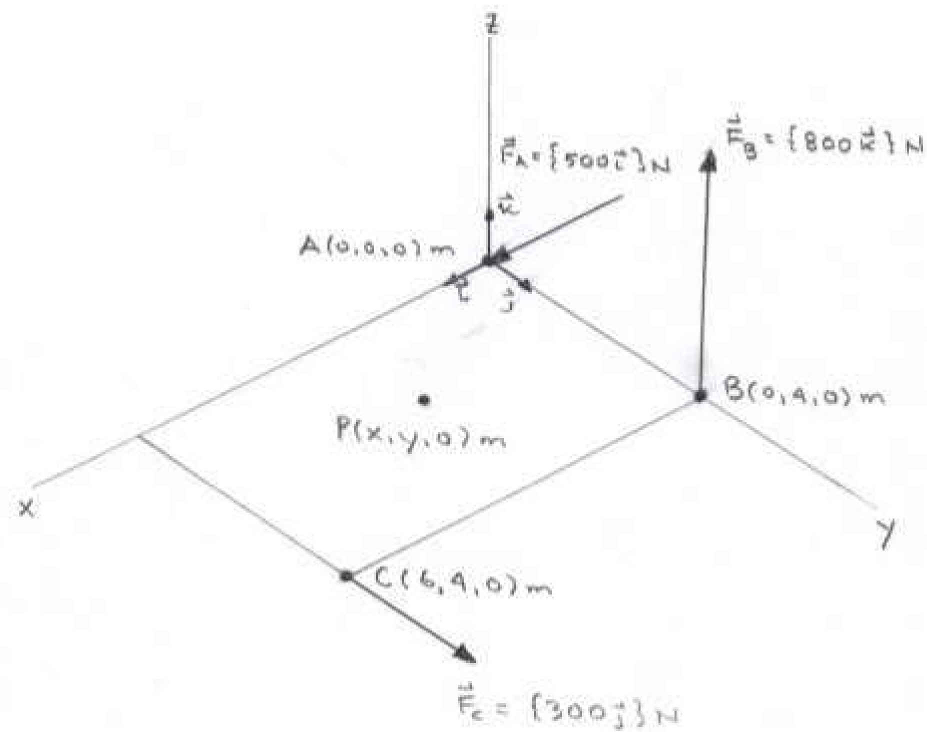
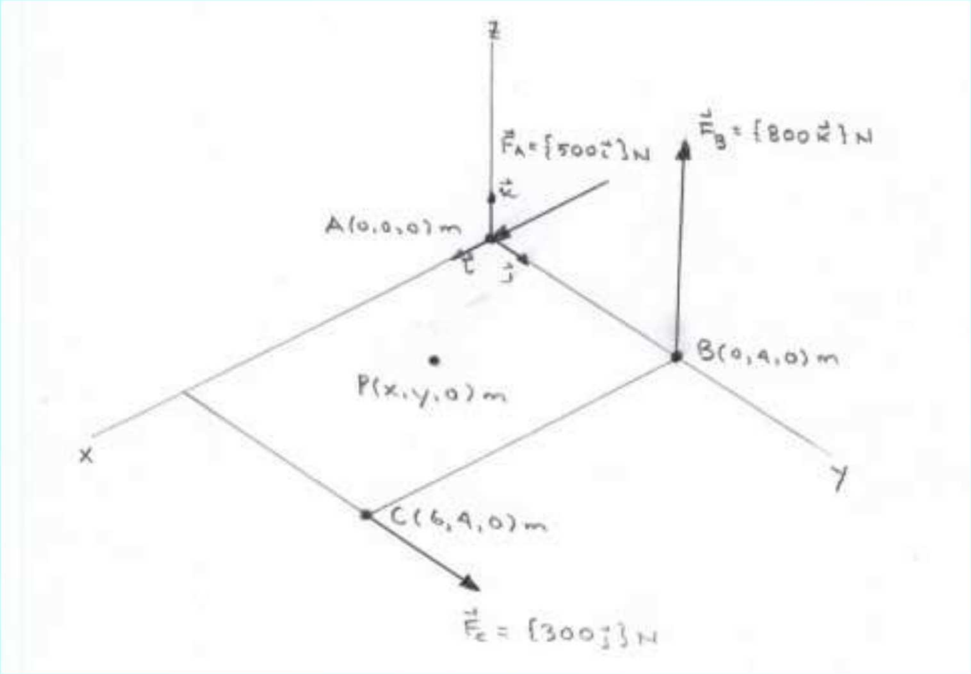




