PHYS 170 Section 101

Lecture 8

September 21, 2018

SEP 21—ANNOUNCEMENTS

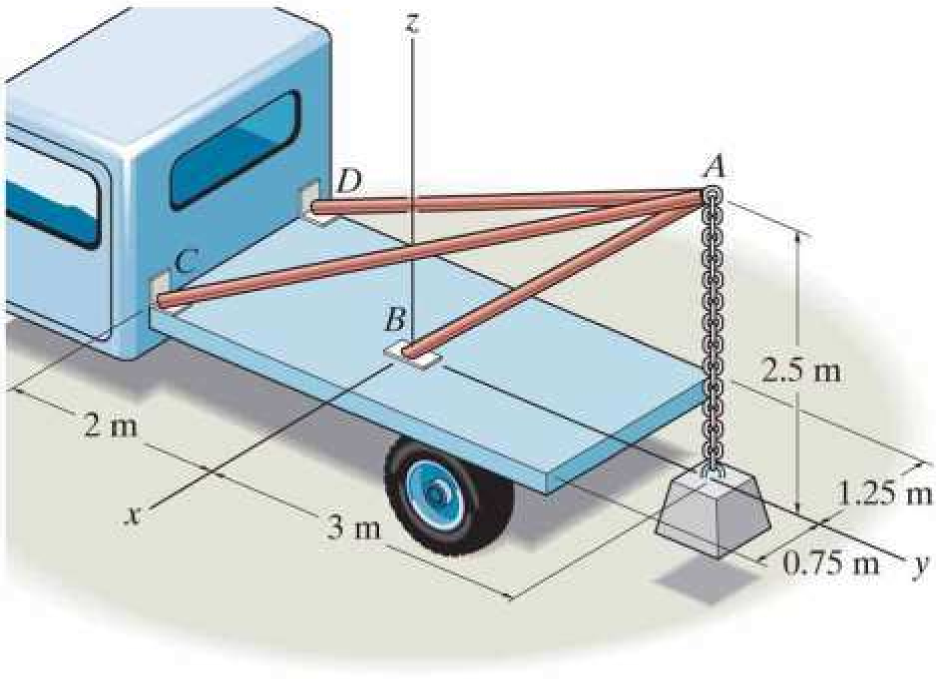
- Homework assignment 2 due tonight, 11:59 PM
- Homework assignment 3 available today at 6:00 PM

Lecture Outline/Learning Goals

- Finish example of static equilibrium of a particle in three dimensions
- Second worked example of static equilibrium of a particle in three dimensions
- Start CHAPTER 4: FORCE SYSTEM RESULTANTS
- Moment of a Force: Scalar Formulation

Problem 3-53 (page 100, 12th edition)

Determine the force acting along the axis of each of the three struts needed to hold the 500 kg block in equilibrium



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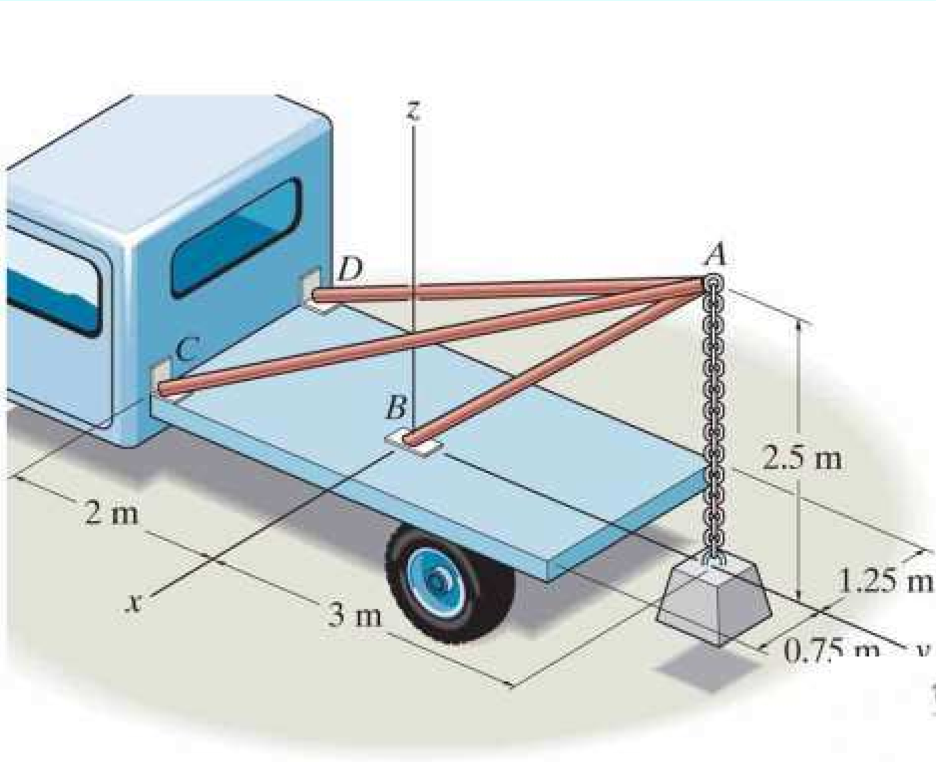

