PHYS 170 Section 101
Lecture 10
September 26, 2018

SEP 26—ANNOUNCEMENTS

- First midterm a bit more than two weeks away—Friday, Oct 12, in class (i.e. from 2:00—2:50 PM)

- A general information sheet concerning exams is available on Canvas in the newly activated "Exams" module

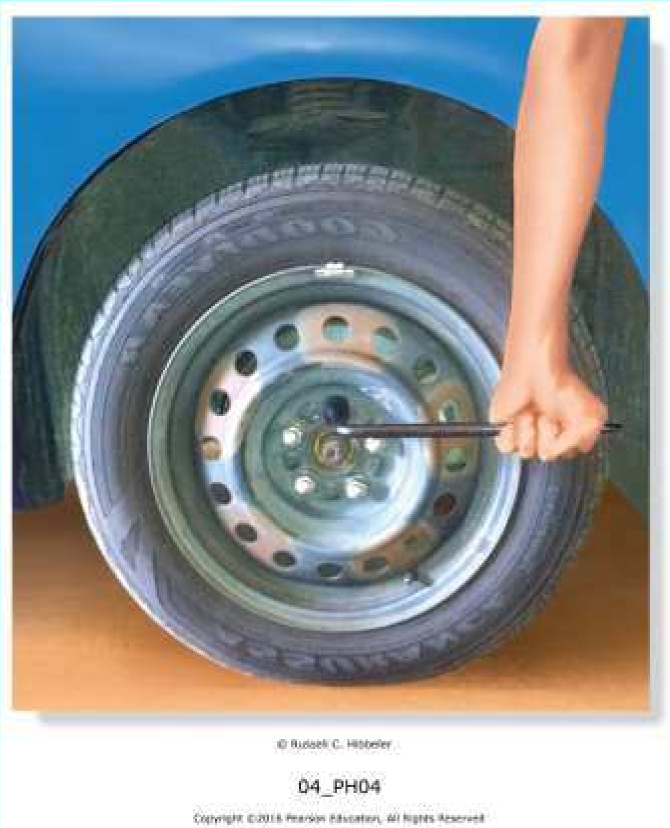
- Exams from previous years, along with solutions, are also available in that module

- Will go over exam information sheet in a future class

Lecture Outline/Learning Goals

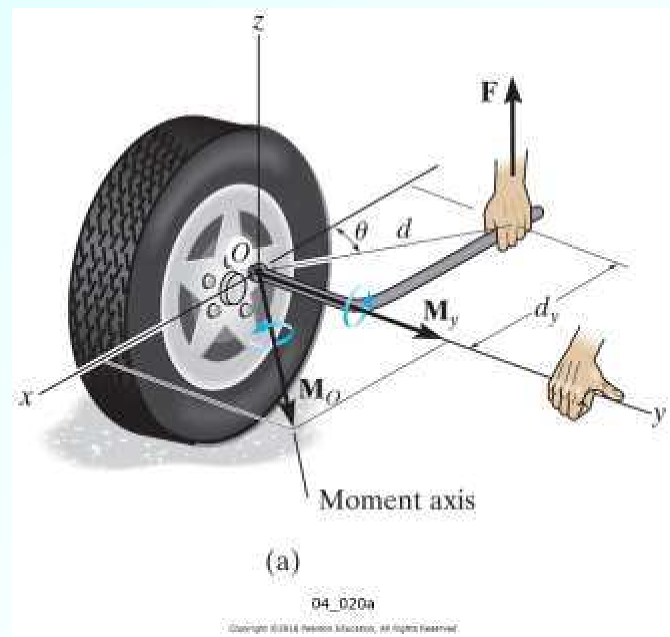
- Moment of a force about a specified axis
- Example problem involving computation of moment, moment arm and moment about an axis
- Moment of a couple (couple moment)
- Simplification of force and couple system
- Resultants of a force and couple system

