PHYS 170 Section 101
Lecture 7
September 19, 2018

Lecture Outline/Learning Goals

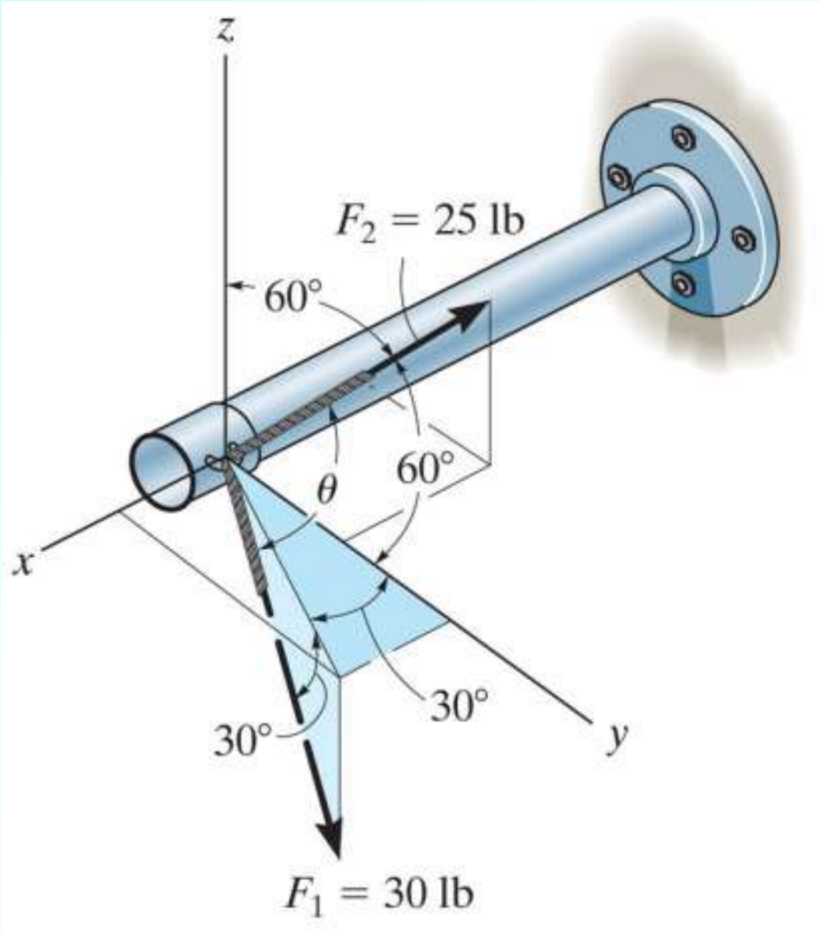
- Finish dot product example
- **Start Chapter 3**
- Quick review of Newton's laws of motion
- Define and discuss equations of equilibrium for coplanar and three dimensional equilibria
- Introduce concept of **FREE BODY DIAGRAM** for a particle
- Discuss different types of forces that will be encountered in equilibrium problems and identify them in various free body diagrams
- Worked example of static equilibrium of a particle in three dimensions

Problems 2-120 and 2-121 (page 78, 14th edition)

Two cables exert forces on the pipe as shown

2-120 Determine the projected component of $\vec{F}_1$ along the line of action of $\vec{F}_2$

2-121 Determine the angle θ between the two cables



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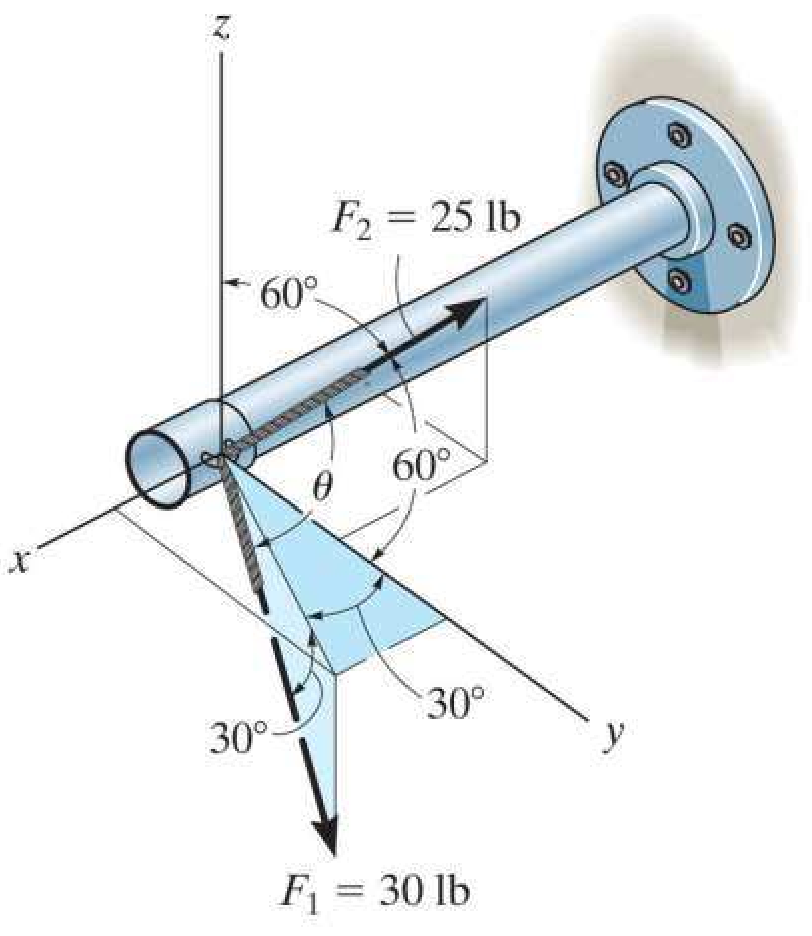
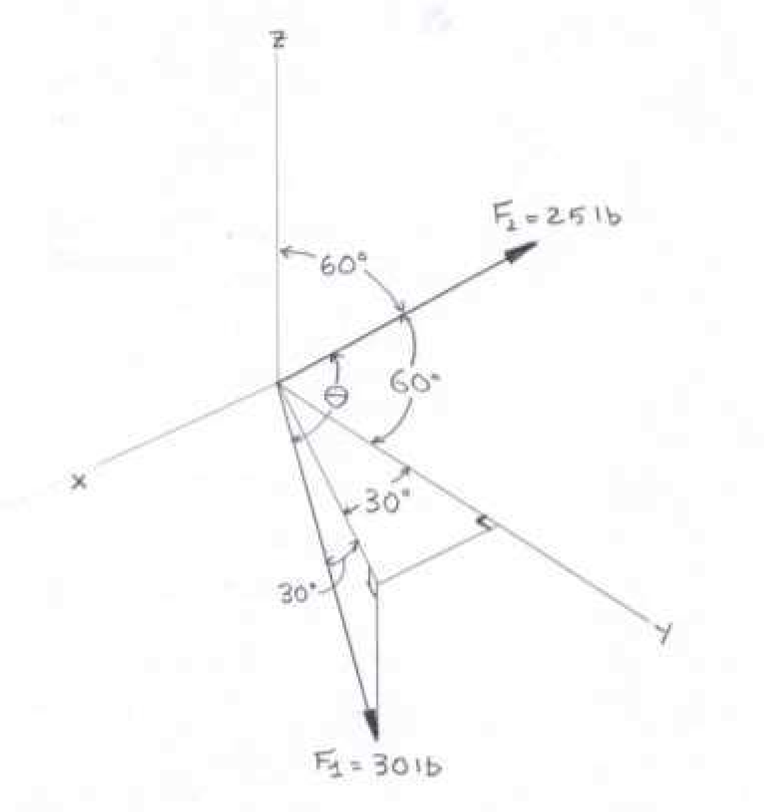


