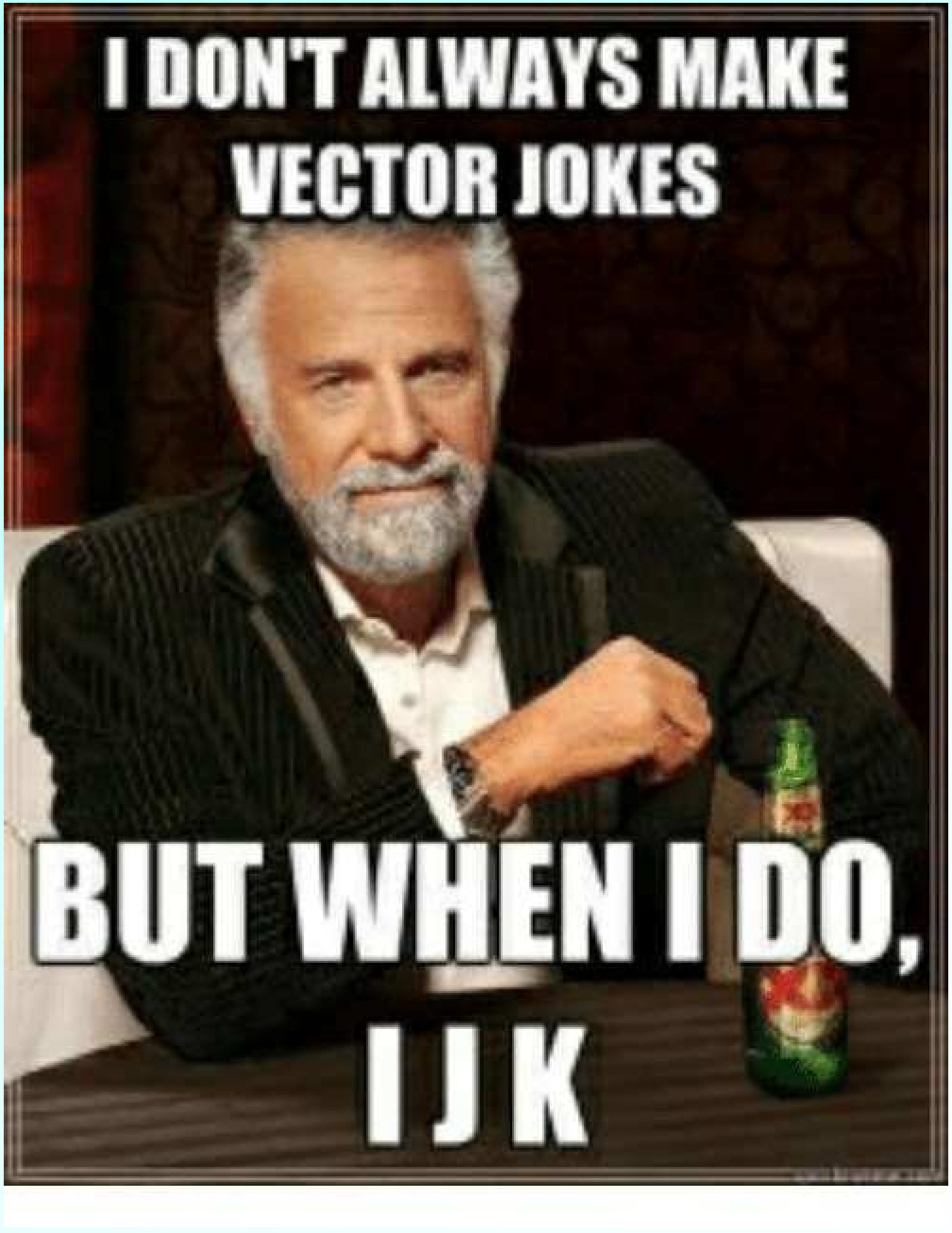
PHYS 170 Section 101
Lecture 6
September 17, 2018

SEPTEMBER 17—ANNOUNCEMENTS

- Homework Assignment 1 due today, 11:59 PM
- Reminder that my office hour is Tuesday, 11:00 AM—noon, in Hennings 403 (see directions on Canvas)
- You can also make an appointment to see me in my office via email

Lecture Outline/Learning Goals

- Finish concurrent force system from last day
- Dot product
  - Laws of operation, Cartesian vector formulation, applications
- Sample problem using dot product
- **Start Chapter 3**
- Quick review of Newton's laws of motion
- Define and discuss equations of equilibrium for coplanar and three dimensional equilibria
- Introduce concept of **FREE BODY DIAGRAM** for a particle


I DON'T ALWAYS MAKE VECTOR JOKES
BUT WHEN I DO,
I J K

Problem 2-109 (page 68, 13th edition)

The magnitude of the resultant force is 1300 N and acts along the axis of the strut from *A* towards *O*

Determine the magnitude of each of the three forces acting on the strut.