4.5 Moment of a Force about a Specified Axis

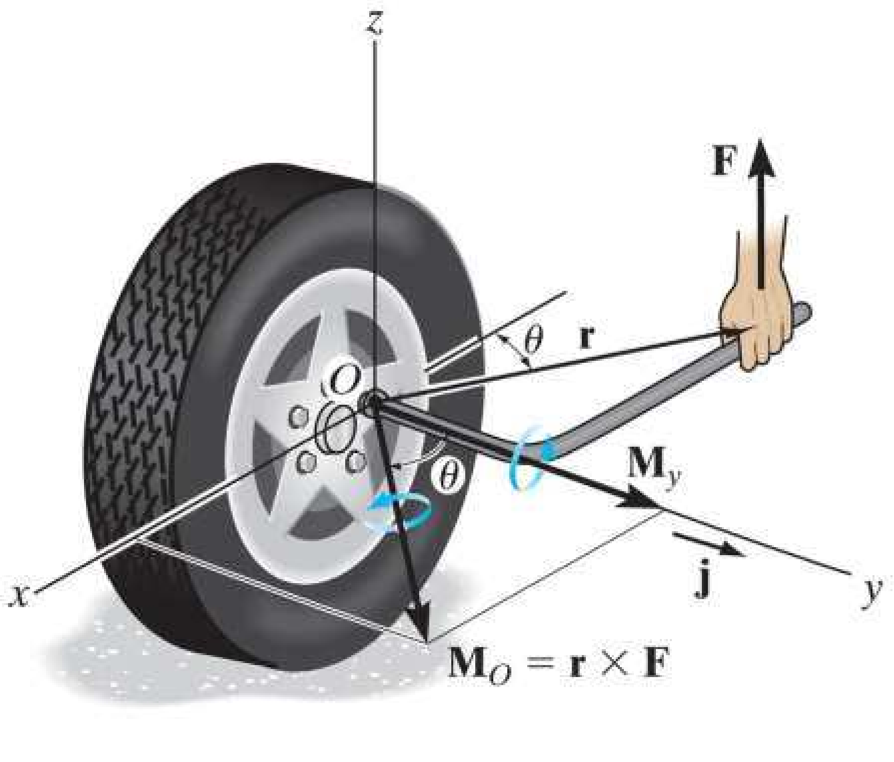


Applied force creates tendency to rotate wrench and nut about moment axis passing through *O* but nut can only rotate about the *y* axis

Thus, to determine the turning effect, only the *y* component of the moment is needed



Will see how to compute such a component of a moment using vector analysis—see text for scalar treatment



(b)

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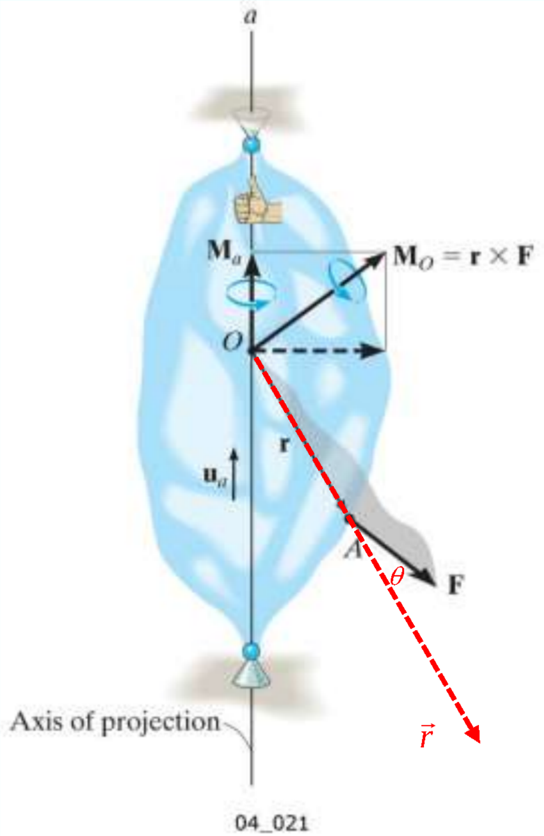
Moment (component) $\mathbf{M}_y$ is the projection of $\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$ along the y axis

Its magnitude, M_y, can be found using the dot product of $\mathbf{j}$ and $\mathbf{M_O}$

$$M_y = \mathbf{j} \cdot \mathbf{M_O} = \mathbf{j} \cdot (\mathbf{r} \times \mathbf{F})$$

Can generalize this analysis to an arbitrary axis …

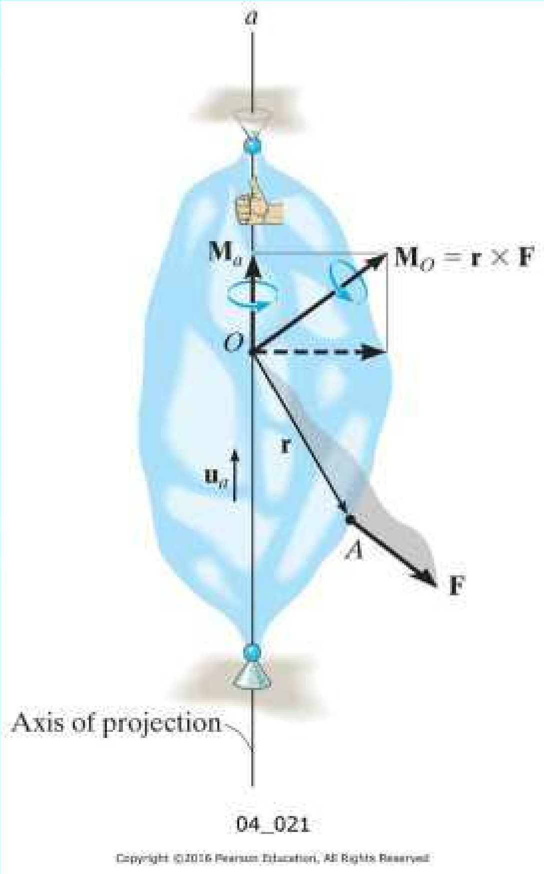
Moment of a Force about a Specified Axis