02_PROB_120-121



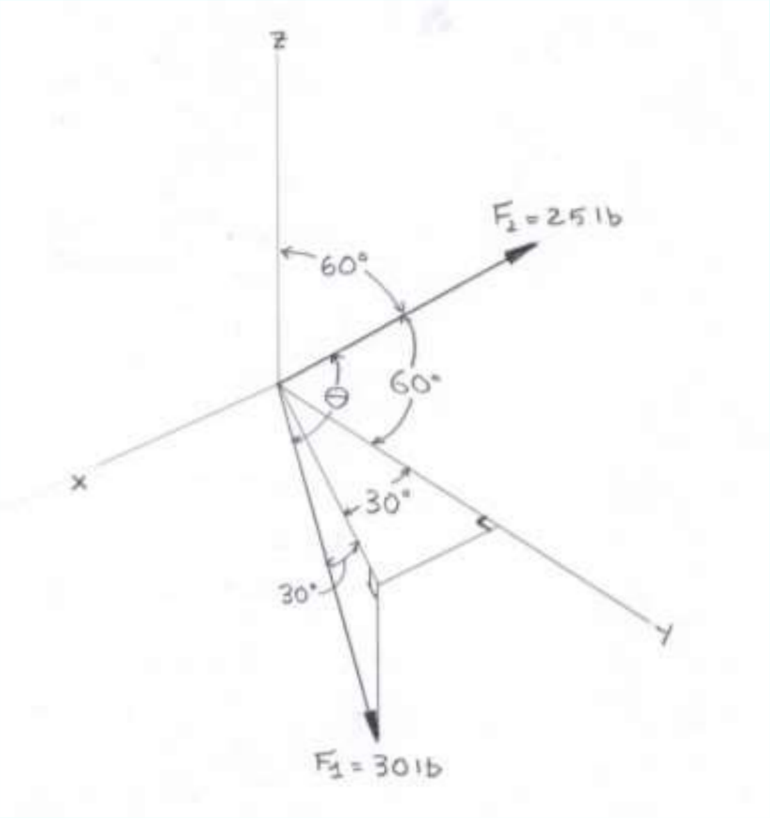
Solution strategy:

(1) Express $\vec{F}_1$ in Cartesian form from given geometry

(2) Express unit vector, $\vec{u}$, in direction of $\vec{F}_2$ in Cartesian components from direction cosines

(3) Compute $\vec{F}_1 \cdot \vec{u}$, the projected component of $\vec{F}_1$ in the direction of $\vec{F}_2$

(4) Compute angle between two cables ($\vec{F}_1$ and $\vec{F}_2$) using fundamental definition of dot product



- **Data** (suppressing units)

$$\vec{F}_1 = 30(\cos 30^\circ \sin 30^\circ \vec{i} + \cos 30^\circ \cos 30^\circ \vec{j} - \sin 30^\circ \vec{k})$$

$$\vec{F}_2 = 25\vec{u}$$

$$\vec{u} = \cos\alpha\,\vec{i} + \cos 60^\circ \vec{j} + \cos 60^\circ \vec{k}$$

- α can be computed from the equation $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1,$ taking into account the fact the x-component of $\vec{F}_2$ is negative

$$\alpha = \cos^{-1}\left(-\sqrt{1 - \cos^2 60^\circ - \cos^2 60^\circ}\right) = 135^\circ$$

- The projected component of $\vec{F}_1$ along the line of action of $\vec{F}_2$ is

$$\vec{F}_1 \cdot \vec{u}$$

- It follows using

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

that

$$\vec{F}_1 \cdot \vec{u} = 30(\cos 30^\circ \sin 30^\circ \cos 135^\circ + \cos 30^\circ \cos 30^\circ \cos 60^\circ - \sin 30^\circ \cos 60^\circ)$$
$$= -5.44 \text{ lb}$$

- The negative sign indicates that the projected component of $\mathbf{F}_1$ along $\mathbf{F}_2$ acts in the opposite sense of direction of $\mathbf{F}_2$, and the magnitude of the projected component is 5.44 lb

- Now, to determine the angle between the two cables, we use the definition of the dot product to write

$$\vec{F}_1 \cdot \vec{u} = F_1 \cos\theta$$

where θ is the angle between $\vec{F}_1$ and $\vec{F}_2$ (same as angle between $\vec{F}_1$ and $\vec{u}$)

- Thus, we have

$$\theta = \cos^{-1}\left(\frac{\vec{F}_1 \cdot \vec{u}}{F_1}\right) = \cos^{-1}\left(\frac{-5.44}{30}\right) = 100^\circ$$

CHAPTER 3
EQUILIBRIUM OF A PARTICLE

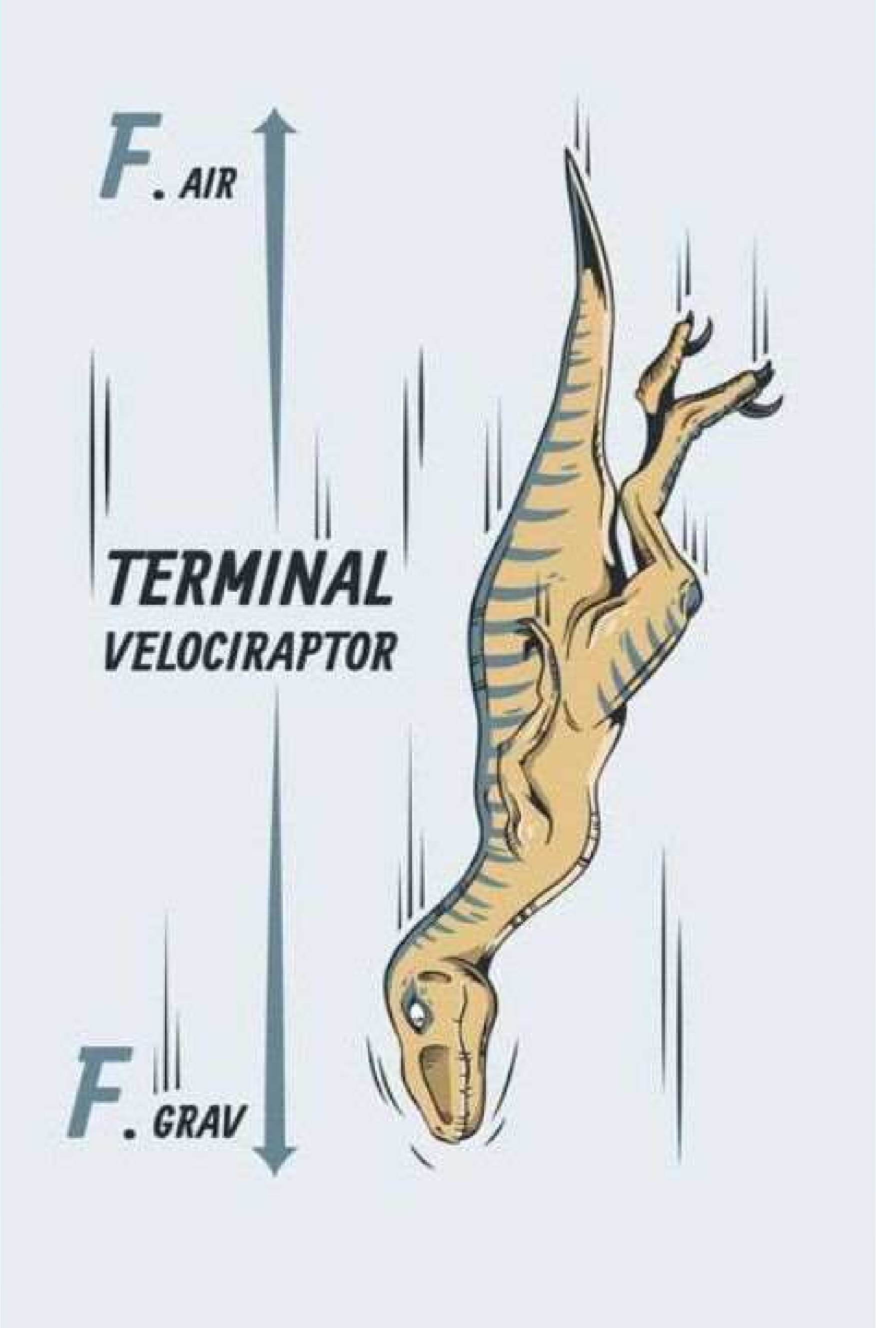

MAY THE
MASS TIMES
ACCELERATION
BE WITH
YOU

Newton's Three Law of Motion (review)