Set $x = 0$ and $z = 5.5$ m

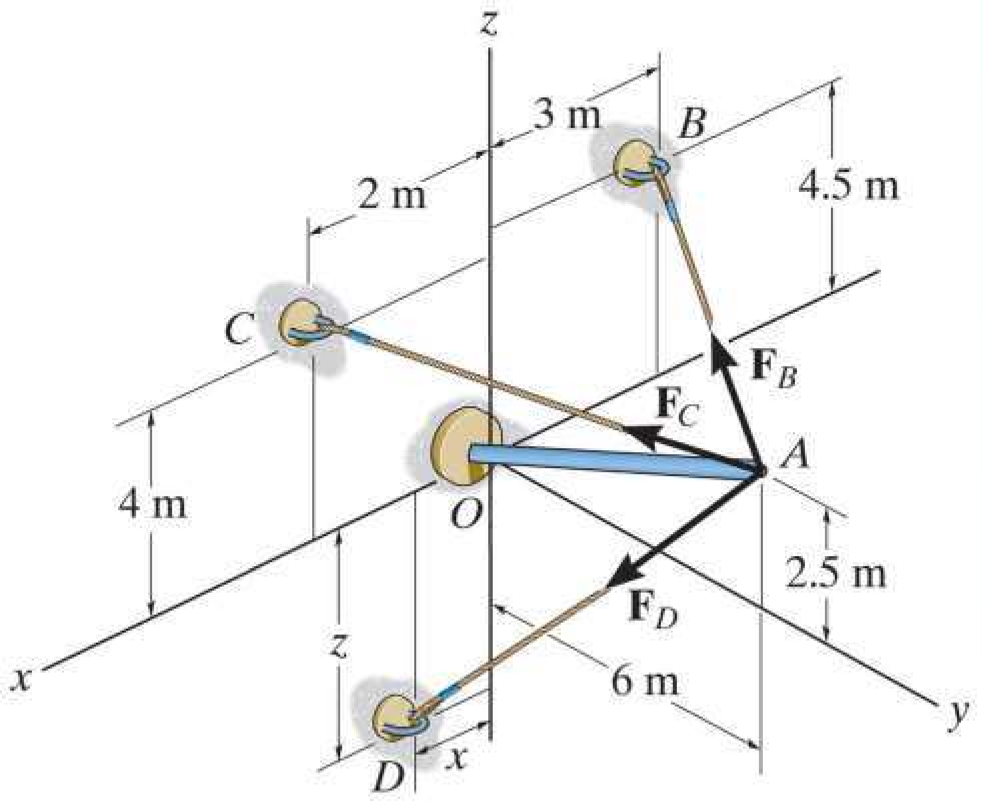


Figure: 02_P108-109

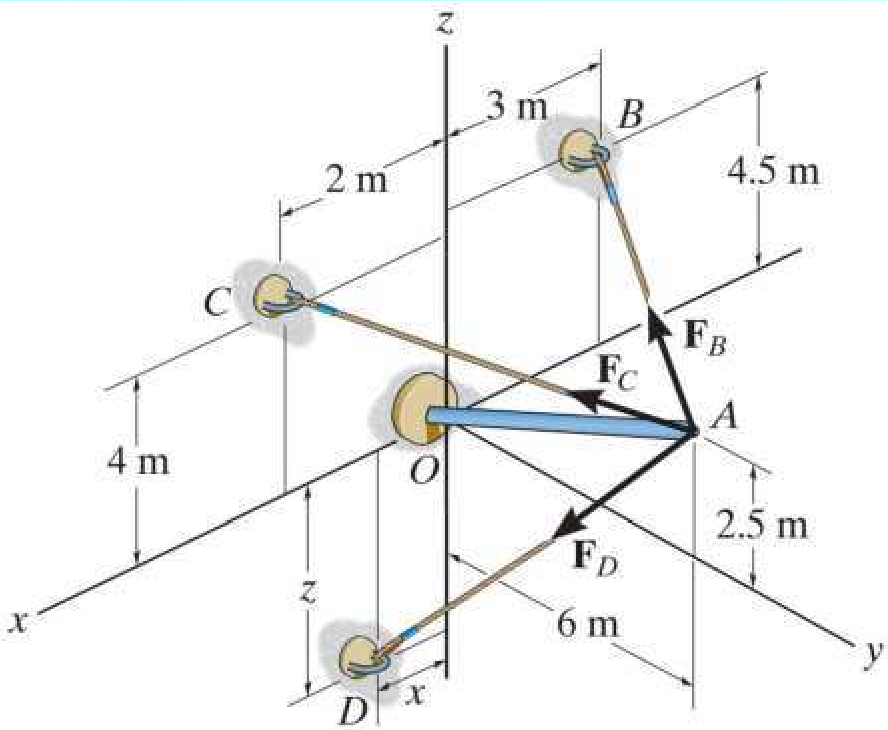
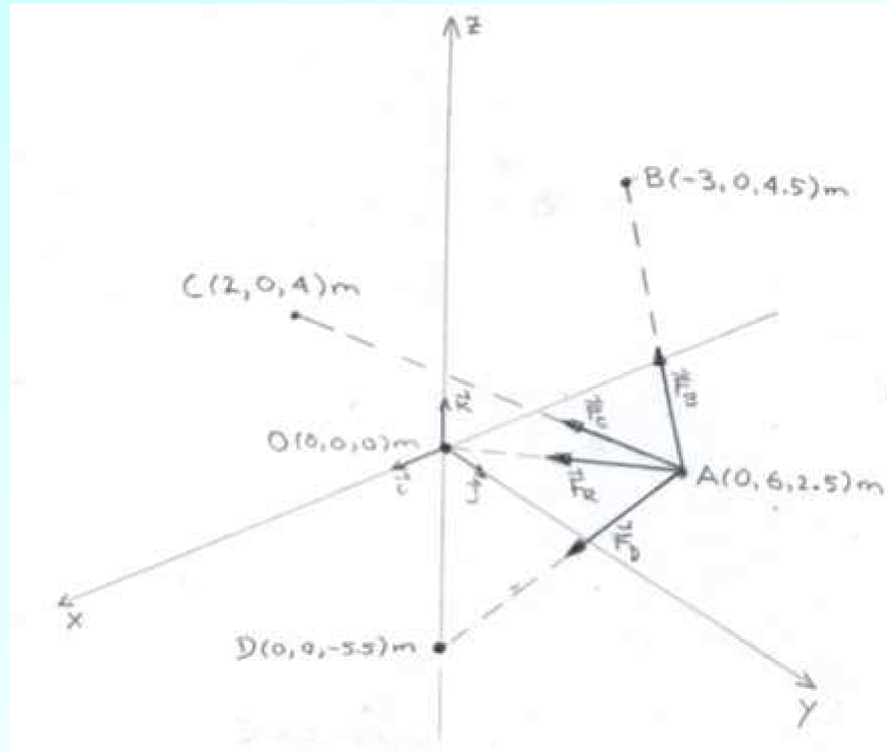