- Consider figure: Force **F** acts on body at point *A*
- We want to compute the tendency of the force to produce rotation about some arbitrary axis, *a*, that passes through the body, i.e. the moment of **F** about *a*, denoted $\mathbf{M}_a$
- Accomplish this with following 3-step process
  - Choose arbitrary point *O* that lies on specified axis *a*, and compute moment, $\mathbf{M}_O$, of **F** about that point
  - Compute component/projection of $\mathbf{M}_O$ onto specified axis using dot product techniques
  - Calculate vector $\mathbf{M}_\text{a}$ from M_a and unit vector in *a* direction

- Start by computing magnitude, M_a. Consider unit vector, $\mathbf{u}_a$, in (positive) direction of *a* axis. Then

$$M_a = \mathbf{M}_O \cdot \mathbf{u}_a = \mathbf{u}_a \cdot \mathbf{M}_O = \mathbf{u}_a \cdot (\mathbf{r} \times \mathbf{F})$$



$$M_a = \mathbf{M}_O \cdot \mathbf{u}_a = \mathbf{u}_a \cdot \mathbf{M}_O = \mathbf{u}_a \cdot (\mathbf{r} \times \mathbf{F})$$

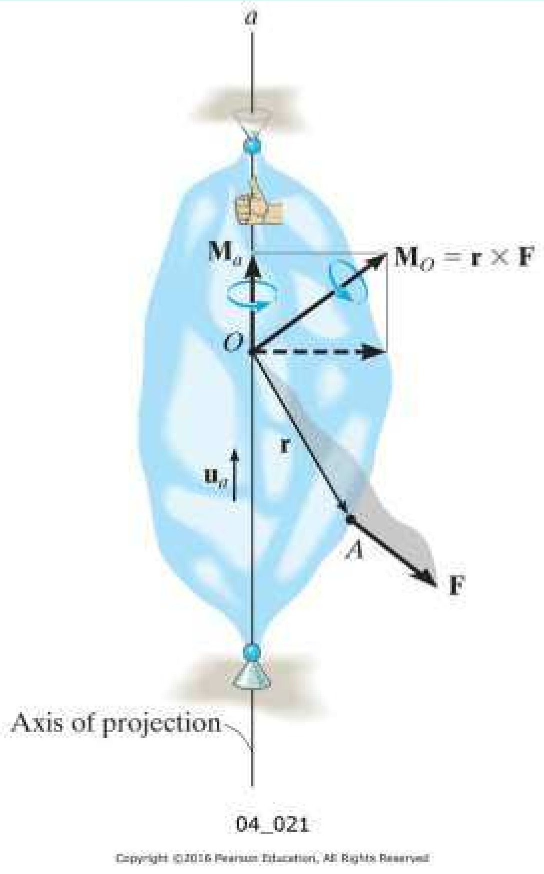
- Above combination of dot and cross product is known as a triple vector product
- Assuming we have a right handed *x, y, z* coordinate system, and we know *x, y, z* components for all of the vectors in the above expression, we have

$$M_a = (u_{a_x}\mathbf{i} + u_{a_y}\mathbf{j} + u_{a_z}\mathbf{k}) \cdot \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

which, as we can show, can be rewritten

Remember that **r** is a vector from *any point, O,* on the axis of interest to *any point, A*, on the line of action of the force, **F**.

$$M_a = \begin{vmatrix} u_{a_x} & u_{a_y} & u_{a_z} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$



- Once we've computed magnitude, can write down actual moment vector

$$\mathbf{M}_a = M_a \mathbf{u}_a = [\mathbf{u}_a \cdot (\mathbf{r} \times \mathbf{F})]\, \mathbf{u}_a$$

- Note: M_a may be of either sign, giving sense of moment as either in same/opposite direction as unit vector $\mathbf{u}_a$

- If we have more than one force, and we are to compute the resultant moment about a given axis, then (projected) moment components due to each force add algebraically, since all of the (projected) moment vectors will be co-linear with the axis
- Thus, will have for the magnitude M_a

$$M_a = \Sigma[\mathbf{u}_a \cdot (\mathbf{r} \times \mathbf{F})] = \mathbf{u}_a \cdot \Sigma(\mathbf{r} \times \mathbf{F})$$

Moment of a Force About a Specified Axis: Special Cases

- Recall that we have

$$M_a = \mathbf{M}_O \cdot \mathbf{u}_a = \mathbf{u}_a \cdot \mathbf{M}_O = \mathbf{u}_a \cdot (\mathbf{r} \times \mathbf{F})$$

- Consider the case when force, **F**, is parallel to axis of interest, i.e. when **F** and $\mathbf{u}_a$ are parallel.
- Since cross product $\mathbf{r} \times \mathbf{F}$ is perpendicular to both **r** and **F**, it is also perpendicular to $\mathbf{u}_a$
- Since dot product of perpendicular vectors is 0, we thus have

$$M_a = \mathbf{u}_a \cdot (\mathbf{r} \times \mathbf{F}) = 0$$

so the moment of a force about any axis that is parallel to the force vanishes!