- **1st law:** Particle originally at rest, or moving in a straight line with constant velocity, will remain in this state provided the particle is not subjected to an unbalanced force
- **2nd law:** A particle acted upon by an unbalanced resultant force experiences an acceleration **a** that has the same direction as the force and a magnitude that is directly proportional to the force, i.e.

$$\Sigma \mathbf{F} = m\mathbf{a}$$

  where $\Sigma \mathbf{F}$ is the vector sum of all forces acting on the particle
- **3rd law:** The mutual forces of action and reaction between two particles are equal, opposite and colinear


F. AIR
TERMINAL
VELOCIRAPTOR
F. GRAV

Condition for Equilibrium of a Particle

- Equilibrium: Particle that satisfies Newton's 1st law is said to be in equilibrium; in statics, most often consider the case where particle is also at rest, which we refer to as static equilibrium.

- Necessary and sufficient condition for equilibrium

$$\Sigma \mathbf{F} = \mathbf{0}$$

Necessary part follows from 1st law, sufficient from 2nd.

Procedure for Drawing Free Body Diagram (FBD)

- Draw outlined shape
  - Abstract the particle as isolated or cut “free” from its environment, by drawing its outlined shape (i.e. without any of the supports, braces, cables, springs etc. etc. that might be attached to it)
- Show all forces
  - Show on your diagram all forces acting on the particle. Includes active forces, which tend to set the particle in motion, as well as reactive forces that are the result of constraints/supports that tend to prevent motion
  - **CRUCIAL POINT:** Must account for all forces, may help to trace around the particle’s boundary. Can often make use of the fact that particle is in equilibrium, i.e. that “forces must balance”.
- Identify/label each force
  - Known forces should be labeled with magnitudes & directions. Letters are used to represent magnitudes and directions of unknown forces

SELF STUDY

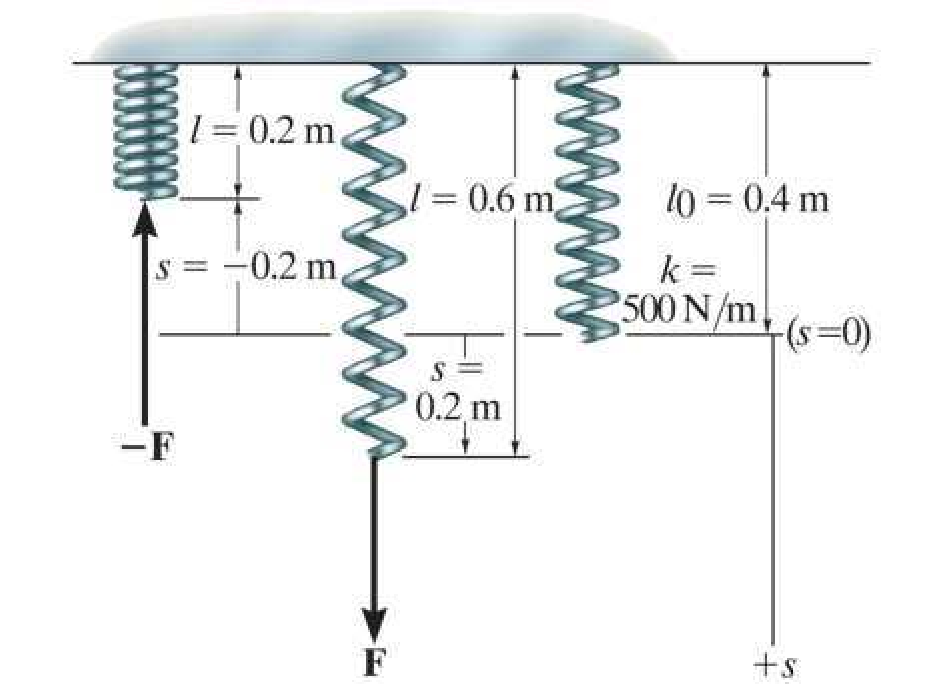
Springs

- Springs used in this course will be idealized as linearly elastic springs: length of spring changes in direct proportion to force acting on it
- Constant of proportionality is known as the spring constant or stiffness and usually denoted k
- Thus, take the following for the definition of the magnitude of the force exerted on (or by, up to a sign, via the 3^{rd} law) a spring

$$F = ks$$

where s is the amount that the spring has been deformed (compressed/elongated) from its unloaded position

- SI units of spring constant: N/m



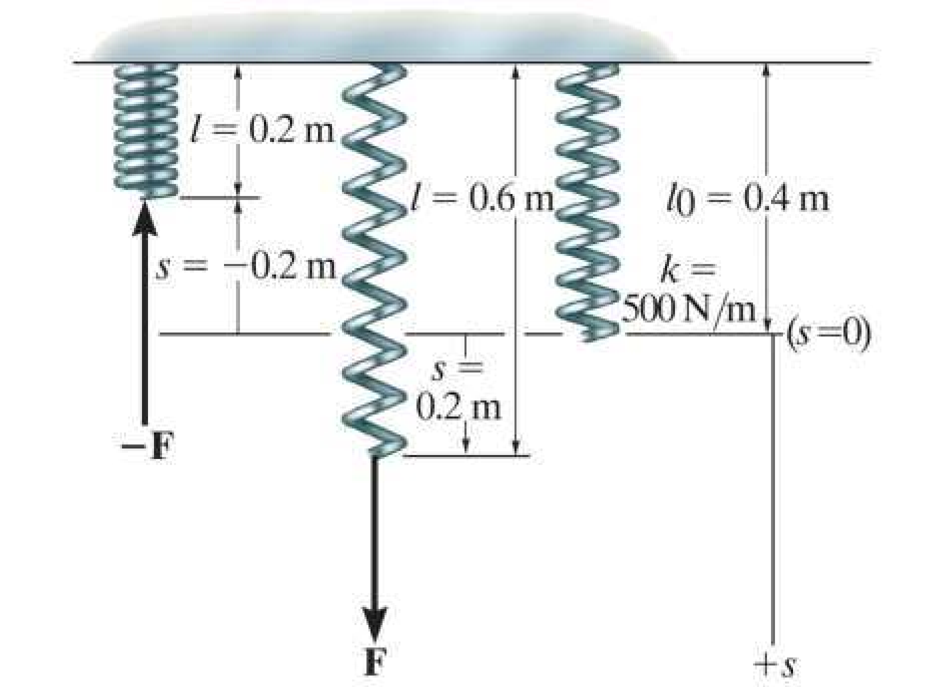
SELF STUDY

$F = ks = k(l - l_0)$

where l_0 and l are the undeformed and deformed lengths of the spring, respectively

If $l > l_0$ then $s > 0$ and $F > 0$, spring is stretched and force must "pull" on spring

If $l < l_0$ then $s < 0$ and $F < 0$, spring is compressed and force must "push" on spring



SELF STUDY

Left case:

$$F = ks = k(l - l_0) = (500)(0.2 - 0.4)\text{ N} = (500)(-0.2) = -100\text{ N}$$