Figure: 02_P108-109



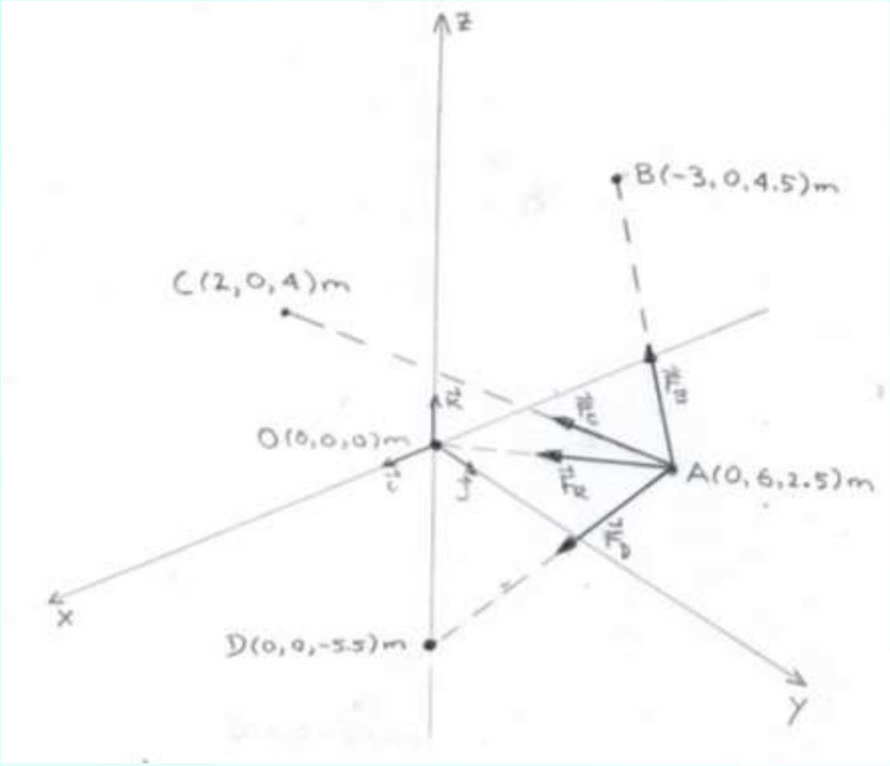
- **Coordinates**

  $A(0,\ 6,\ 2.5)$ m

  $B(-3,\ 0,\ 4.5)$ m

  $C(2,\ 0,\ 4)$ m

  $D(0,\ 0,\ -5.5)$ m



- **Force example** (suppressing units)

$$\vec{F}_B = F_B \vec{u}_{AB} = F_B \left(\frac{\vec{r}_{AB}}{r_{AB}} \right) = \vec{r}_{AB} X \qquad \text{where} \qquad X = \frac{F_B}{r_{AB}}$$

$$= \left((-3-0)\,\vec{i} + (0-6)\,\vec{j} + (4.5-2.5)\,\vec{k} \right) X$$

$$= \left(-3\vec{i} - 6\vec{j} + 2\vec{k} \right) X \quad \text{where} \quad X = F_B / r_{AB} = F_B / \sqrt{3^2 + 6^2 + 2^2}$$

- **Forces** (suppressing units)

$$\vec{F}_B = \left(-3\vec{i} - 6\vec{j} + 2\vec{k}\right)X \qquad X = F_B / \sqrt{3^2 + 6^2 + 2^2}$$

$$\vec{F}_C = \left(2\vec{i} - 6\vec{j} + 1.5\vec{k}\right)Y \qquad Y = F_C / \sqrt{2^2 + 6^2 + 1.5^2}$$

$$\vec{F}_D = \left(-6\vec{j} - 8\vec{k}\right)Z \qquad Z = F_D / \sqrt{6^2 + 8^2}$$

$$\vec{F}_R = \left(-6\vec{j} - 2.5\vec{k}\right)A \qquad A = 1300 / \sqrt{6^2 + 2.5^2}$$

Store in memory A

- **Resultant force** (suppressing units)

$$\vec{F}_R = F_{Rx}\vec{i} + F_{Ry}\vec{j} + F_{Rz}\vec{k}$$

$$\sum F_x = F_{Rx} : \qquad -3X + 2Y = 0$$

$$\sum F_y = F_{Ry} : \qquad -6X - 6Y - 6Z = -6A$$

$$\sum F_z = F_{Rz} : \qquad 2X + 1.5Y - 8Z = -2.5A$$

- **Solving:** $X = 45.36 \qquad Y = 68.04 \qquad Z = 86.60$

$$\text{Then} \quad F_B = \left(\sqrt{3^2 + 6^2 + 2^2}\right) X = 318\ \text{N}$$

$$F_C = \left(\sqrt{2^2 + 6^2 + 1.5^2}\right) Y = 442\ \text{N}$$

$$F_D = \left(\sqrt{6^2 + 8^2}\right) Z = 866\ \text{N}$$

Solving the equations using the reduced row echelon form method

- Equations for X, Y and Z :

$$-3X + 2Y = 0 \qquad (1)$$

$$X + Y + Z = A \qquad (2)$$

$$2X + 1.5Y - 8Z = -2.5A \qquad (3)$$

where $A = 1300 / \sqrt{6^2 + 2.5^2} = 200$

- Solve (1) to (3) using the reduced echelon form matrix program **rref([*M*])** on a TI graphing calculator, where [*M*] is the 3 x 4 matrix

$$[M]=\begin{bmatrix} -3 & 2 & 0 & 0 \\ 1 & 1 & 1 & 200 \\ 2 & 1.5 & -8 & -500 \end{bmatrix}$$