Solution strategy

(1) Compute equivalent resultant force at P using $\vec{F}_R = \sum \vec{F}$ and resultant couple moment at P using $(\vec{M}_R)_P = \sum \vec{M} + \sum (\vec{r} \times \vec{F})$

(2) Determine x, y and $(M_R)_P$ using fact that, for a wrench, $\vec{F}_R$ and $(\vec{M}_R)_P$ must be parallel or anti-parallel.

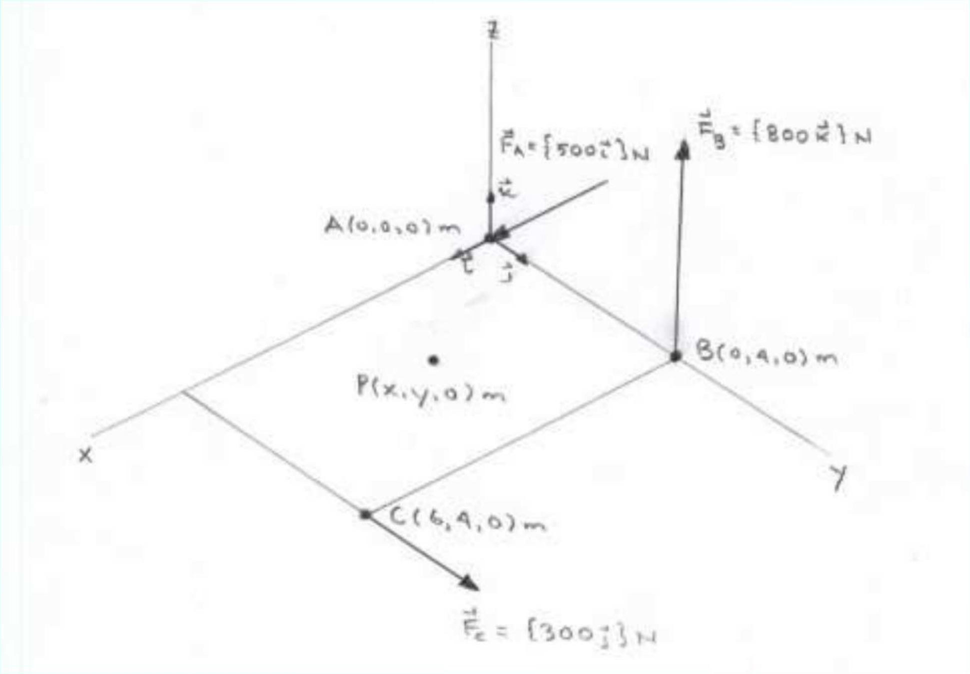
Coordinates (suppressing units until further notice)

$A(0,0,0)$

$B(0,4,0)$

$C(6,4,0)$

$P(x,y,0)$



Position vectors

$$\vec{r}_{PA} = -x\vec{i} - y\vec{j}$$

$$\vec{r}_{PB} = -x\vec{i} + (4-y)\vec{j}$$

$$\vec{r}_{PC} = (6-x)\vec{i} + (4-y)\vec{j}$$

Resultant force $\vec{F}_R$ at point P

$$\vec{F}_R = \sum \vec{F} = \vec{F}_A + \vec{F}_B + \vec{F}_C = 500\,\vec{i} + 300\,\vec{j} + 800\vec{k}$$