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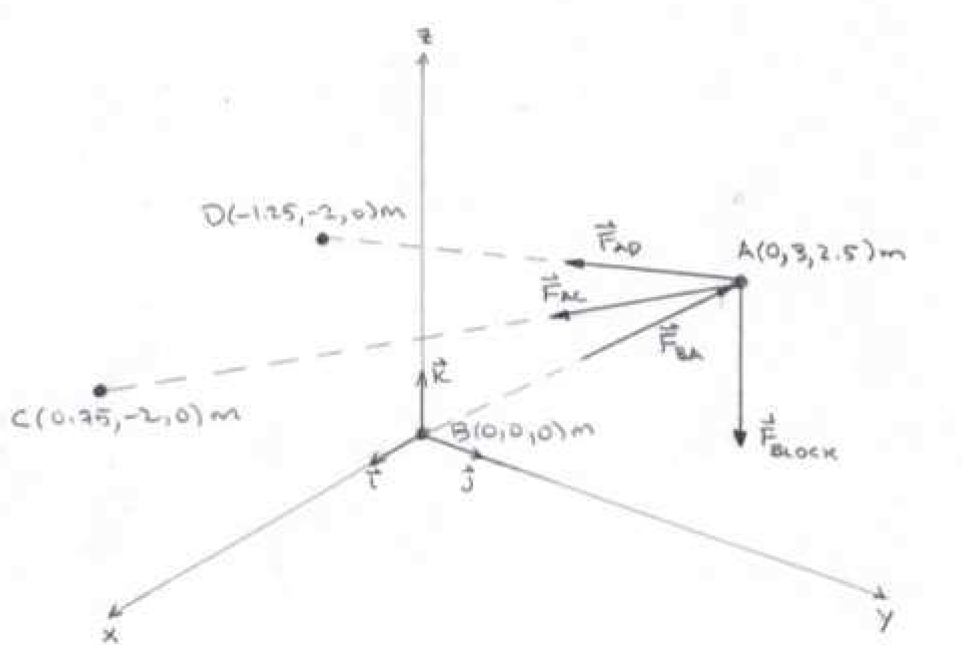
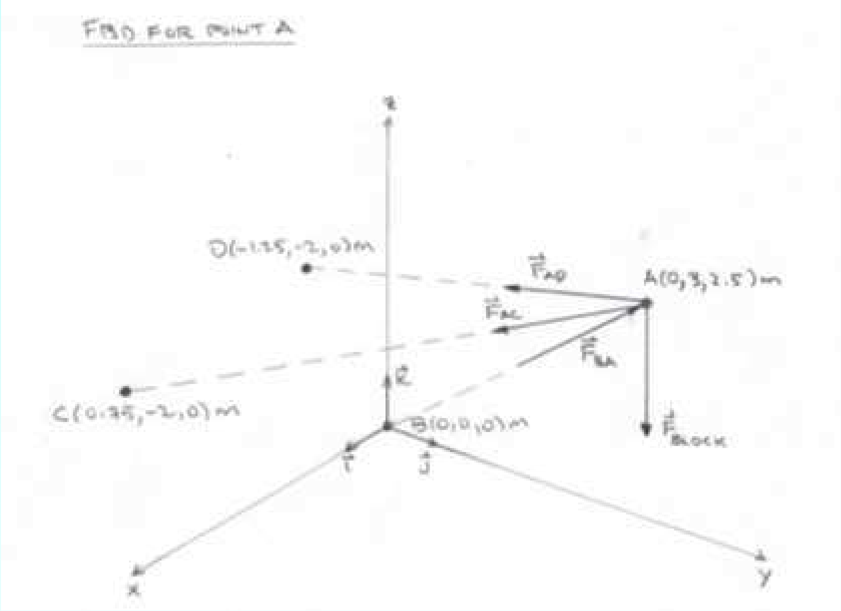


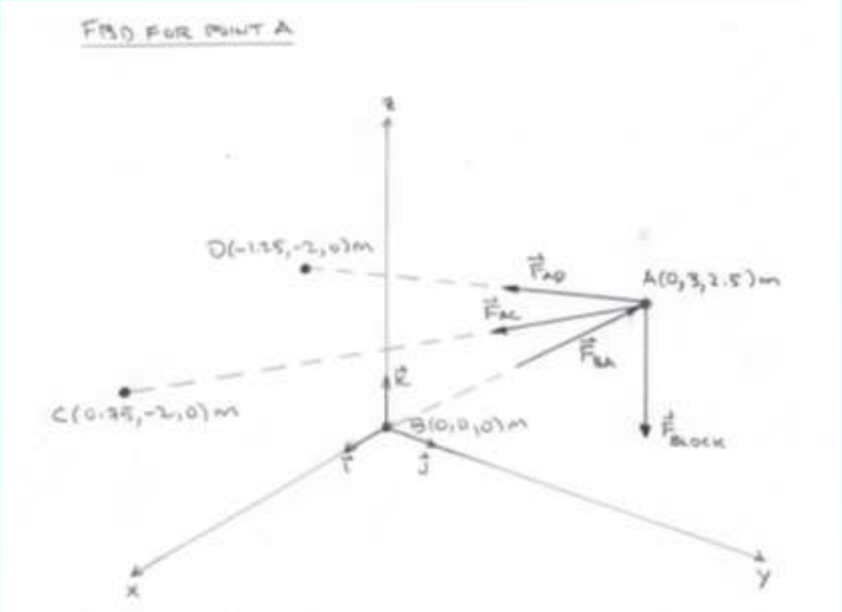
Problem 3-53 (page 100, 12th edition)

- Note that as shown in the free-body diagram, struts AD and AC are under tension (forces directed *away* from A), while strut AB is under compression (force directed *towards* A).