This yields

$$\begin{bmatrix} 1 & 0 & 0 & X \\ 0 & 1 & 0 & Y \\ 0 & 0 & 1 & Z \end{bmatrix}=\begin{bmatrix} 1 & 0 & 0 & 45.36 \\ 0 & 1 & 0 & 68.04 \\ 0 & 0 & 1 & 86.60 \end{bmatrix}$$

from which we read the solution

$$X = 45.36 \quad Y = 68.04 \quad Z = 86.60$$

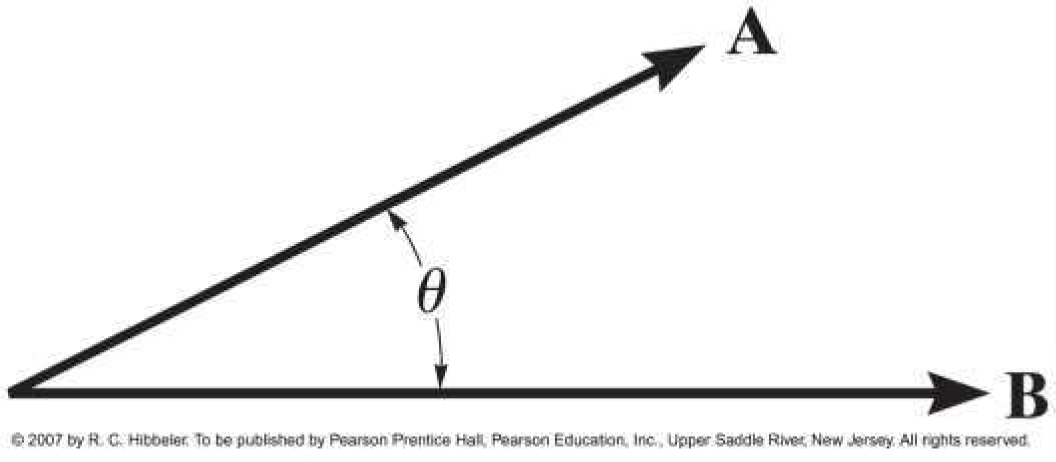
- There's a worked example of solving a 3 x 3 system using **rref** on Canvas in the file "TI-3x3-solve-example.pdf" in "Course information -> Guidebooks and examples for TI Calculators"

2.9 DOT PRODUCT

- One of two vector "multiplication" operations that we will consider
  - **DOT PRODUCT:** Takes two vectors, produces a scalar (hence also sometimes called scalar product)
  - **CROSS PRODUCT:** Takes two vectors, produces another vector
- Both have many important applications in mechanics (and other areas that use vector analysis)

DOT PRODUCT

- DEFINITION: Dot product of A and B is the product of the magnitudes of A and B and the cosine of the angle θ between their tails (the least angle between the vectors)



$$\mathbf{A} \cdot \mathbf{B} = AB\cos\theta$$

NOTE: $0 \le \theta \le 180^{\circ}$

Dot product is a *scalar*

DOT PRODUCTS: LAWS OF OPERATION

- Commutative law

$$\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A}$$

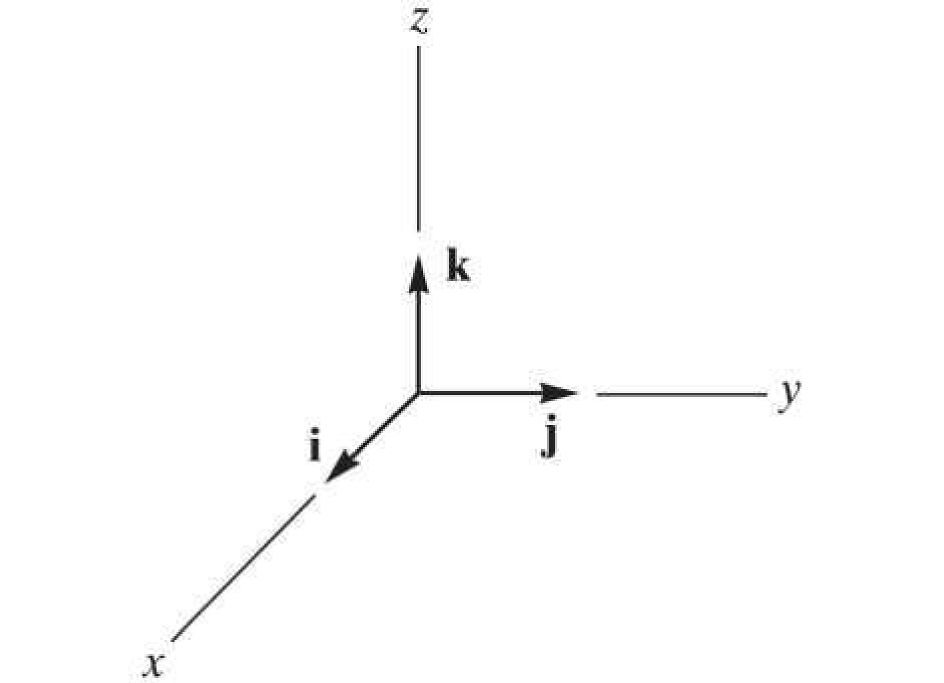
- Multiplication by a scalar, *a*

$$a(\mathbf{A} \cdot \mathbf{B}) = (a\mathbf{A}) \cdot \mathbf{B} = \mathbf{A} \cdot (a\mathbf{B}) = (\mathbf{A} \cdot \mathbf{B})a$$