Magnitude of resultant force

$$F_R = \sqrt{500^2 + 300^2 + 800^2} = 990\ \text{N}$$

Resultant couple moment $(\vec{M}_R)_P$ at point P

Exercise: verify algebra by evaluating the three determinants

$$(\vec{M}_R)_P = \sum \vec{M} + \sum (\vec{r} \times \vec{F}) = \vec{r}_{PA} \times \vec{F}_A + \vec{r}_{PB} \times \vec{F}_B + \vec{r}_{PC} \times \vec{F}_C$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -x & -y & 0 \\ 500 & 0 & 0 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -x & 4-y & 0 \\ 0 & 0 & 800 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 6-x & 4-y & 0 \\ 0 & 300 & 0 \end{vmatrix}$$

$$= (3200 - 800y)\vec{i} + 800x\,\vec{j} + (500y + 1800 - 300x)\vec{k} \qquad (1)$$

For a wrench, $(\vec{M}_R)_P$ must be parallel or antiparallel to $\vec{F}_R$. Let $\vec{u}$ be the unit vector in the direction of $\vec{F}_R$ and let M_R denote the magnitude of $(\vec{M}_R)_P$. Then

$$(\vec{M}_R)_P = M_R\vec{u} = M_R \frac{500\vec{i} + 300\vec{j} + 800\vec{k}}{990} = M_R(0.5051\vec{i} + 0.3030\vec{j} + 0.8081\vec{k}) \qquad (2)$$

Now equate $\vec{i}$, $\vec{j}$ and $\vec{k}$ components of the right hand sides of equations (1) and (2). This will give us a system of 3 linear equations in the three unknowns M_R, x and y. With some rearranging of terms we have

$$0.5051M_R + 800y = 3200$$
$$0.3030M_R - 800x = 0$$
$$0.8081M_R + 300x - 500y = 1800$$

Solving this system we find (reinserting units)

$M_R = (M_R)_P = 3.07\ \text{kNm}$ $\qquad$ $(x, y) = (1.16,\ 2.06)\ \text{m}$

END OF MATERIAL FOR FIRST MIDTERM

Chapter 5—Equilibrium of a Rigid Body

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It is important to be able to determine the forces in the cables used to support this boom to ensure that it does not fail. In this chapter we will study how to apply equilibrium methods to determine the forces acting on the supports of a rigid body such as this.

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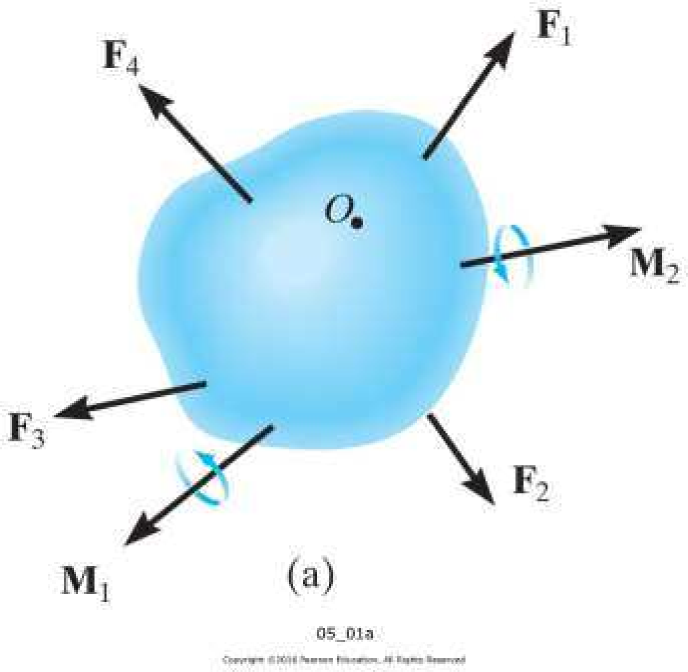
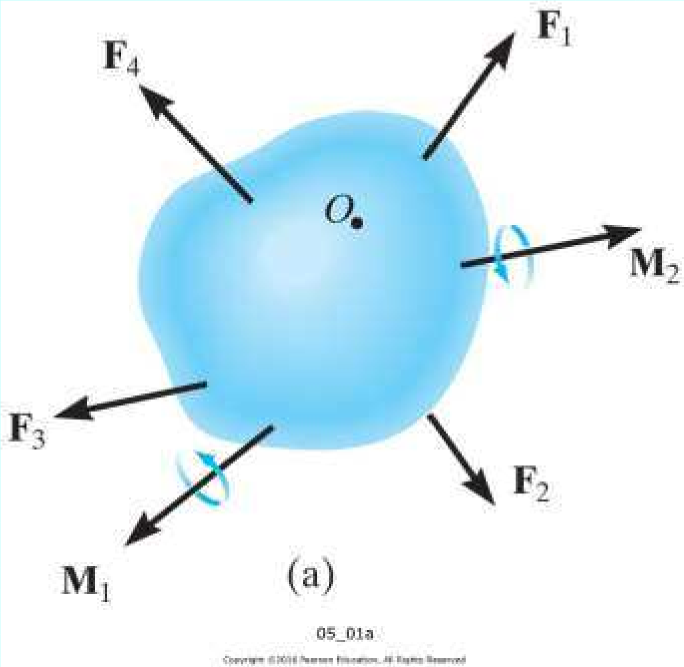
5.1 Conditions for Rigid Body Equilibrium