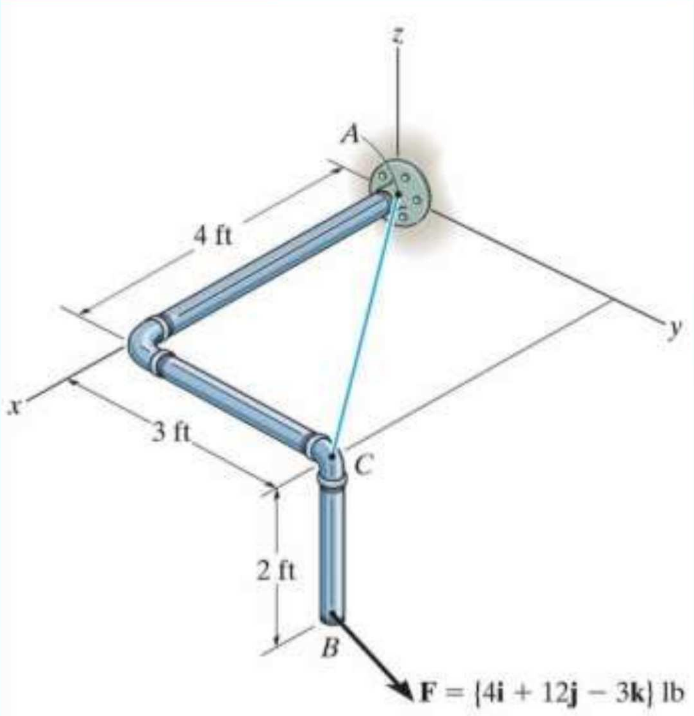
- Recall that we also know that the moment of a force about any point that lies on the line of action of the force also vanishes

Problem 4-53 (page 146, 13th edition)

1) Determine the moment of $\vec{F}$ about A

2) Determine the moment arm (lever arm) for $\vec{F}$

3) Determine the magnitude of the moment of $\vec{F}$ about an axis extending from A to C



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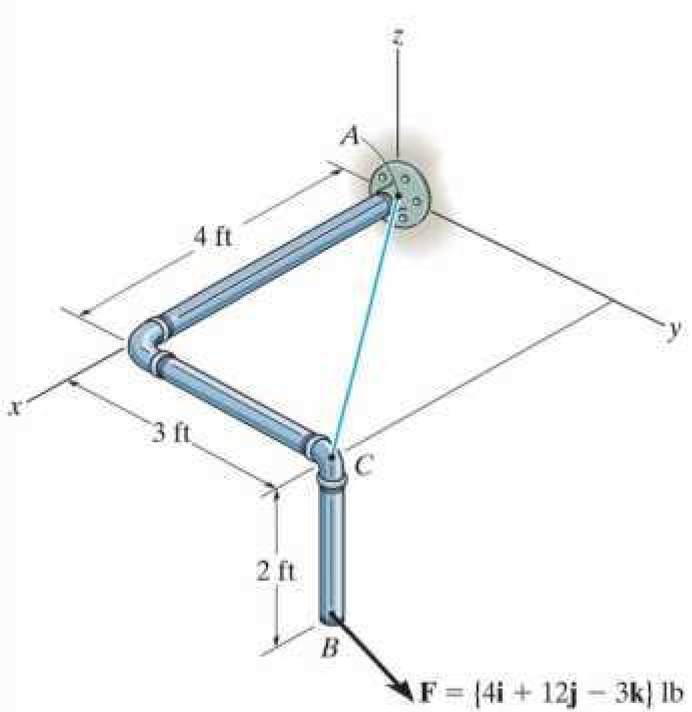