- Distributive law

$$\mathbf{A} \cdot (\mathbf{B} + \mathbf{C}) = (\mathbf{A} \cdot \mathbf{B}) + (\mathbf{A} \cdot \mathbf{C})$$

CARTESIAN VECTOR FORMULATION



- Consider dot product applied to Cartesian unit vectors

- Examples:

$$\mathbf{i} \cdot \mathbf{i} = |\mathbf{i}||\mathbf{i}|\cos 0^\circ$$
$$= (1)(1)(1) = 1$$

$$\mathbf{i} \cdot \mathbf{j} = |\mathbf{i}||\mathbf{j}|\cos 90^\circ$$
$$= (1)(1)(0) = 0$$

CARTESIAN VECTOR FORMULATION

- Considering all possible combinations of unit vectors, we have

$$\mathbf{i}\cdot\mathbf{i}=1 \quad \mathbf{j}\cdot\mathbf{j}=1 \quad \mathbf{k}\cdot\mathbf{k}=1$$

$$\mathbf{i}\cdot\mathbf{j}=0 \quad \mathbf{i}\cdot\mathbf{k}=0 \quad \mathbf{j}\cdot\mathbf{k}=0$$

- Can now use above results to work out dot product in Cartesian vector form. Recall that we have

$$\mathbf{A}=A_x\mathbf{i}+A_y\mathbf{j}+A_z\mathbf{k}$$

$$\mathbf{B}=B_x\mathbf{i}+B_y\mathbf{j}+B_z\mathbf{k}$$

- Thus, we have

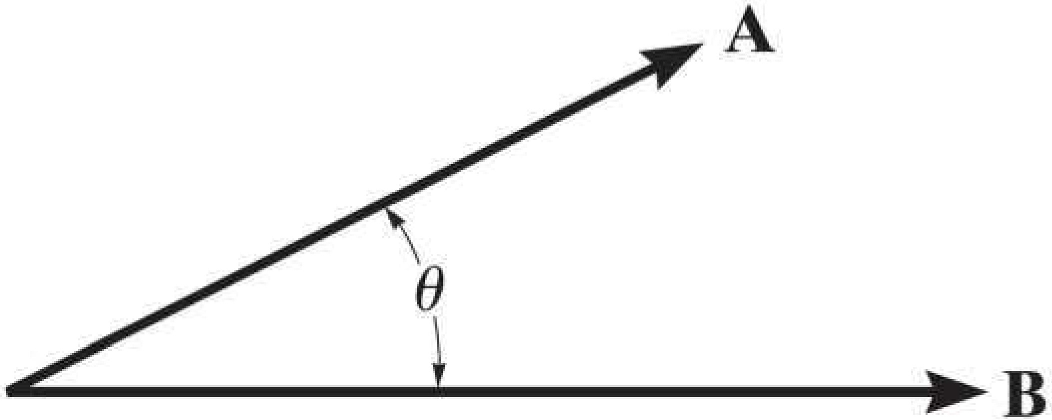
$$\begin{aligned}\mathbf{A}\cdot\mathbf{B} &= (A_x\mathbf{i}+A_y\mathbf{j}+A_z\mathbf{k})\cdot(B_x\mathbf{i}+B_y\mathbf{j}+B_z\mathbf{k})\\ &= A_xB_x(\mathbf{i}\cdot\mathbf{i})+A_xB_y(\mathbf{i}\cdot\mathbf{j})+A_xB_z(\mathbf{i}\cdot\mathbf{k})\\ &\quad+A_yB_x(\mathbf{j}\cdot\mathbf{i})+A_yB_y(\mathbf{j}\cdot\mathbf{j})+A_yB_z(\mathbf{j}\cdot\mathbf{k})\\ &\quad+A_zB_x(\mathbf{k}\cdot\mathbf{i})+A_zB_y(\mathbf{k}\cdot\mathbf{j})+A_zB_z(\mathbf{k}\cdot\mathbf{k})\end{aligned}$$

- Only the terms in red are non-zero, and so we find

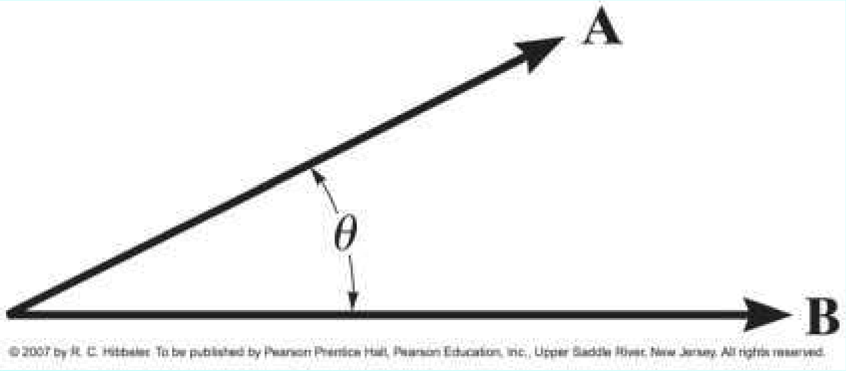
$$\mathbf{A}\cdot\mathbf{B} = A_xB_x + A_yB_y + A_zB_z$$