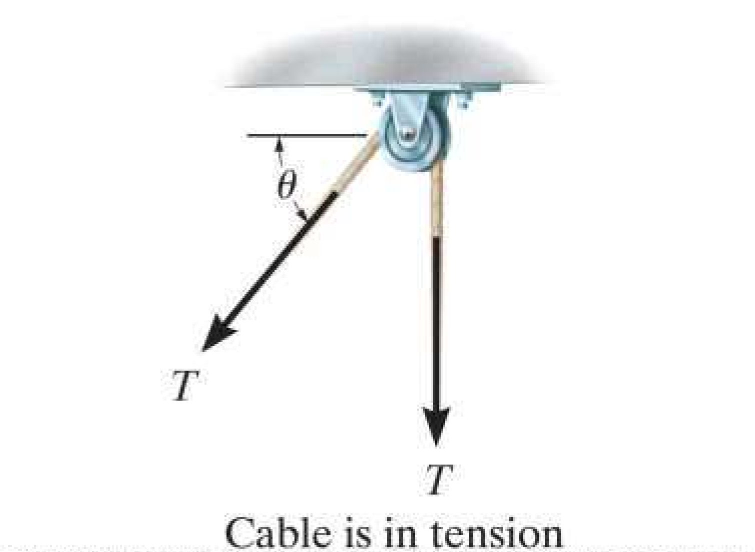
Right case:

$$F = ks = k(l - l_0) = (500)(0.6 - 0.4)\text{ N} = (500)(+0.2) = 100\text{ N}$$

NOTE: These are the forces that must act ON the spring so that it deforms with the given displacement (either compressed or stretched)

Cables and Pulleys

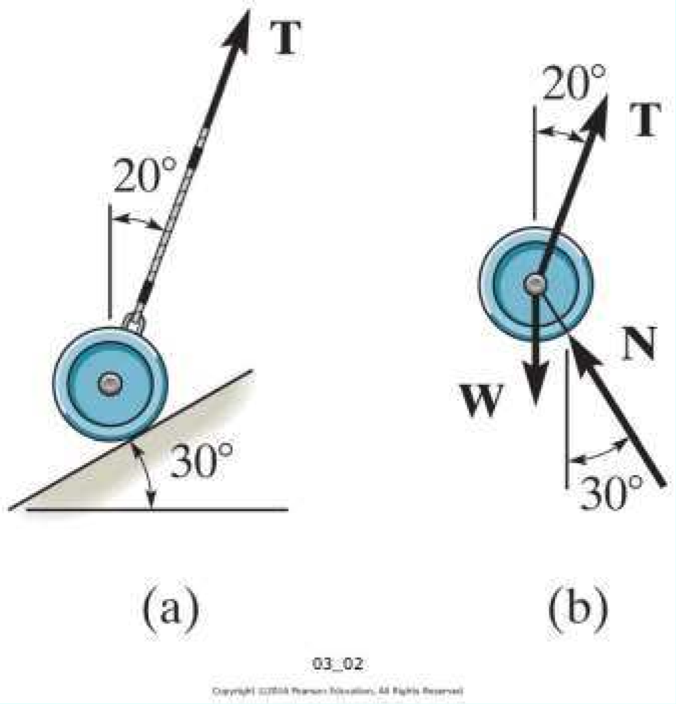
- Unless otherwise noted, cables (cords) are assumed to be massless (negligible weight), and they cannot stretch



- Cable can only support tension ("pulling force") and this force always acts in the direction of the cable

- Tension in a continuous cable, passing over a massless pulley, must have a constant magnitude to keep the cable in equilibrium

Smooth Contact



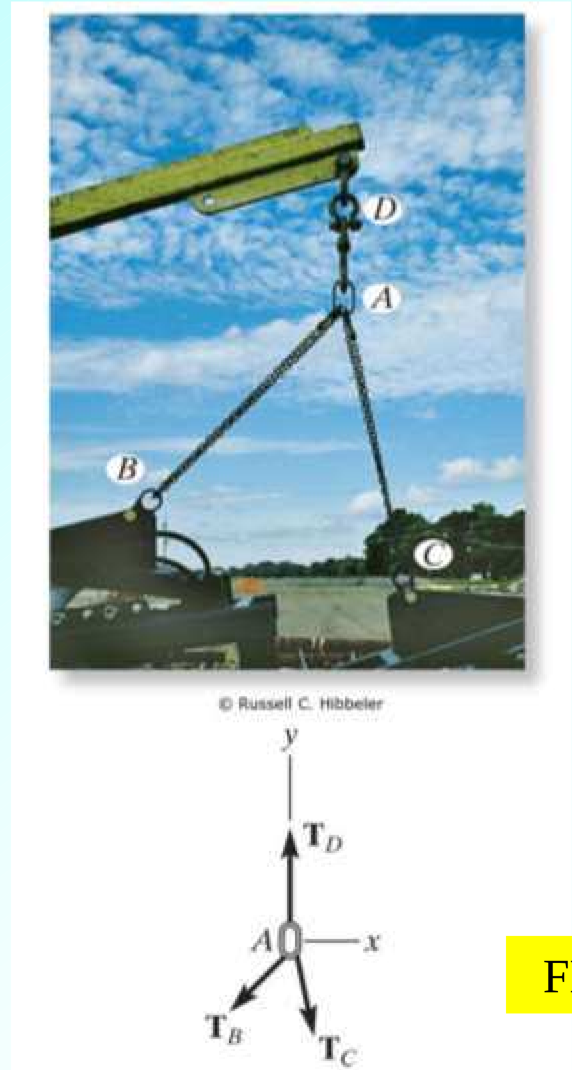
Object resting on smooth surface: smooth surface will exert a force which is normal to the point of contact

Example at left: inclined plane exerts normal force **N** on cylinder

Cylinder is also subject to its weight **W** and the tension **T** in the cord. Since all three forces acting on the cylinder are concurrent at its center we can treat the cylinder as a "particle" and apply the equations of equilibrium to it.

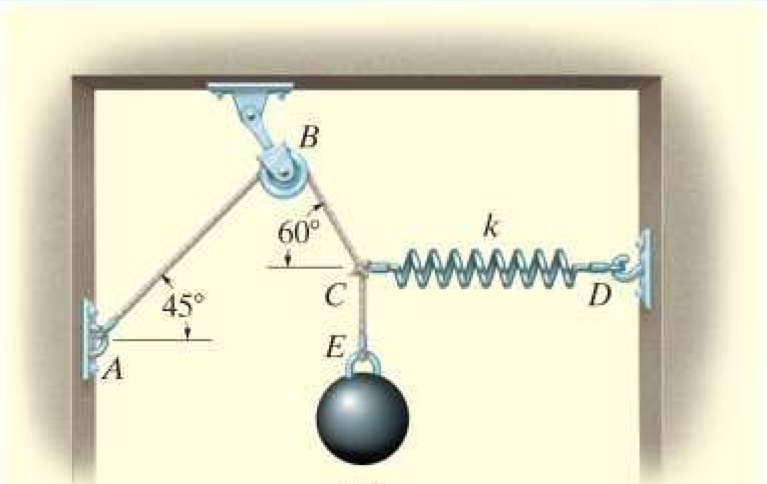
Coplanar Free Body Diagrams: Examples

FBD of bucket



FBD of ring A

Coplanar Free Body Diagrams: Examples



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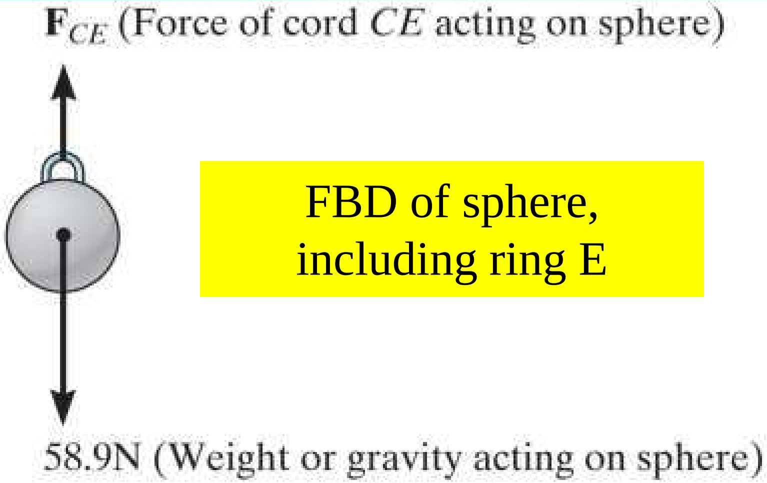