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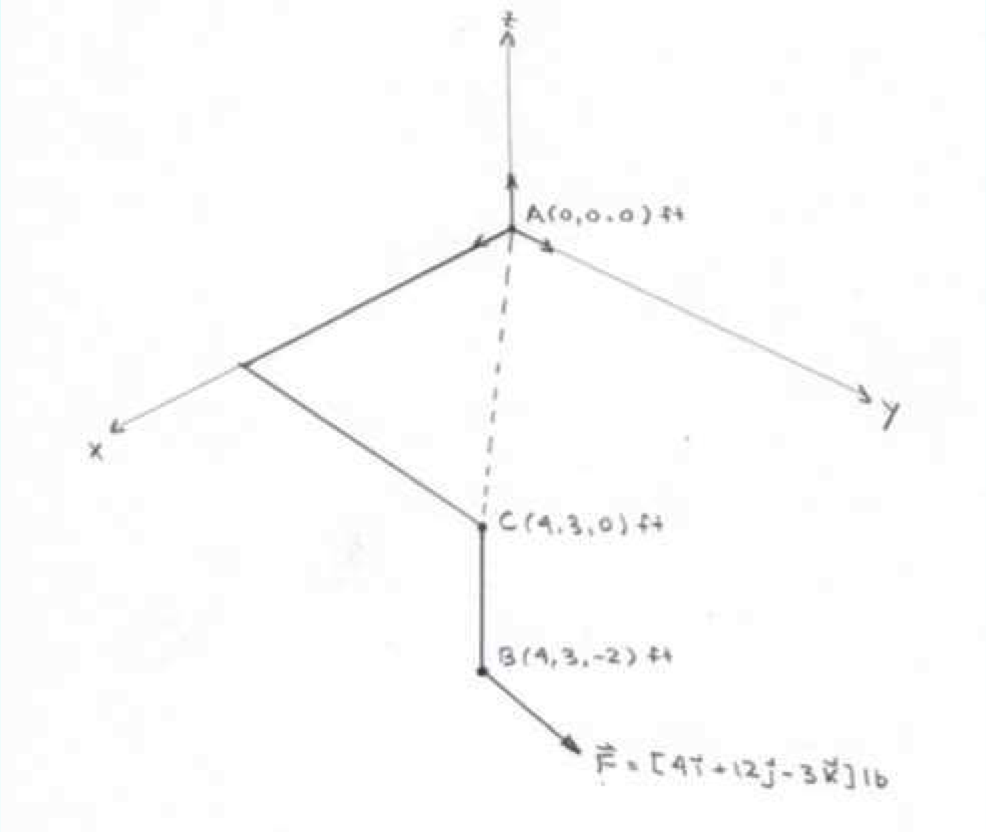
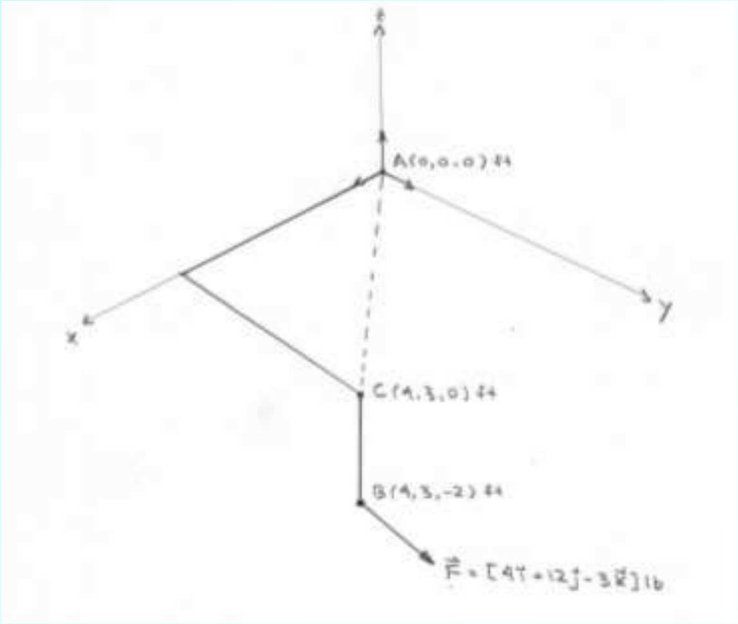


Solution strategy



(1) Compute moment $\vec{M}_A$ of $\vec{F}$ about A using cross product $\vec{r}_{AB} \times \vec{F}$

(2) Compute magnitudes of $\vec{M}_A$ and $\vec{F}$, compute moment arm, d, from definition $M_A = Fd$

(3) Compute moment of $\vec{F}$ about axis AC using

$$M_{AC} = \vec{u}_{AC} \cdot \vec{M}_A$$

(1) Determine the moment of $\vec{F}$ about A (suppressing units)

$$\vec{M}_A = \vec{r}_{AB} \times \vec{F} = \left(4\vec{i} + 3\vec{j} - 2\vec{k}\right) \times \left(4\vec{i} + 12\vec{j} - 3\vec{k}\right)$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & 3 & -2 \\ 4 & 12 & -3 \end{vmatrix} = \vec{i}\,[3(-3) - 12(-2)] - \vec{j}\,[4(-3) - 4(-2)] + \vec{k}\,[4(12) - 4(3)]$$

$$= 15\vec{i} + 4\vec{j} + 36\vec{k} \text{ lb} \cdot \text{ft}$$

(2) Determine the moment arm for $\vec{F}$

Magnitude of $\vec{M}_A$: $M_A = \sqrt{15^2 + 4^2 + 36^2} = 39.20 \text{ lb} \cdot \text{ft}$

Magnitude of $\vec{F}$: $F = \sqrt{4^2 + 12^2 + 3^2} = 13 \text{ lb}$

Moment arm : $M_A = Fd \;\rightarrow\; d = \dfrac{M_A}{F} = \dfrac{39.20}{13} = 3.02 \text{ ft}$

3) Determine the magnitude of the moment of $\vec{F}$ about an axis extending from A to C

$$M_{AC} = \vec{u}_{AC} \cdot \vec{M}_A$$

where $\vec{u}_{AC}$ is the unit vector directed from A to C.