DOT PRODUCT: APPLICATION 1

1. Angle formed between two vectors, or intersecting lines



DOT PRODUCT: APPLICATION 1



- Start from definition

$$\mathbf{A} \cdot \mathbf{B} = AB\cos\theta$$

- so we have

$$\cos\theta = \frac{\mathbf{A} \cdot \mathbf{B}}{AB}$$

and

$$\theta = \cos^{-1}\left(\frac{\mathbf{A} \cdot \mathbf{B}}{AB}\right)$$

Assumption is that **A** and **B** are known in Cartesian component form. Also, two vectors are always coplanar (or colinear) so this “works in 3D”

- Important special case:

$$\mathbf{A} \cdot \mathbf{B} = 0$$

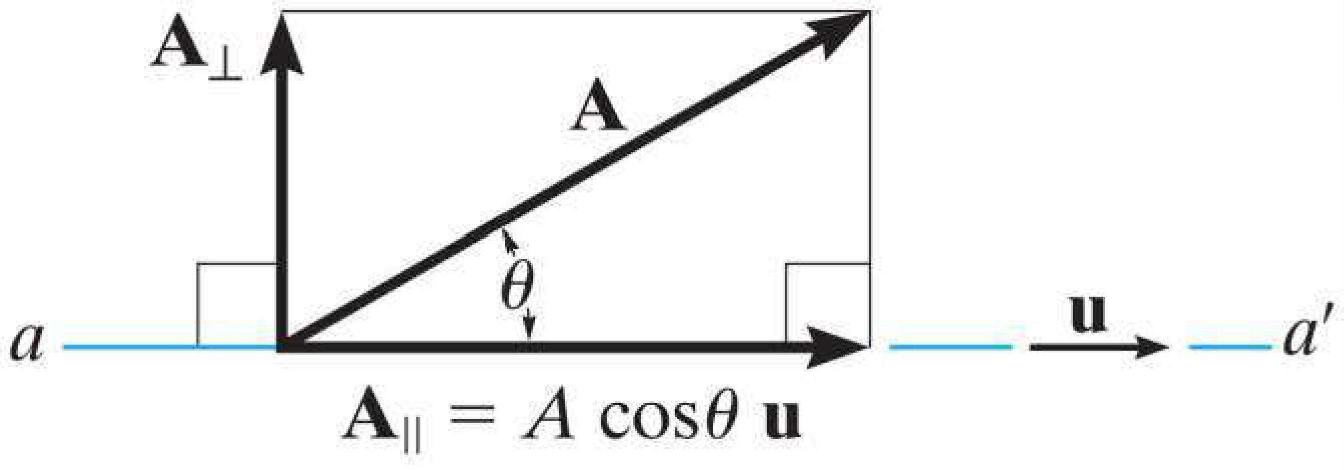
- Then have

$$\theta = \cos^{-1}\left(\frac{\mathbf{A} \cdot \mathbf{B}}{AB}\right) = \cos^{-1} 0 = 90^{\circ}$$

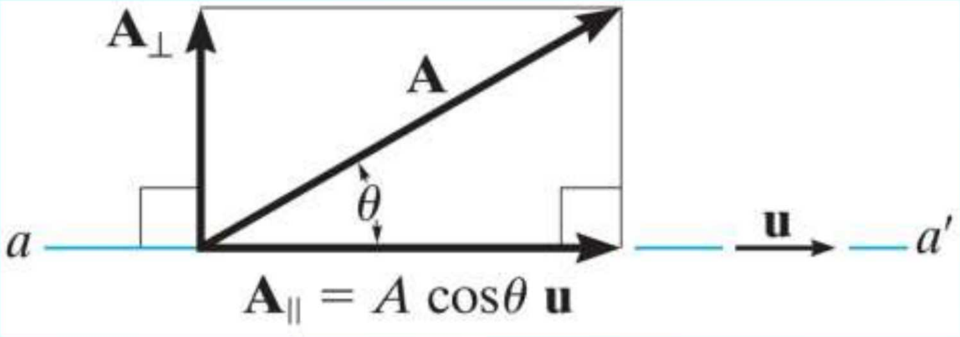
so two non-vanishing vectors are perpendicular (orthogonal) if their dot product vanishes

DOT PRODUCT: APPLICATION 2

- Components of a vector parallel and perpendicular to a line, *aa’*



- Note: Orientation of *aa’* is arbitrary, i.e. not necessarily horizontal, vertical, etc.



- Parallel component, $\mathbf{A}_{||}$, (vector) has magnitude

$$A_{||} = A\cos\theta$$

- Unit vector in *aa'* direction is $\mathbf{u}$, therefore have

$$\mathbf{A}_{||} = A\cos\theta\,\mathbf{u}$$

- Moreover, from definition of dot product have

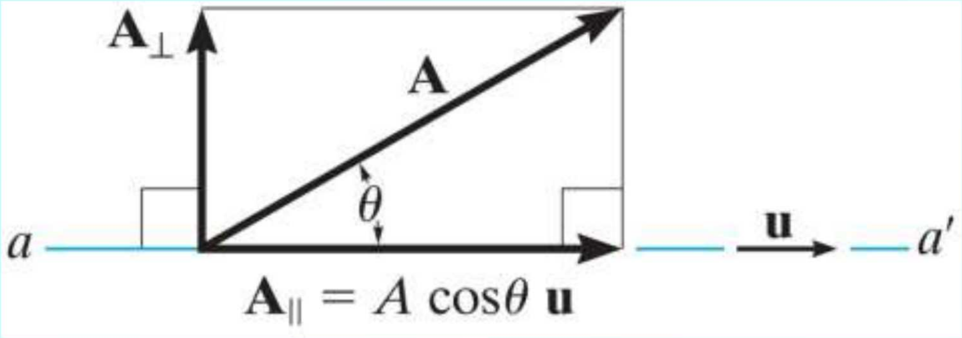
$$\mathbf{A}\cdot\mathbf{u} = Au\cos\theta = A\cos\theta$$