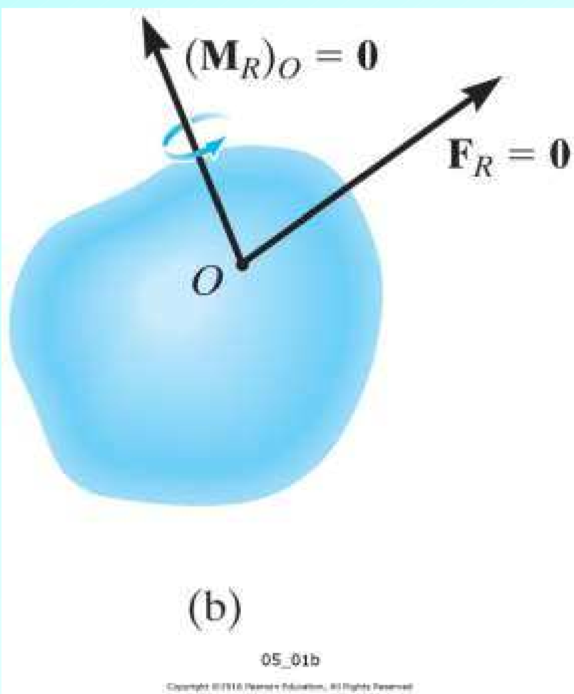


- Rigid body: Body which does not deform when subjected to loads (forces, moments). Idealization, but good one for many engineering materials (steel, concrete, e.g.)

- View rigid body as being composed of particles with interactions between adjacent particles

- Consider free body diagram (FBD) for arbitrary particle—particle is subjected to 2 types of forces
  - Internal forces, due to interactions with neighboring particles
  - External forces, due to e.g. gravitational, electric/magnetic, contact forces between particle and anything not included in body
- Don't need to consider internal forces when considering equilibrium as they always occur in action/reaction pairs and thus "cancel" in the equilibrium equations, as do the moments they produce
- Thus, consider only external forces and couple moments as in the figure





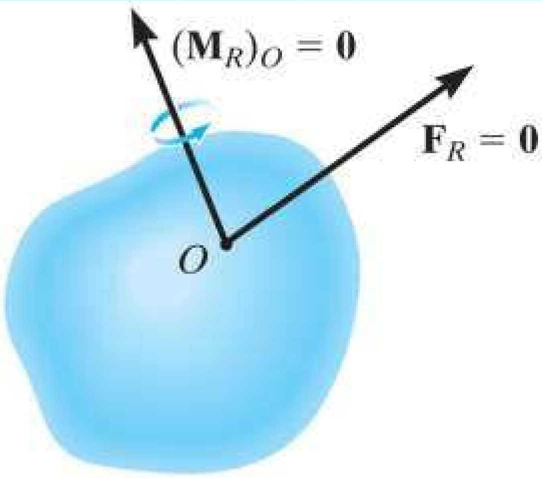
- As we have seen, arbitrary system of external forces and couple moments acting on a body can always be replaced by an equivalent resultant force and couple moment acting at some arbitrary point, *O*, typically located within the body

- If the resultant force and resultant couple moment both vanish:

$$\mathbf{F}_R = \sum \mathbf{F} = \mathbf{0}$$
$$(\mathbf{M}_R)_O = \sum \mathbf{M}_O = \mathbf{0}$$

then we say that the body is in equilibrium.