$$M_{AC} = \frac{\vec{r}_{AC}}{r_{AC}} \cdot \vec{M}_A = \frac{4\vec{i} + 3\vec{j}}{\sqrt{4^2 + 3^2}} \cdot \left(15\vec{i} + 4\vec{j} + 36\vec{k}\right)$$

$$= \frac{4\vec{i} + 3\vec{j}}{5} \cdot \left(15\vec{i} + 4\vec{j} + 36\vec{k}\right)$$

$$= 12 + \frac{12}{5} = 14.4 \text{ lb} \cdot \text{ft}$$

3) Alternative method using scalar triple product (suppressing units)

$$M_{AC} = \vec{u}_{AC} \cdot \vec{M}_A = \vec{u}_{AC} \cdot \left(\vec{r}_{AB} \times \vec{F} \right)$$

$$\vec{u}_{AC} = 0.8\vec{i} + 0.6\vec{j}$$

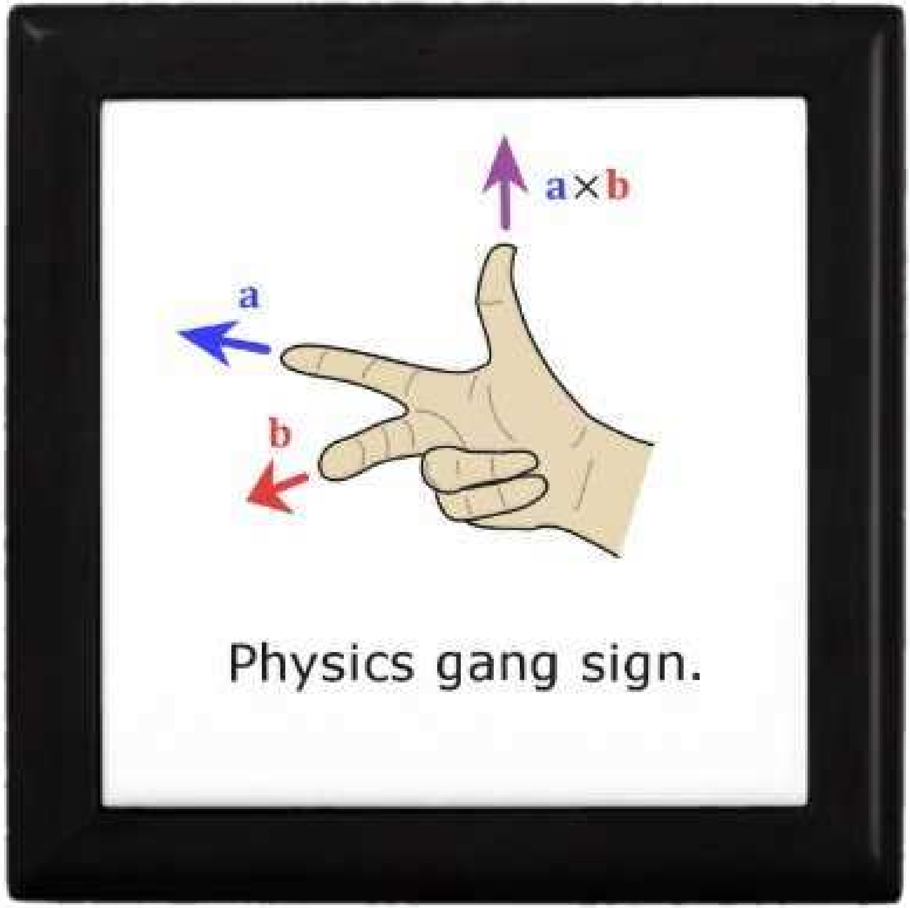
$$\vec{r}_{AB} = \left(4\vec{i} + 3\vec{j} - 2\vec{k} \right)$$

$$\vec{F} = \left(4\vec{i} + 12\vec{j} - 3\vec{k} \right)$$

$$M_{AC} = \begin{vmatrix} (u_{AC})_x & (u_{AC})_y & (u_{AC})_z \\ (r_{AB})_x & (r_{AB})_y & (r_{AB})_z \\ F_x & F_y & F_z \end{vmatrix}$$