so magnitude of parallel component is

$$A_{||} = \mathbf{A}\cdot\mathbf{u}$$

and

$$\mathbf{A}_{||} = (\mathbf{A}\cdot\mathbf{u})\mathbf{u}$$



- Perpendicular component, $\mathbf{A}_\perp$, (vector) can be computed indirectly by observing that

$$\mathbf{A} = \mathbf{A}_{\parallel} + \mathbf{A}_\perp$$

so

$$\mathbf{A}_\perp = \mathbf{A} - \mathbf{A}_{\parallel} = \mathbf{A} - (\mathbf{A}\cdot\mathbf{u})\mathbf{u}$$

- Magnitude, $A_\perp$, can be computed in at least two ways

$$A_\perp = A\sin\theta \quad \text{where} \quad \theta = \cos^{-1}\left(\frac{\mathbf{A}\cdot\mathbf{u}}{A}\right)$$

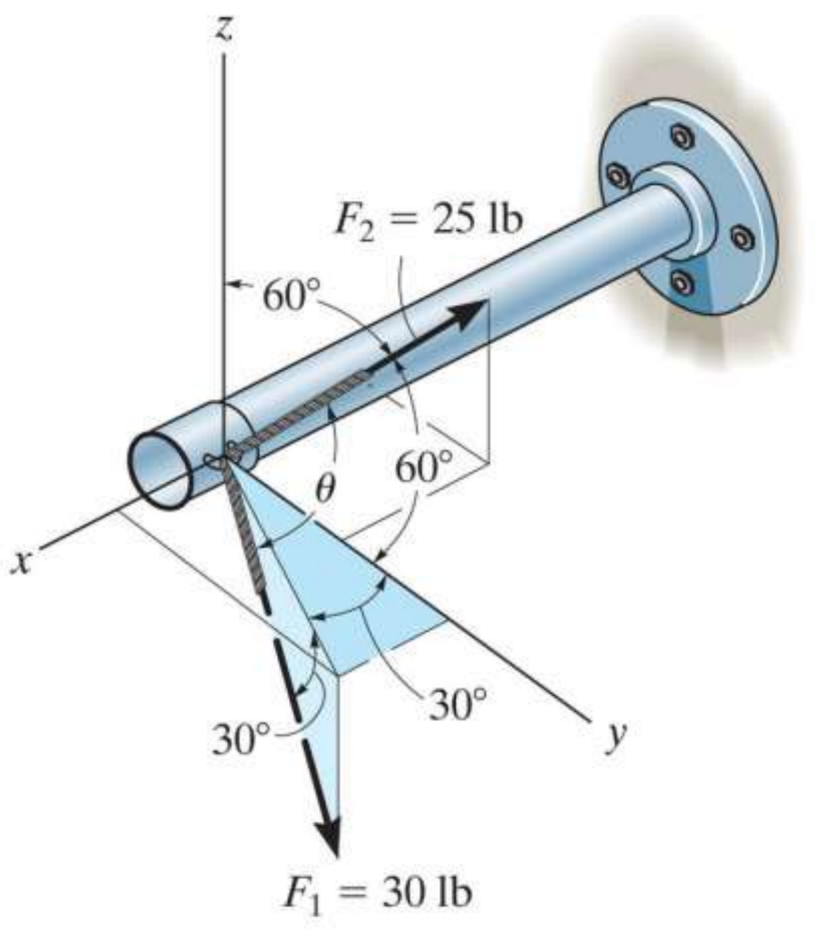
$$A_\perp = \sqrt{A^2 - A_{\parallel}^2}$$

Problems 2-120 and 2-121 (page 78, 14th edition)

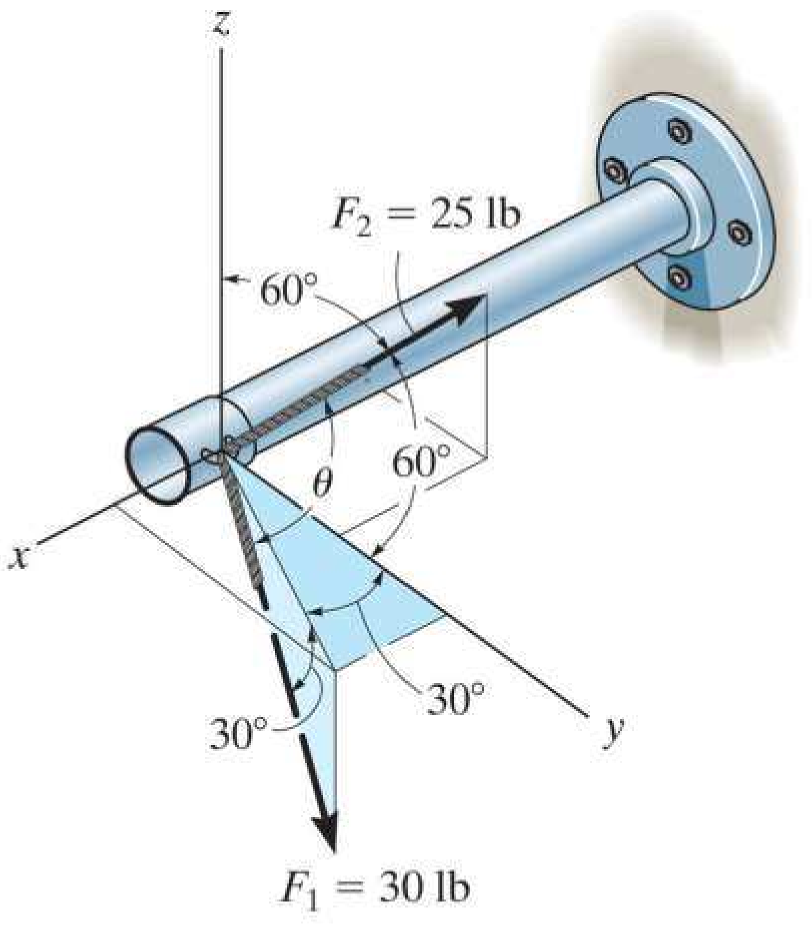
Two cables exert forces on the pipe as shown