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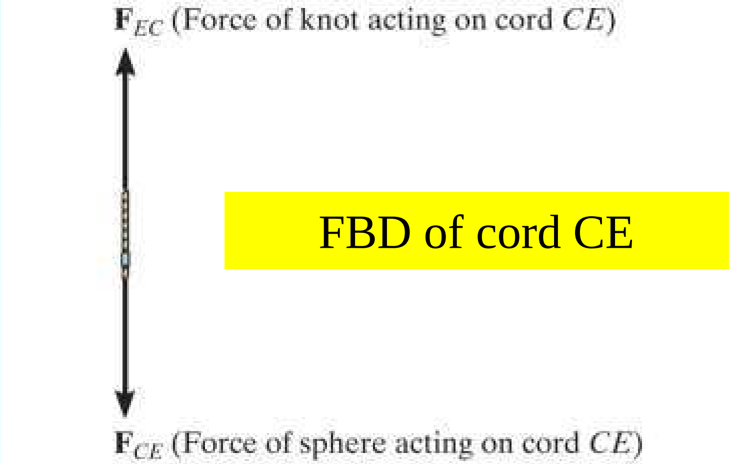

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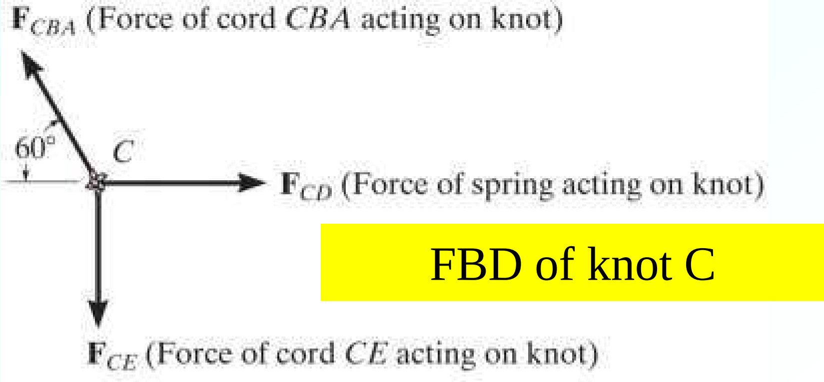
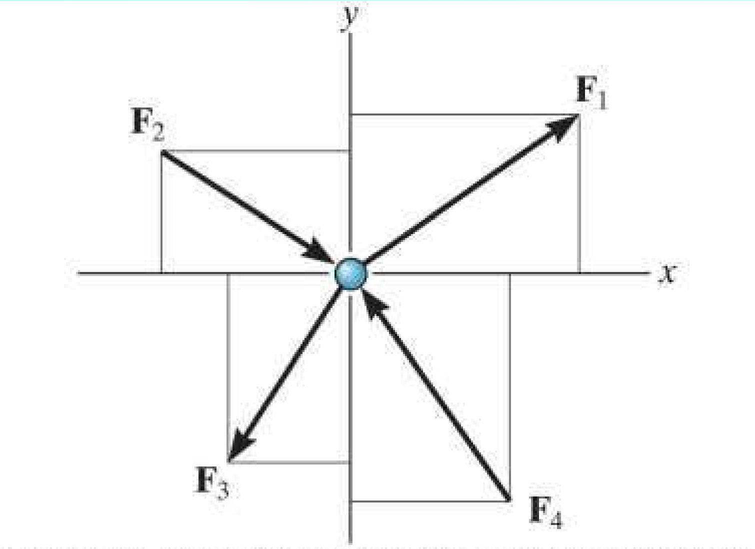


(d)

SELF STUDY

SELF STUDY

Coplanar Force Systems (2D systems)



- Resolve all forces into x and y components

$$\Sigma \mathbf{F} = \mathbf{0}$$
$$\Sigma F_x \mathbf{i} + \Sigma F_y \mathbf{j} = \mathbf{0}$$

- So have scalar equations of equilibrium

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$

- NOTE: Two equations, can solve for at most two unknowns! (angles, magnitudes, components of forces in FBD)

SELF STUDY

Scalar Notation / Sense of Force

- Account for sense of direction of each force component with algebraic sign

- For forces with unknown magnitudes, assume sign according to direction of force arrow in free body diagram

- If solution yields negative scalar, force (or force component) acts in opposite direction than was assumed

$$\Sigma F_x = 0$$

$$+F + 10 \text{ N} = 0$$

$$\rightarrow F = -10 \text{ N}$$

so force acts to left

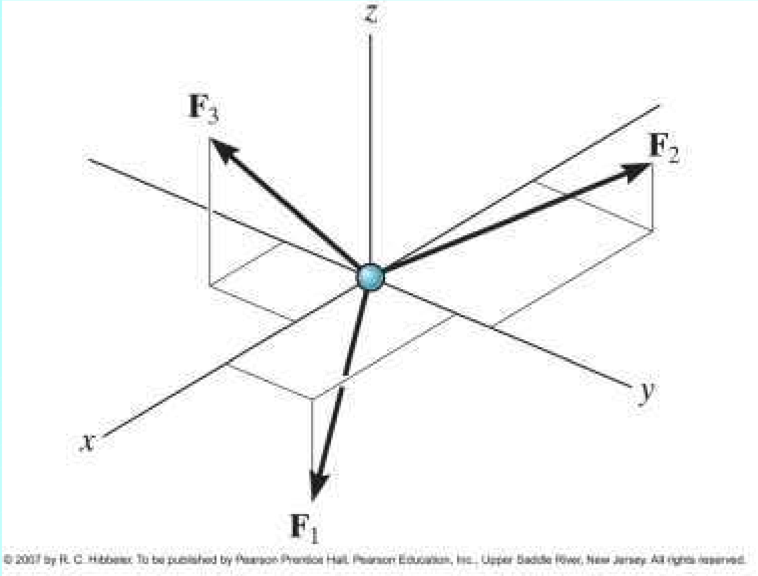
Coplanar Equilibrium

SELF STUDY

- Don’t have time to go into any details/examples of 2D equilibrium calculations.

- However, strongly recommend that you practice a few problems from the text (3-1 through 3-42)

Three-Dimensional Force Systems (3D)



- Particle equilibrium requires

$$\Sigma \mathbf{F} = \mathbf{0}$$

$$\Sigma F_x \mathbf{i} + \Sigma F_y \mathbf{j} + \Sigma F_z \mathbf{k} = \mathbf{0}$$