  We incorporate this information in our solution—in particular in how we define the directions of the various force vectors—with the anticipation that the unknowns representing the magnitudes of the vectors will come out positive. However, we could equally well simply make arbitrary choices for the directions and then the signs of the corresponding unknowns (when we have solved the equations) will tell us whether we made the correct assumptions or not.



Solution strategy:



(1) Express $\vec{F}_{BA}$, $\vec{F}_{AC}$, $\vec{F}_{AD}$ and $\vec{F}_{block}$ in Cartesian form using coordinates of points A, B, C and D. Also adopt calculational trick used previously to simplify linear equations and solution thereof

(2) Use definition of resultant and equations of equilbrium to formulate 3 linear equations in 3 unknowns (essentially the magnitudes of the three forces F_{BA}, F_{AC} and F_{AD})

(3) Solve linear system (use of calculator recommended), and then determine force magnitudes from inverse relation of calculational trick.

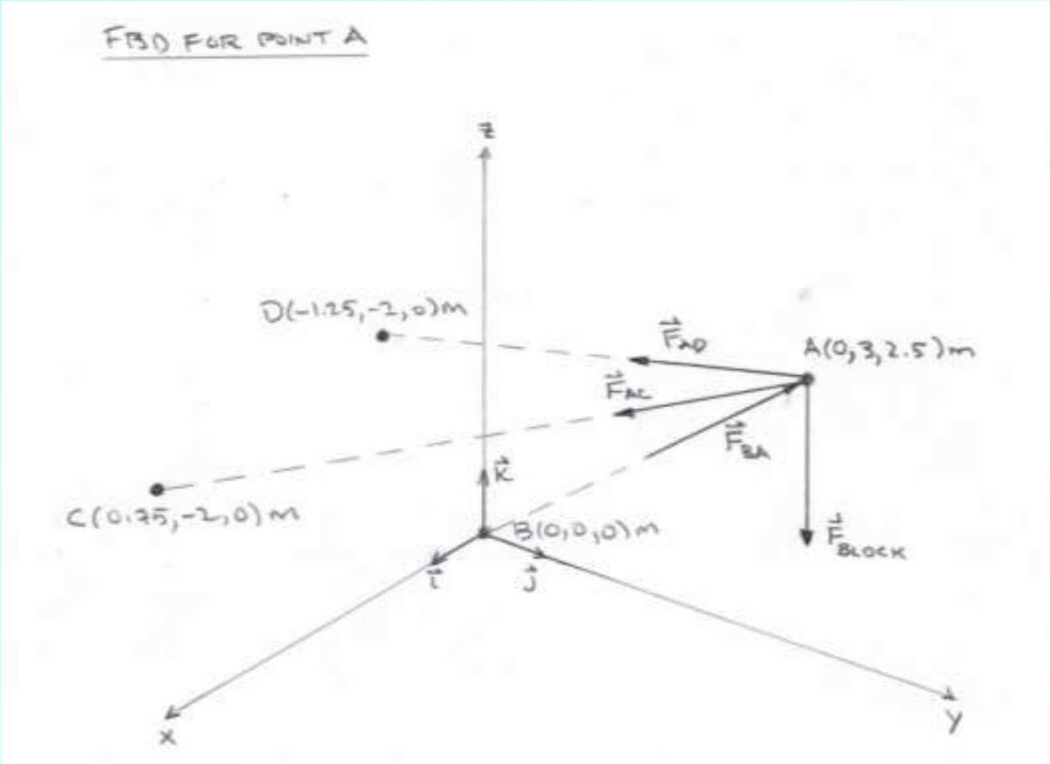
- **Coordinates**

$A(0,\ 3,\ 2.5)\,\text{m}$

$B(0,\ 0,\ 0)\ \text{m}$

$C(0.75,\ -2,\ 0)\ \text{m}$

$D(-1.25,\ -2,\ 0)\ \text{m}$



- **Forces** (suppressing units)
  - Example

$$\vec{r}_{BA} = \vec{r}_A - \vec{r}_B = \left((0-0)\vec{i} + (3-0)\vec{j} + (2.5-0)\vec{k}\right) = 3\vec{j} + 2.5\vec{k}$$

$$\vec{F}_{BA} = F_{BA}\left(\frac{\vec{r}_{BA}}{r_{BA}}\right) = (3\vec{j} + 2.5\vec{k})X \qquad \text{where} \quad X = F_{BA}/r_{BA} = F_{BA}/\sqrt{3^2 + 2.5^2}$$

Once we determine X, then we calculate F_{BA} from

$$F_{BA} = X\,r_{BA} = X\sqrt{3^2 + 2.5^2}$$

- **Coordinates**

$A(0,\ 3,\ 2.5)\ \text{m}$

$B(0,\ 0,\ 0)\ \text{m}$

$C(0.75,\ -2,\ 0)\ \text{m}$

$D(-1.25,\ -2,\ 0)\ \text{m}$

- **Forces** (suppressing units)

$$\vec{F}_{BA} = (3\vec{j} + 2.5\vec{k})X \qquad X = F_{BA} / \sqrt{3^2 + 2.5^2}$$

$$\vec{F}_{AC} = (0.75\vec{i} - 5\vec{j} - 2.5\vec{k})Y \qquad Y = F_{AC} / \sqrt{0.75^2 + 5^2 + 2.5^2}$$