- In words: A body is in equilibrium if the sum of the external forces acting on it vanishes and the sum of the moments about some point due to those forces added to all the couple moments also vanishes.



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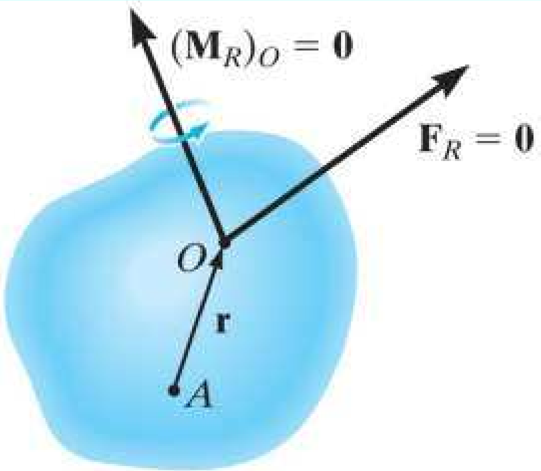

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$$\mathbf{F}_R = \sum \mathbf{F} = \mathbf{0}$$
$$(\mathbf{M}_R)_O = \sum \mathbf{M}_O = \mathbf{0}$$

- These equations are necessary conditions for equilibrium.

- They are also sufficient conditions, as we can see by considering summing the moments about some other point, *A*. If the conditions *are* sufficient, we must have

$$\sum \mathbf{M}_A = \mathbf{r} \times \mathbf{F}_R + (\mathbf{M}_R)_O = \mathbf{0}$$