- Thus have following 3 scalar component equations of particle equilibrium

$$\Sigma F_x = 0$$

$$\Sigma F_y = 0$$

$$\Sigma F_z = 0$$

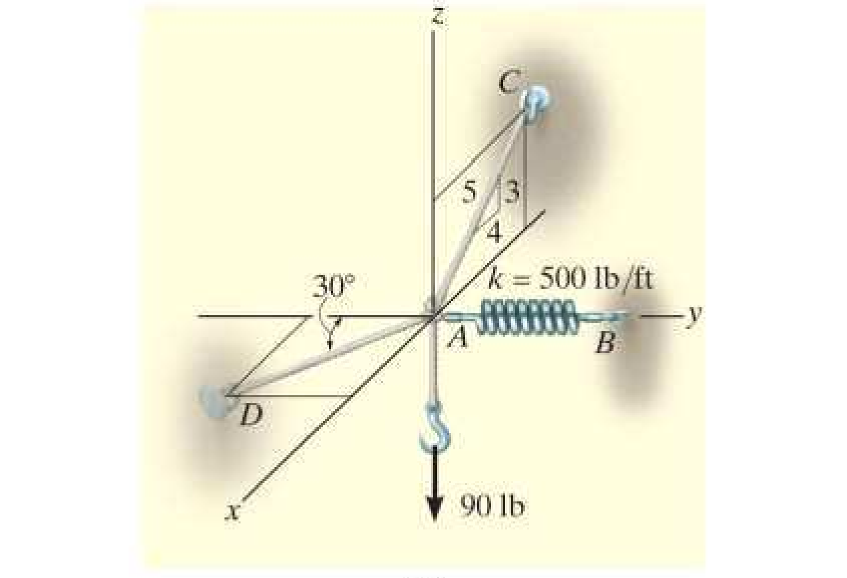
NOTE: Three equations, can solve for at most three unknowns! (angles, magnitudes, components of forces in FBD)

3D Free Body Diagrams: Examples

FBD of ring A

3D Free Body Diagrams: Examples

FBD of knot A



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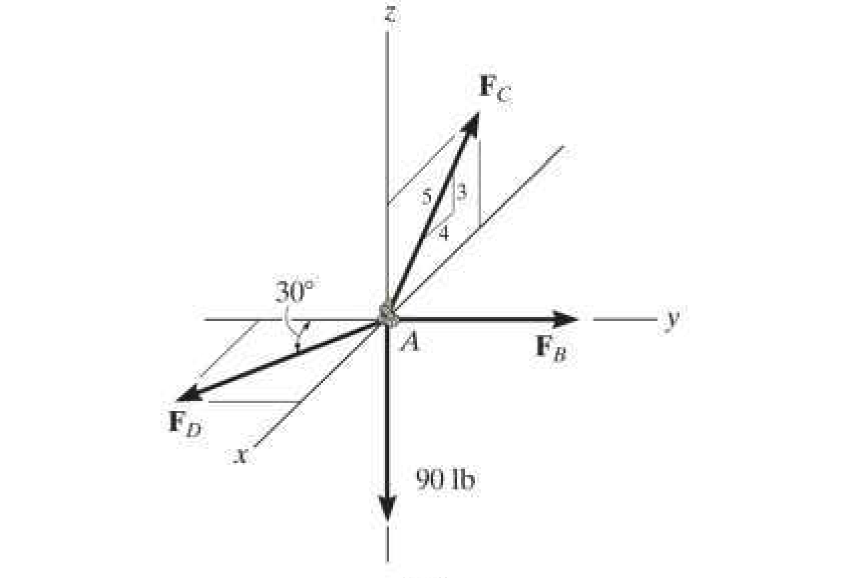
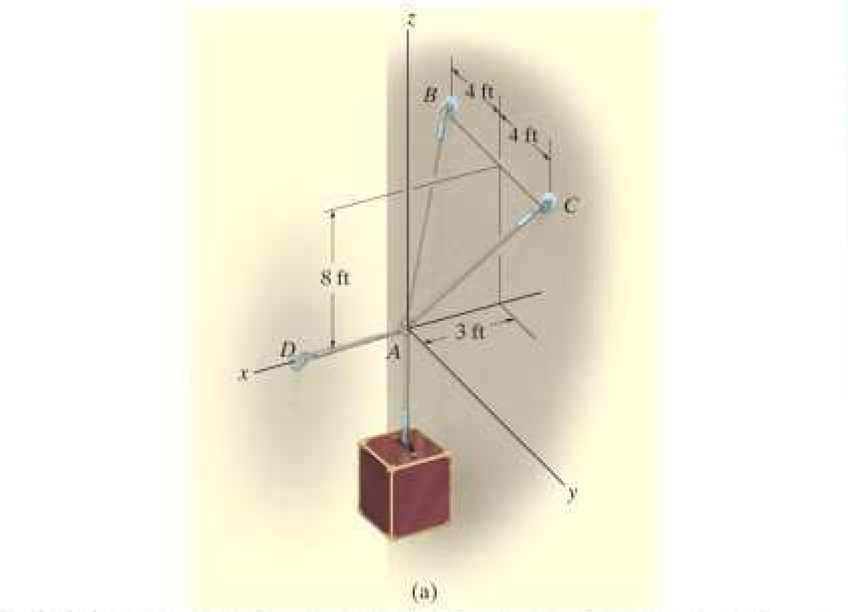
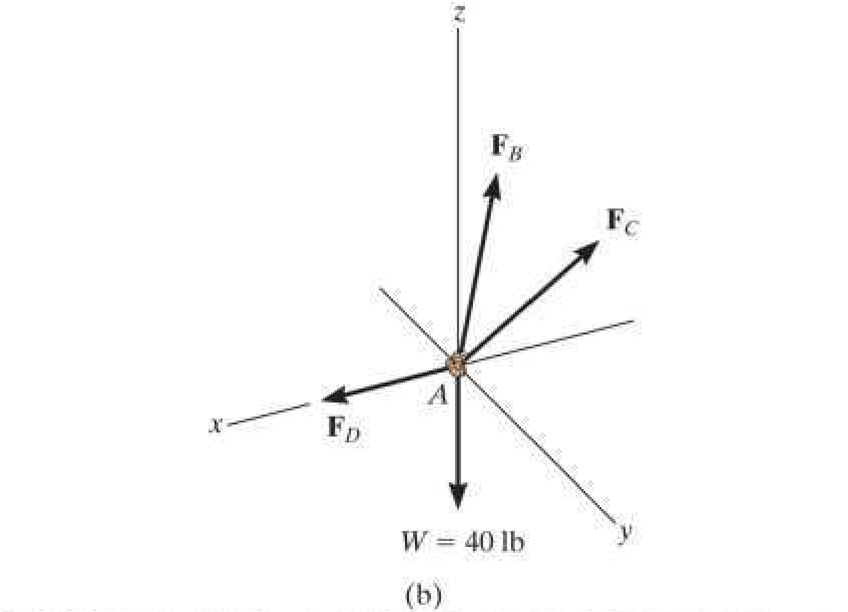


(b)