$$\vec{F}_{AD} = (-1.25\vec{i} - 5\vec{j} - 2.5\vec{k})Z \qquad Z = F_{AD} / \sqrt{1.25^2 + 5^2 + 2.5^2}$$

$$\vec{F}_{\text{block}} = (-500)(9.81)\vec{k}$$

- **Equations of equilibrium**

$$\vec{F}_R = F_{Rx}\,\vec{i} + F_{Ry}\,\vec{j} + F_{Rz}\,\vec{k} = 0$$

$$F_{Rx} = \sum F_x = 0: \qquad 0.75Y - 1.25Z = 0$$

$$F_{Ry} = \sum F_y = 0: \qquad 3X - 5Y - 5Z = 0$$

$$F_{Rz} = \sum F_z = 0: \qquad 2.5X - 2.5Y - 2.5Z - (500)(9.81) = 0$$

- **Solving**

$$X = 4905 \qquad Y = 1839 \qquad Z = 1104$$

Then, using

$$F_{BA} = X\sqrt{3^2 + 2.5^2} \qquad F_{AC} = Y\sqrt{0.75^2 + 5^2 + 2.5^2} \qquad F_{AD} = Z\sqrt{1.25^2 + 5^2 + 2.5^2}$$

we have

$$F_{BA} = 19.2\,\text{kN} \qquad F_{AC} = 10.4\,\text{kN} \qquad F_{AD} = 6.32\,\text{kN}$$

Does this answer make sense?

Solving the equations using the reduced row echelon form method

- Equations for *X*, *Y* and *Z* :

$$0.75Y - 1.25Z = 0 \qquad (1)$$

$$3X - 5Y - 5Z = 0 \qquad (2)$$

$$2.5X - 2.5Y - 2.5Z = 4905 \qquad (3)$$

- Solve (1) to (3) using the reduced row echelon form matrix program **rref(**[*M*]**)** on a TI graphing calculator, where [*M*] is the 3 x 4 matrix

$$[M] = \begin{bmatrix} 0 & 0.75 & -1.25 & 0 \\ 3 & -5 & -5 & 0 \\ 2.5 & -2.5 & -2.5 & 4905 \end{bmatrix}$$

Each **row** of the matrix corresponds to one equation; row elements must be entered consistently (i.e. *X* coefficients in first column, *Y* coefficients in second etc.); right hand side of the equation is always the last element of the row.

This yields

$$\begin{bmatrix} 1 & 0 & 0 & X \\ 0 & 1 & 0 & Y \\ 0 & 0 & 1 & Z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 4905 \\ 0 & 1 & 0 & 1839 \\ 0 & 0 & 1 & 1104 \end{bmatrix}$$

Comparison with "direct" method for specifying forces

- **Forces (suppressing units)**

$$\vec{F}_{BA} = F_{BA}\left(\frac{\vec{r}_{BA}}{r_{BA}}\right) = F_{BA}\frac{3\,\vec{j} + 2.5\vec{k}}{\sqrt{3^2 + 2.5^2}} = F_{BA}\left(0.7682\,\vec{j} + 0.6402\,\vec{k}\right)$$

$$\vec{F}_{AC} = F_{AC}\left(\frac{\vec{r}_{AC}}{r_{AC}}\right) = F_{AC}\frac{0.75\vec{i} - 5\,\vec{j} - 2.5\vec{k}}{\sqrt{0.75^2 + 5^2 + 2.5^2}} = F_{AC}\left(0.1330\,\vec{i} - 0.8865\,\vec{j} - 0.4432\,\vec{k}\right)$$

$$\vec{F}_{AD} = F_{AD}\left(\frac{\vec{r}_{AD}}{r_{AD}}\right) = F_{AD}\frac{-1.25\vec{i} - 5\,\vec{j} - 2.5\vec{k}}{\sqrt{1.25^2 + 5^2 + 2.5^2}} = F_{AD}\left(-.2182\,\vec{i} - 0.8729\,\vec{j} - 0.4364\,\vec{k}\right)$$

- **Equations of equilibrium**

$$\vec{F}_R = F_{Rx}\,\vec{i} + F_{Ry}\,\vec{j} + F_{Rz}\,\vec{k} = 0$$

$$F_{Rx} = \sum F_x = 0: \qquad 0.1330F_{AC} - 0.2182F_{AD} = 0$$

$$F_{Ry} = \sum F_y = 0: \qquad 0.7682F_{AB} - 0.8865F_{AC} - 0.8729F_{AB} = 0$$

$$F_{Rz} = \sum F_z = 0: \qquad 0.6402F_{AB} - 0.4432F_{AC} - 0.4364F_{AD} = 4905$$