$$= \begin{vmatrix} 0.8 & 0.6 & 0 \\ 4 & 3 & -2 \\ 4 & 12 & -3 \end{vmatrix}$$

$$= 14.4\,\text{lb} \cdot \text{ft}$$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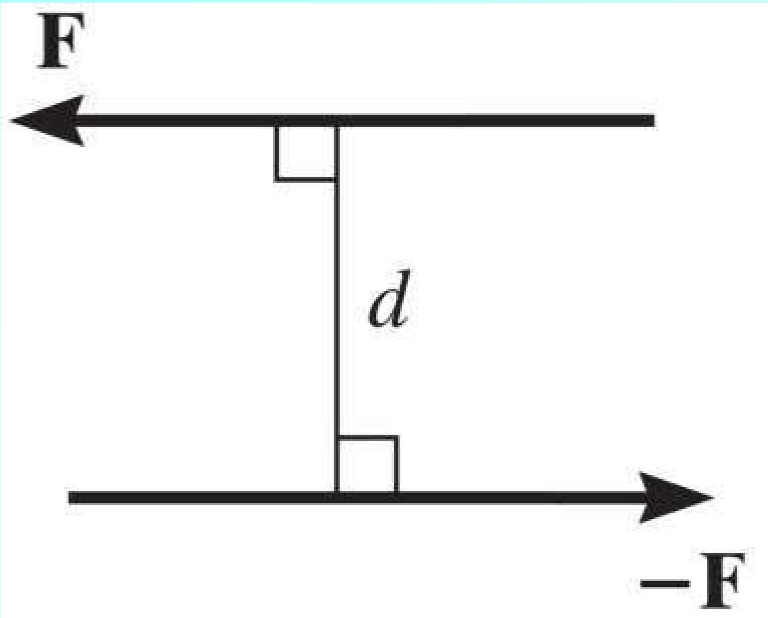
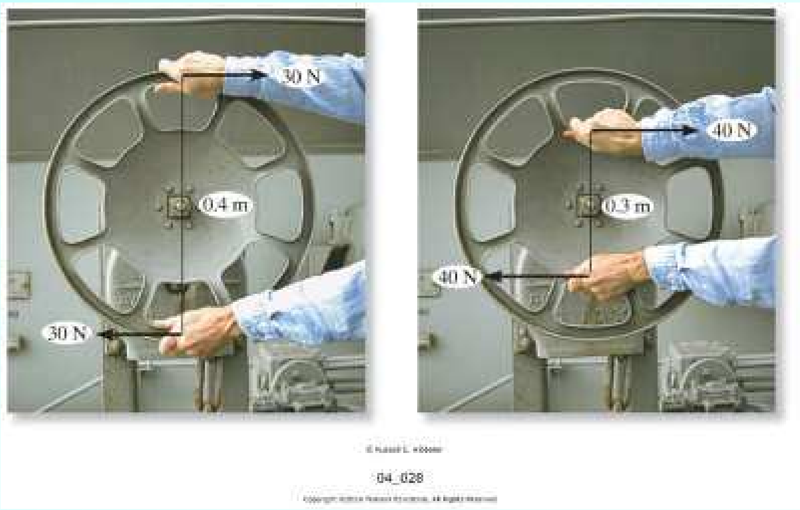
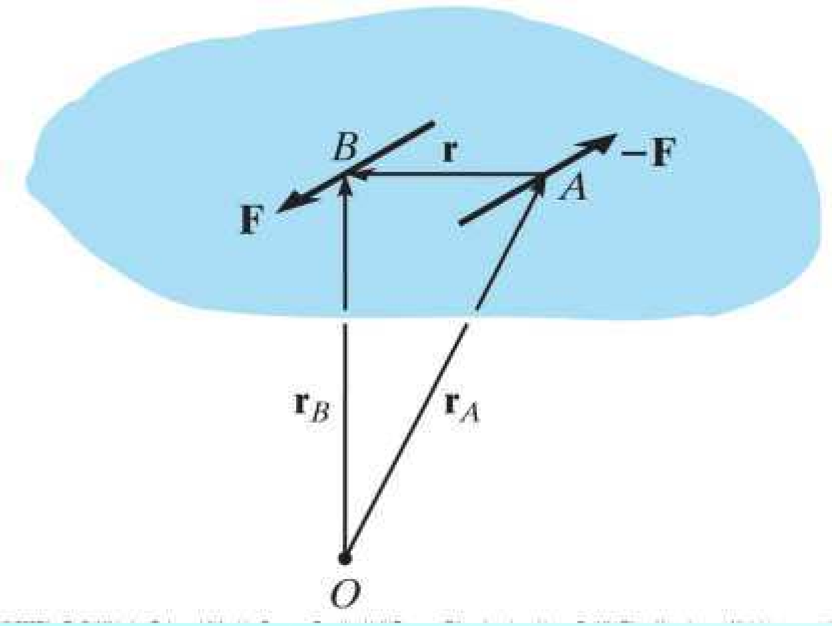
4.6 Moment of a Couple





- **Couple (definition):** Two parallel forces that have the same magnitude, opposite directions, and which are separated by a perpendicular distance *d*
- Resultant force (net force) due to a couple is 0, so only effect of couple is to produce rotation or tendency for rotation in some direction
- **Couple moment (definition):** The moment produced by a couple



- Interestingly, couple moment is independent of point about which moments are computed
- Consider figure, and compute moments about *O*

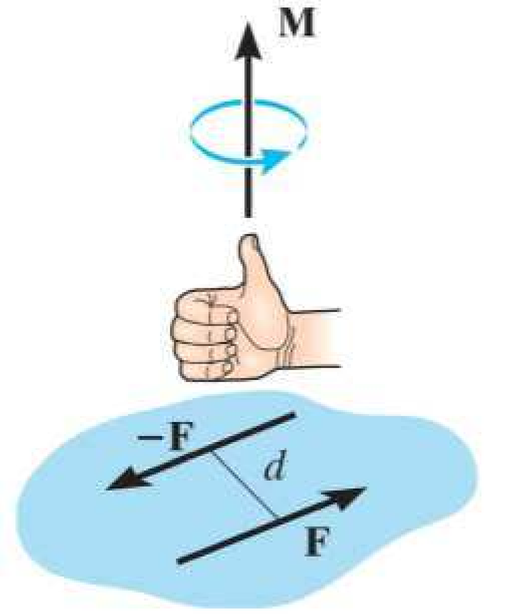
$$\mathbf{M} = \mathbf{r}_A \times (-\mathbf{F}) + \mathbf{r}_B \times \mathbf{F}$$