3D Free Body Diagrams: Examples

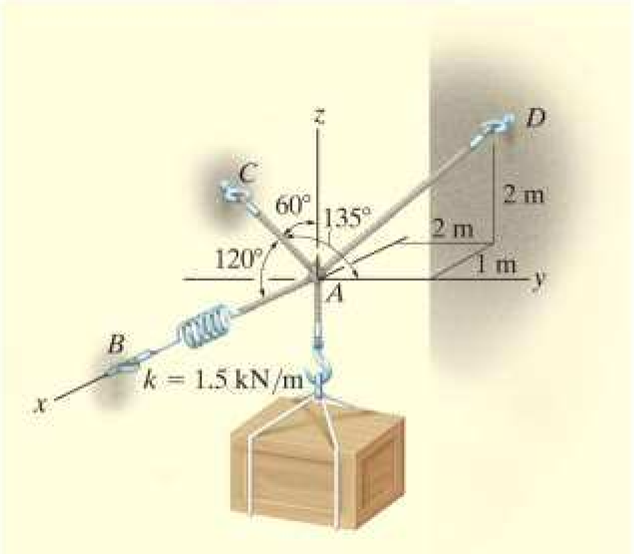
FBD of knot A





3D Free Body Diagrams: Examples

FBD of knot A



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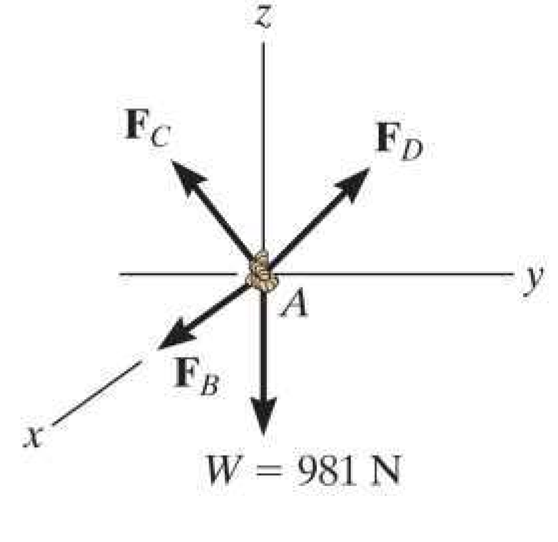
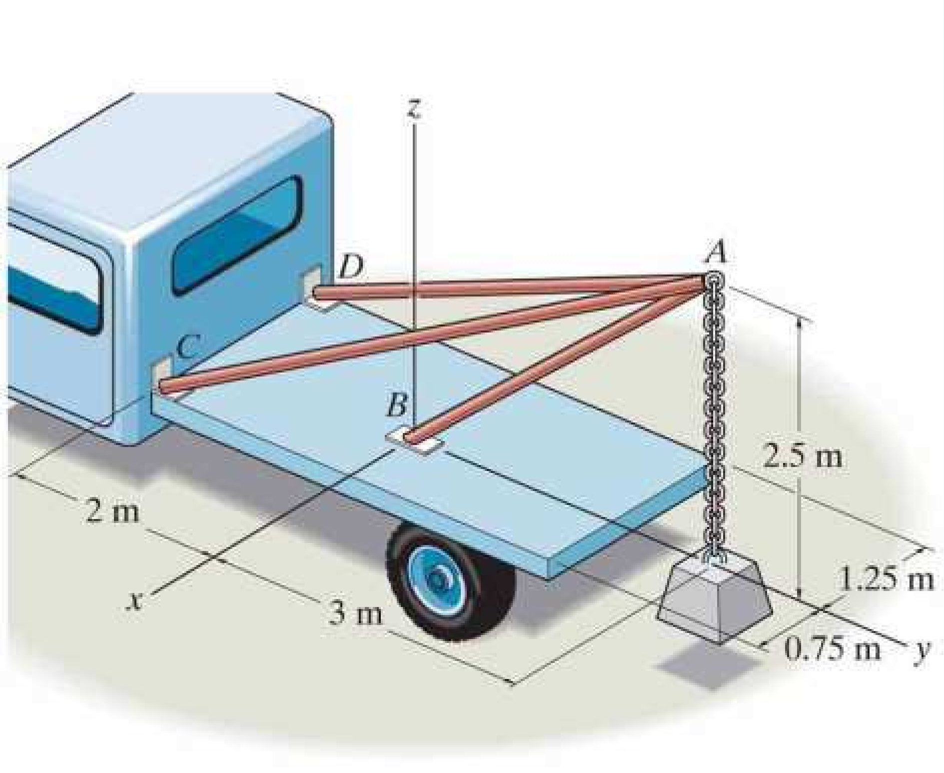
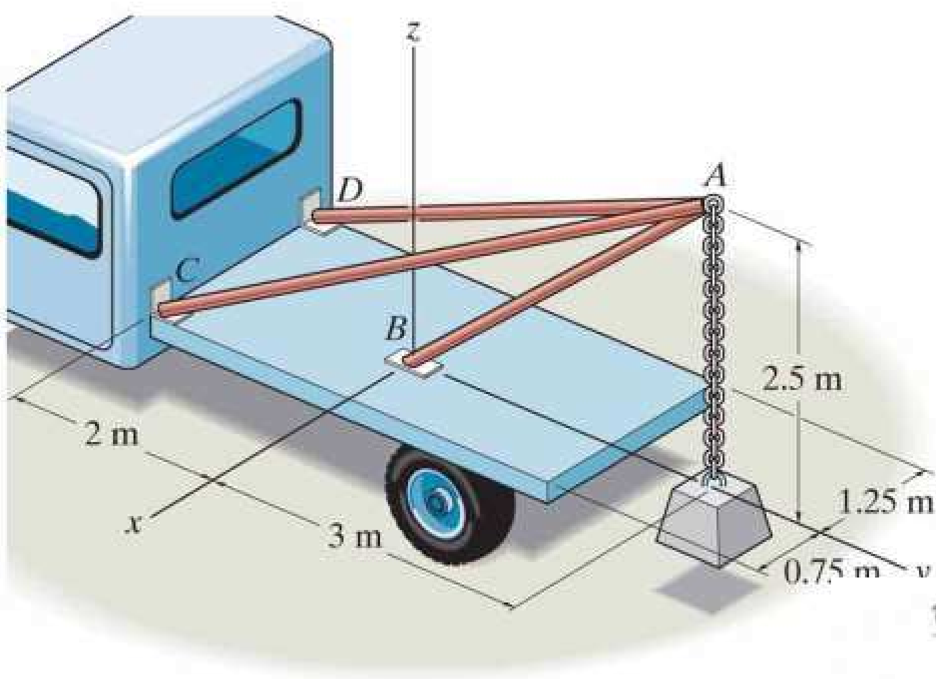


(b)

Problem 3-53 (page 100, 12th edition)

Determine the force acting along the axis of each of the three struts needed to hold the 500 kg block in equilibrium





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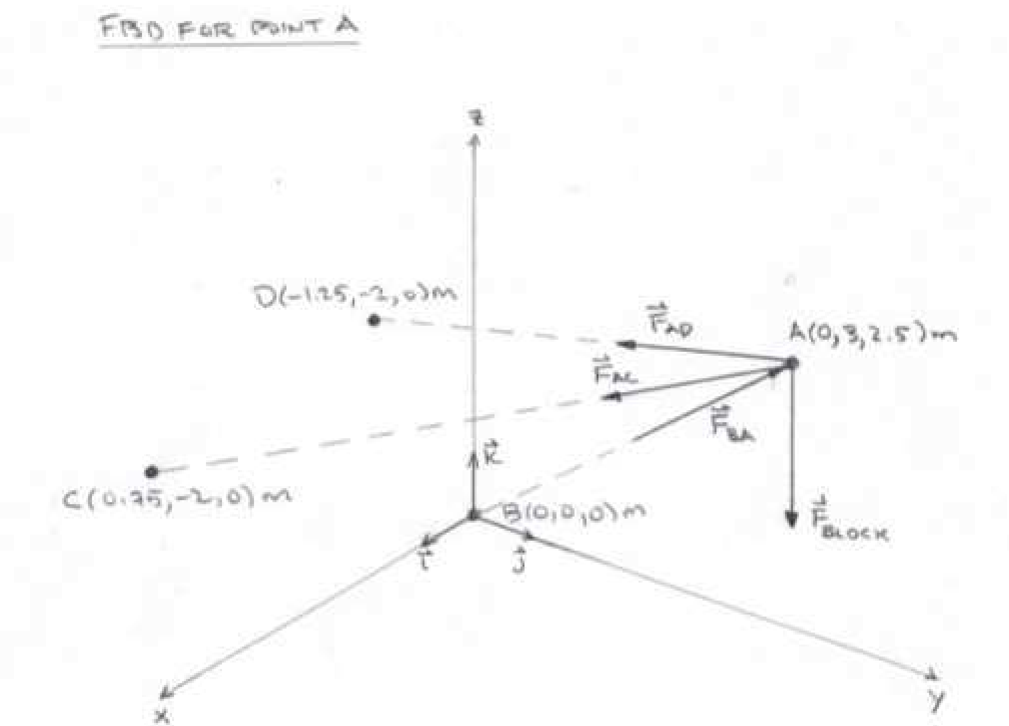
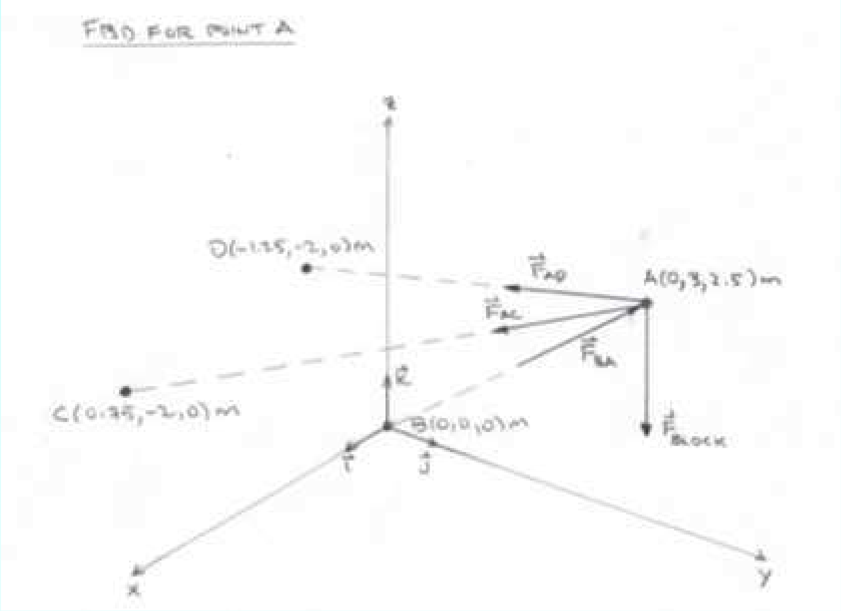