- **Solving**

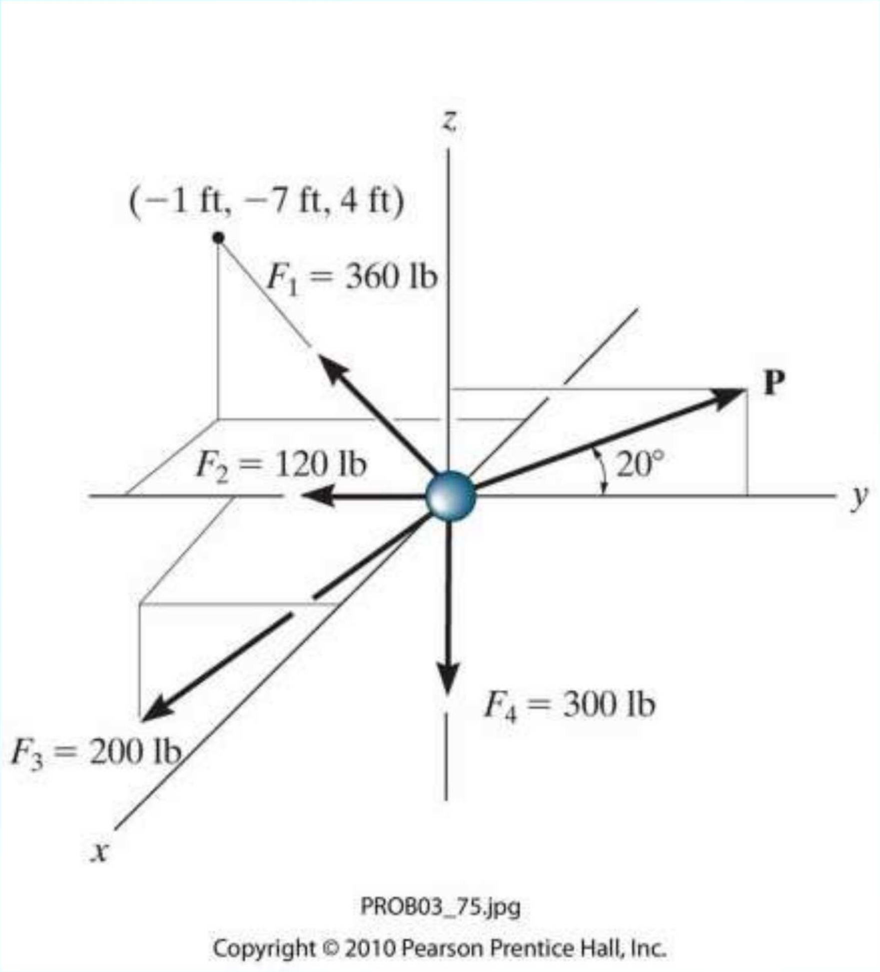
$$F_{AB} = 19.1\text{ kN} \qquad F_{AC} = 10.4\text{ kN} \qquad \mathrm{F}_{AD} = 6.32\text{ kN}$$

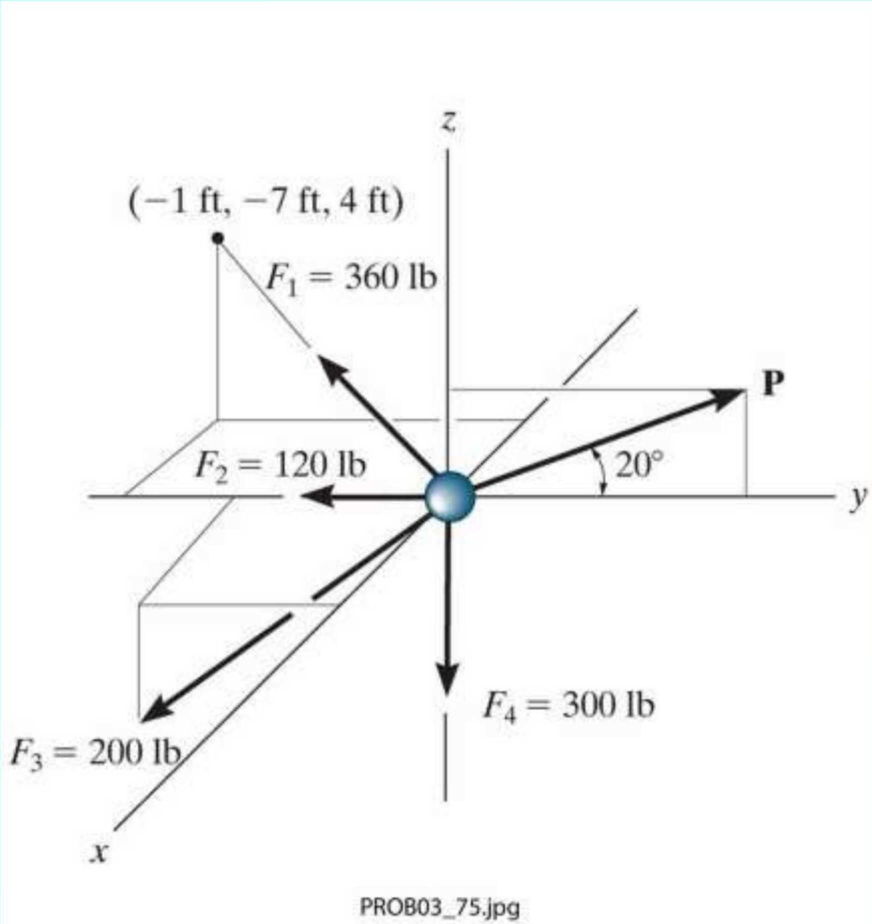
- Note the significantly increased complexity of the numerical coefficients that must be computed, recorded and re-entered into the calculator relative to the "transformed" method.