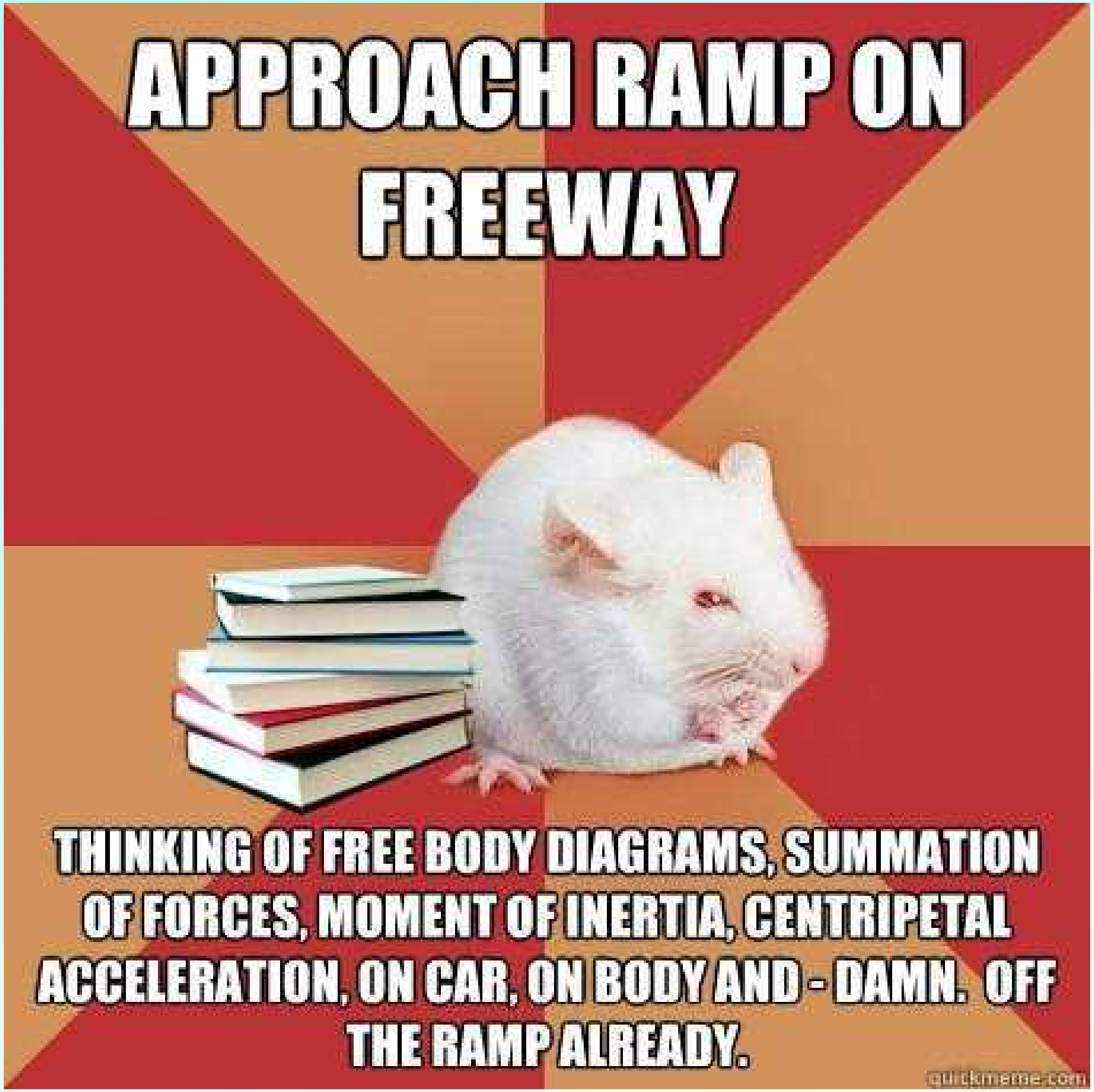
- Since **r** is non-zero, this equation is satisfied in general if and only if the equilibrium equations are satisfied

Equilibrium in Two Dimensions

- We will not study two-dimensional equilibrium in class, but you are encouraged to read through the relevant sections of the text (5.2 and 5.3), and try a few problems, particularly If you encounter difficulties with the three dimensional calculations.

Equilibrium in Three Dimensions

Free Body Diagrams



5.5 Free-Body diagrams

- Will be considering rigid bodies that are acted on by a variety of forces and moments

- These bodies will typically be supported in various ways, and the particulars of the forces and moments to which the body is subjected will depend on the nature of the supports

- In general

  - A force is developed by a support that restricts the translation of its attached member

  - A couple moment is developed when rotation of the attached member is prevented

- The following is a table of various supports, along with a description of the forces and moments that are developed when attached to another body

TABLE 5–2 Supports for Rigid Bodies Subjected to Three-Dimensional Force Systems

Types of Connection	Reaction	Number of Unknowns
(1)  cable		One unknown. The reaction is a force which acts away from the member in the known direction of the cable.
(2)  smooth surface support		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(3)  roller		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.

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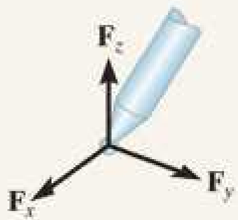
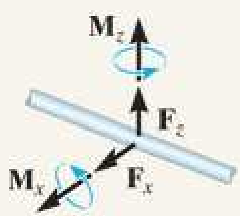
TABLE 5–2 Supports for Rigid Bodies Subjected to Three-Dimensional Force Systems

Types of Connection	Reaction	Number of Unknowns
(4)  ball and socket		Three unknowns. The reactions are three rectangular force components.
(5)  single journal bearing		Four unknowns. The reactions are two force and two couple-moment components which act perpendicular to the shaft. Note: The couple moments are *generally not applied* if the body is supported elsewhere. See the examples.

continued

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