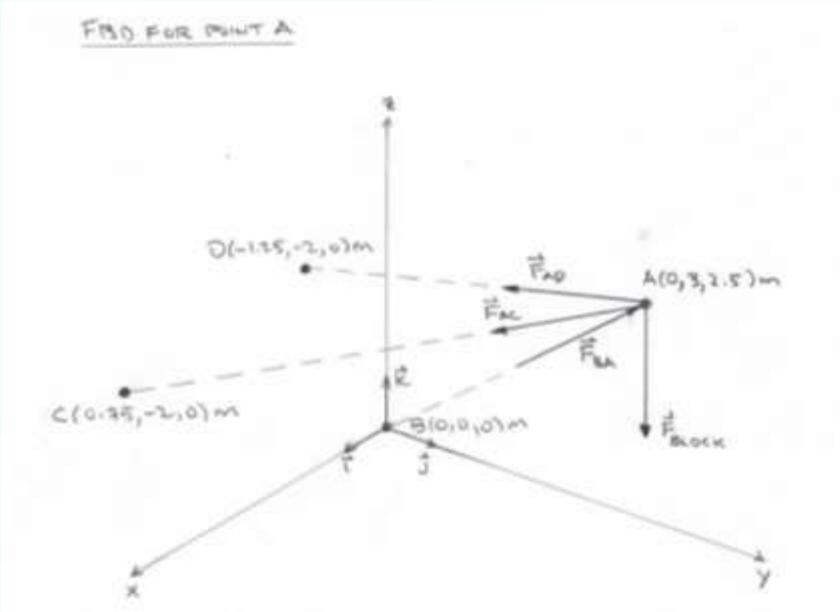
Problem 3-53 (page 100, 12th edition)

- Note that as shown in the free-body diagram, struts AD and AC are under tension (forces directed *away* from A), while strut AB is under compression (force directed *towards* A).

  We incorporate this information in our solution—in particular in how we define the directions of the various force vectors—with the anticipation that the unknowns representing the magnitudes of the vectors will come out positive. However, we could equally well simply make arbitrary choices for the directions and then the signs of the corresponding unknowns (when we have solved the equations) will tell us whether we made the correct assumptions or not.



Solution strategy:



(1) Express $\vec{F}_{BA}$, $\vec{F}_{AC}$, $\vec{F}_{AD}$ and $\vec{F}_{block}$ in Cartesian form using coordinates of points A, B, C and D. Also adopt calculational trick used previously to simplify linear equations and solution thereof

(2) Use definition of resultant and equations of equilbrium to formulate 3 linear equations in 3 unknowns (essentially the magnitudes of the three forces F_{BA}, F_{AC} and F_{AD})

(3) Solve linear system (use of calculator recommended), and then determine force magnitudes from inverse relation of calculational trick.

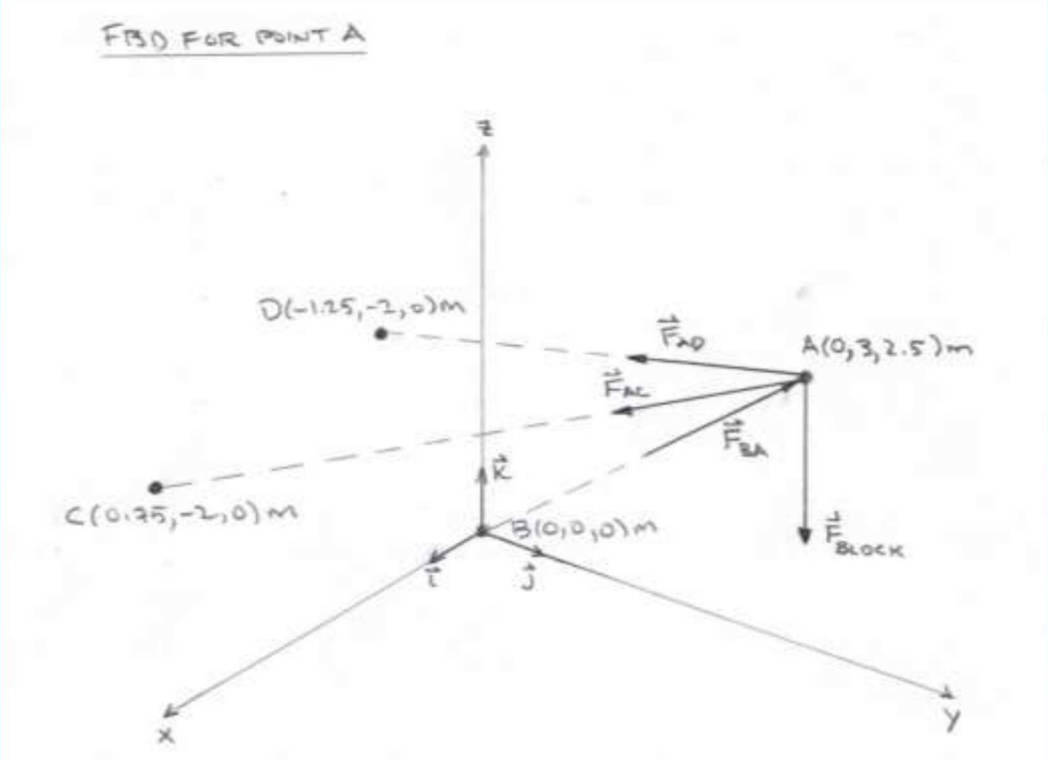
- **Coordinates**

$A(0,\ 3,\ 2.5)$ m

$B(0,\ 0,\ 0)$ m

$C(0.75,\ -2,\ 0)$ m

$D(-1.25,\ -2,\ 0)$ m



- **Forces** (suppressing units)
  - Example

$$\vec{r}_{BA} = \vec{r}_A - \vec{r}_B = \left((0-0)\vec{i} + (3-0)\vec{j} + (2.5-0)\vec{k}\right) = 3\vec{j} + 2.5\vec{k}$$

$$\vec{F}_{BA} = F_{BA}\left(\frac{\vec{r}_{BA}}{r_{BA}}\right) = (3\vec{j} + 2.5\vec{k})X \qquad \text{where} \quad X = F_{BA}/r_{BA} = F_{BA}/\sqrt{3^2 + 2.5^2}$$

Once we determine X, then we calculate F_{BA} from

$$F_{BA} = X\, r_{BA} = X\sqrt{3^2 + 2.5^2}$$

Solution continues in Lecture 8