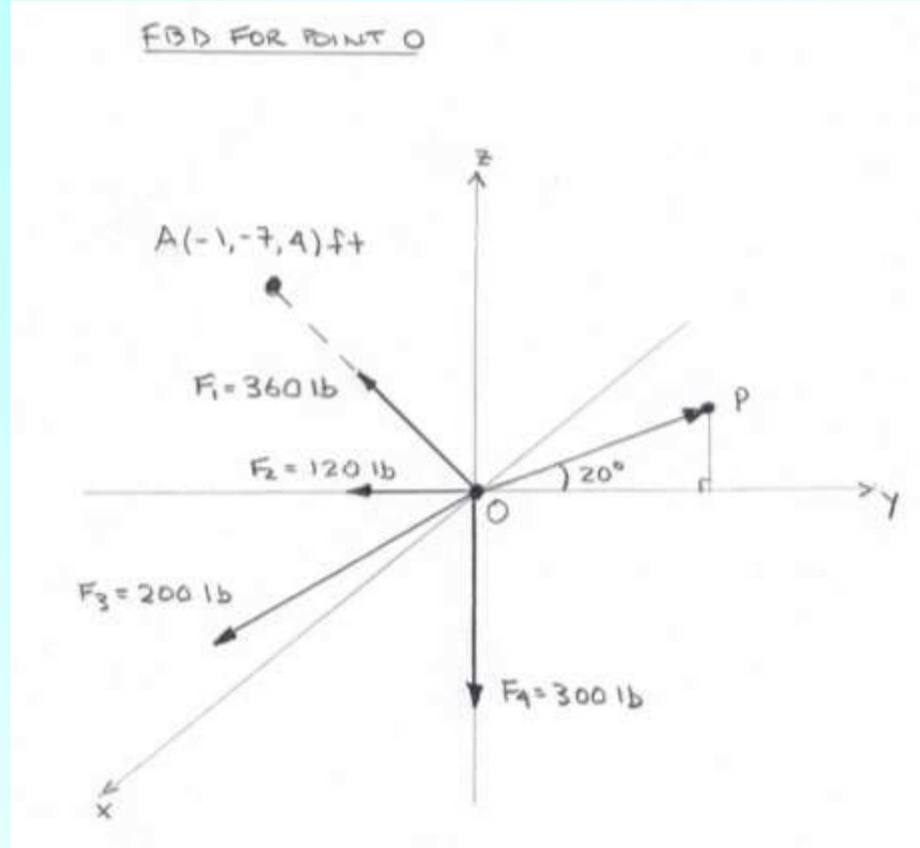
  Also observe that due to the nature of the solution of the linear equations (which tends to become less accurate as the size of the system increases), we should really use 5 digit accuracy for the coefficients to be guaranteed 3 digit accuracy of the final result, and you might want to do so when working Mastering Engineering assignments if you use the direct technique (for exams, 4 digit intermediate results will suffice)

Problem 3-75 (page 115, 12th edition)

Determine the magnitude of $\vec{P}$ and the coordinate direction angles of $\vec{F}_3$ required for equilibrium of the particle. Note that $\vec{F}_3$ acts in the octant shown.







Unknown vectors: $\vec{F}_3, \vec{P}$: write as

$$\vec{F}_3 = 200\left(\cos\alpha\,\vec{i} + \cos\beta\,\vec{j} + \cos\gamma\,\vec{k}\right)$$

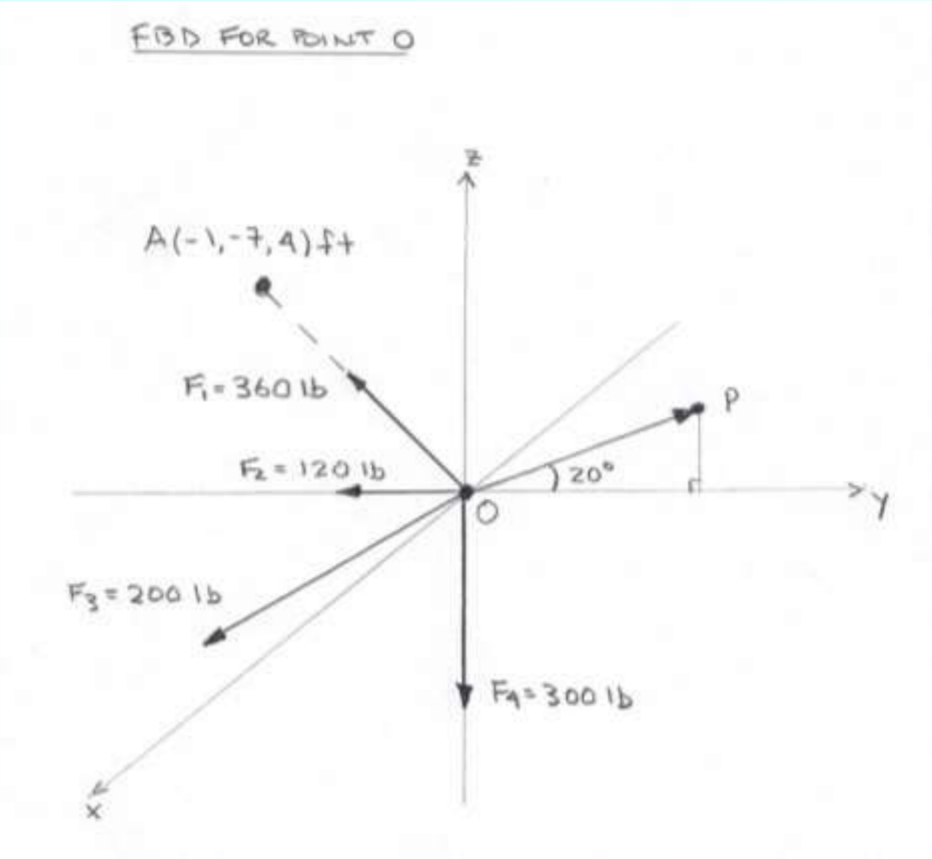
$$\vec{P} = P\,(\cos 20^\circ\,\vec{j} + \sin 20^\circ\,\vec{k})$$

Solution strategy

(1) Have 4 unknowns $(P, \alpha, \beta, \gamma)$ and 4 equations: three from equilibrium, plus $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$. Naively, at least, should be able to find a solution.