- Assuming couple moment *is* independent of where moments are computed about, can more easily compute **M** by taking moments about *A*. Since moment of **–F** about *A* vanishes, we have

$$\mathbf{M} = \mathbf{r} \times \mathbf{F}$$

- To see that these *do* give the same result for the couple moment, note that we have $\mathbf{r}_B = \mathbf{r}_A + \mathbf{r}$ or $\mathbf{r} = \mathbf{r_B} - \mathbf{r_A}$. Thus,

$$\mathbf{M} = \mathbf{r}_A \times (-\mathbf{F}) + \mathbf{r}_B \times \mathbf{F} = (\mathbf{r_B} - \mathbf{r_A}) \times \mathbf{F} = \mathbf{r} \times \mathbf{F}$$

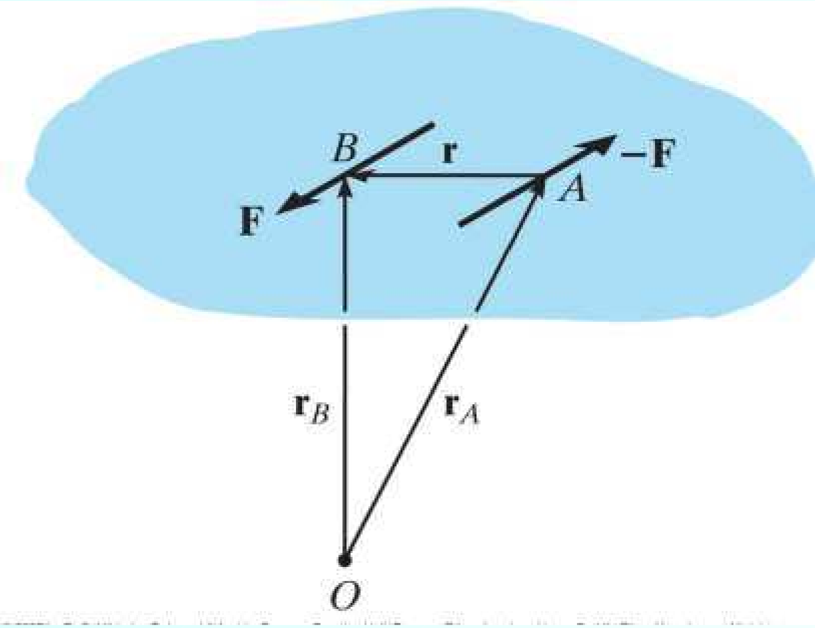


SCALAR FORMULATION

- **Magnitude of couple moment**

$$M = Fd$$

- **Direction**
  - Given by right hand rule, where fingers curl with sense of rotation produced by couple, thumb points in direction of couple moment



VECTOR FORMULATION

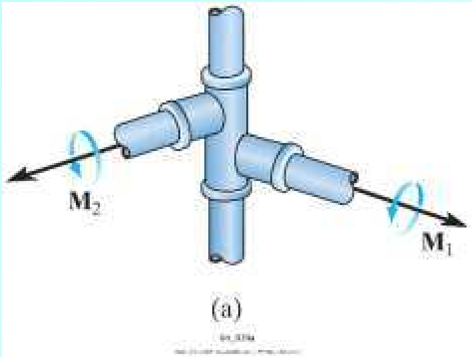
- Using the vector cross product, we have

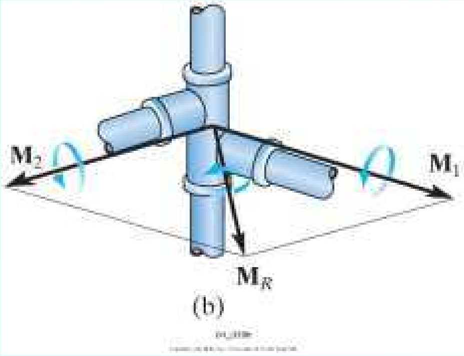
$$\mathbf{M} = \mathbf{r} \times \mathbf{F}$$

- Mnemonic: Consider moments about a point on line of action of one of the forces (*A* e.g.)
- Note: **r** is an *arbitrary* vector from the line of action of –**F** to line of action of **F**

- Couple moments as **free vectors**: Since a couple moment is independent of the point that we choose to compute moments, it can be viewed as a *free vector*; i.e. it can act at *any* point (contrast with case of moment of a force which requires choice of *specific* point (axis) about which moment is to be computed)

- Equivalent couples: We say that two couples are equivalent if they produce the same moment. Since direction of couple moment is always perpendicular to plane containing two forces defining the couple, equivalent couples must either lie in same plane, or in planes that are *parallel* to one another





- Resultant couple moment: Since couple moments are free vectors, if more than one such moment acts on a body, they can be relocated to any point, *P*, on the body, and added vectorally as in the figure

$$\mathbf{M}_R = \mathbf{M}_1 + \mathbf{M}_2$$

- For a general system of couple moments we have

$$\mathbf{M}_R = \Sigma(\mathbf{r} \times \mathbf{F})$$