(2) Express vectors in Cartesian form, use definition of resultant and equations of equilibrium to formulate 3 equations, each of which contains one of $200\cos\alpha$, $200\cos\beta$ and $200\cos\gamma$.

(3) Rewrite equations with those terms (direction cosines) on the left hand sides, square both sides of all equations, add all equations and use $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$



(4) Get single nonlinear (quadratic) equation for P; solve by hand, or better, using solver function of calculator (need to find two solutions).

Can then also find coordinate direction angles of $\vec{F}_3$

- **Forces** (suppressing units)

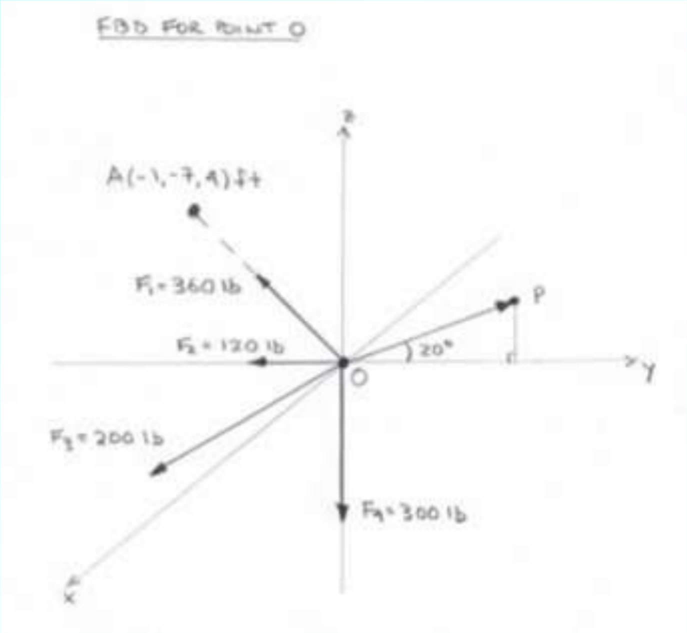
$$\vec{F}_1 = (-\vec{i} - 7\vec{j} + 4\vec{k})\,A \qquad A = 360/\sqrt{1^2 + 7^2 + 4^2}$$

$$\vec{F}_2 = -120\,\vec{j}$$

$$\vec{F}_3 = 200(\cos\alpha\,\vec{i} + \cos\beta\,\vec{j} + \cos\gamma\,\vec{k})$$

$$\vec{F}_4 = -300\,\vec{k}$$

$$\vec{P} = P(\cos 20^\circ\,\vec{j} + \sin 20^\circ\,\vec{k})$$



- **Equations of equilibrium**

$$\vec{F}_R = F_{Rx}\,\vec{i} + F_{Ry}\,\vec{j} + F_{Rz}\,\vec{k} = 0$$

$$F_{Rx} = \sum F_x = 0: \qquad -A + 200\cos\alpha = 0$$

$$F_{Ry} = \sum F_y = 0: \qquad -7A - 120 + 200\cos\beta + P\cos 20^\circ = 0$$

$$F_{Rz} = \sum F_z = 0: \qquad 4A + 200\cos\gamma - 300 + P\sin 20^\circ = 0$$

Solving the equations of equilibrium

- Equations for P, α, β, γ:

$$200\cos\alpha = A = 360/\sqrt{1^2+7^2+4^2} \qquad (1)$$

$$200\cos\beta = -P\cos 20^\circ + 7A + 120 \qquad (2)$$

$$200\cos\gamma = -P\sin 20^\circ - 4A + 300 \qquad (3)$$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 \qquad (4)$$

- From (1), $\alpha = \cos^{-1}(A/200) = 77.2^\circ$

- Square equations (1) to (3), add them and use (4)

$$200^2 = A^2 + (-P\cos 20^\circ + 7A + 120)^2 + (-P\sin 20^\circ - 4A + 300)^2$$

- Could expand and solve resulting quadratic equation for *P*. However, as previously, since we want a numeric answer, we can also use the **solver** program on a TI plotting calculator. Find two solutions:

- **Solutions**

$$P = 254 \text{ lb}$$

$$\beta = \cos^{-1}((-P\cos 20^{\circ} + 7A + 120)/200) = 16.6^{\circ}$$

$$\gamma = \cos^{-1}((-P\sin 20^{\circ} - 4A + 300/200) = 79.6^{\circ}$$

$$P = 639 \text{ lb}$$

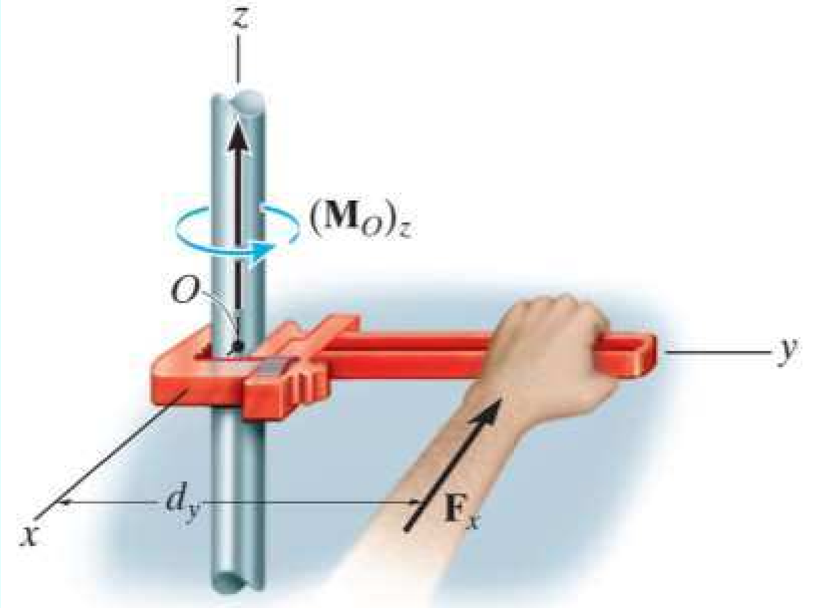
$$\beta = \cos^{-1}((-P\cos 20^{\circ} + 7A + 120)/200) = 148^{\circ}$$

$$\gamma = \cos^{-1}((-P\sin 20^{\circ} - 4A + 300/200) = 119^{\circ}$$

The second solution is the correct one since $\vec{F}_3$ acts in the octant shown.

CHAPTER 4: Force System Resultants

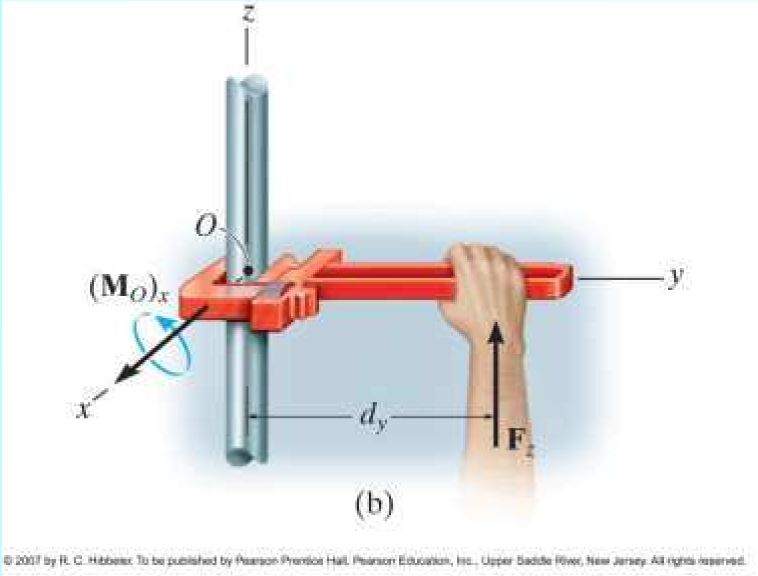
4.1 Moment of a Force: Scalar Formulation



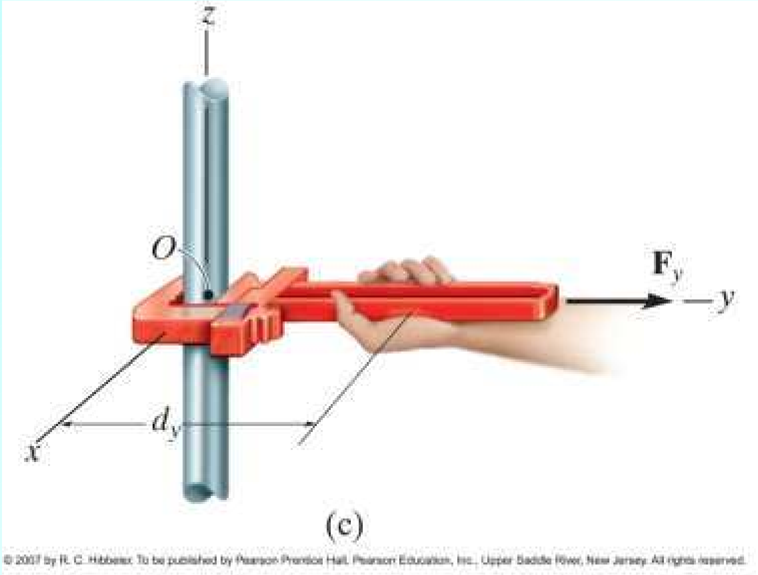
- **MOMENT OF A FORCE**
  - MOMENT
  - TORQUE
- Moment of a force about a point or axis provides a measure of the tendency of the force to cause a body to rotate about the point or axis
- **NOTE: a moment is a vector!**

- Referring to the diagram, the larger the force, or the perpendicular distance from the axis, the greater the turning effect


THAT MOMENT YOU REALIZE
YOU HAVE MORE TORQUE
THAN YOUR HONDA
STARECAT.COM



- $\mathbf{F}_z$ tends to rotate pipe about x axis
- Even if pipe cannot turn, the applied force still produces a moment about point O



- No moment produced in this case: force $\mathbf{F}_y$ acts through point O, no tendency for rotation possible