2-120 Determine the projected component of $\vec{F}_1$ along the line of action of $\vec{F}_2$

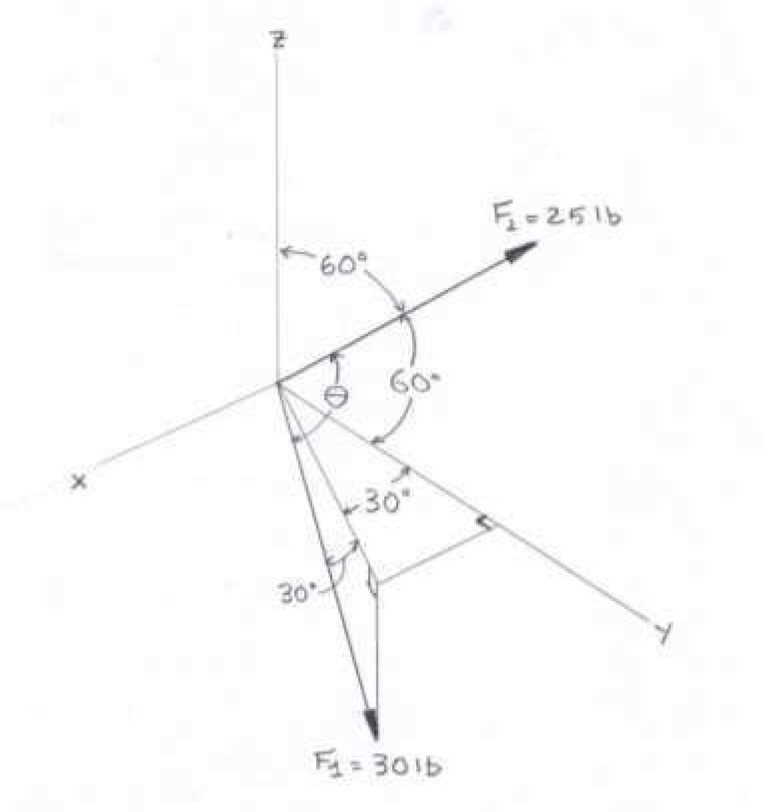
2-121 Determine the angle θ between the two cables



02_PROB_120-121



02_PROB_120-121



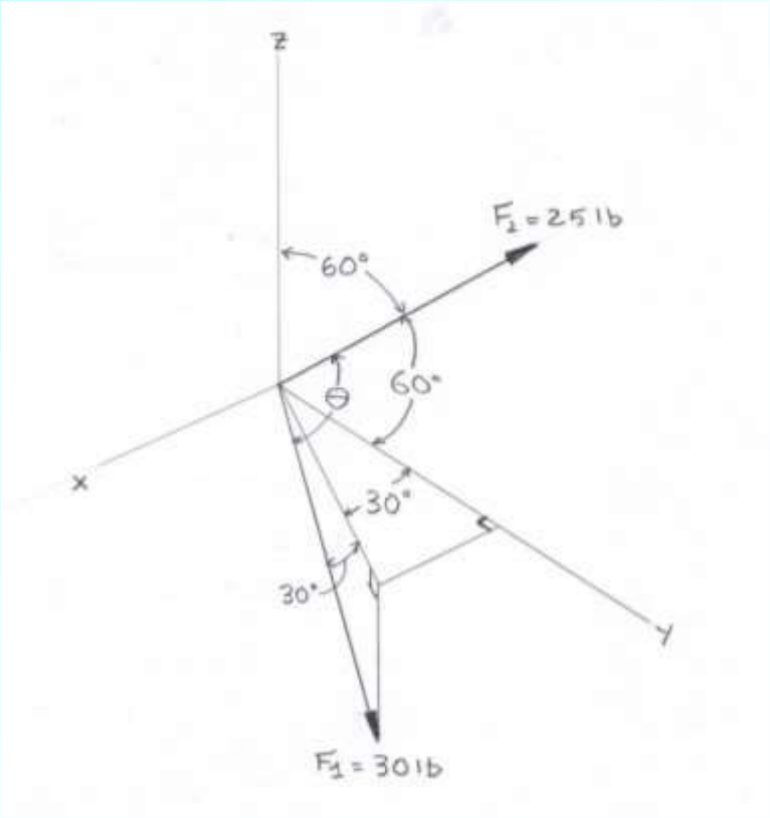
Solution strategy:

(1) Express $\vec{F}_1$ in Cartesian form from given geometry

(2) Express unit vector, $\vec{u}$, in direction of $\vec{F}_2$ in Cartesian components from direction cosines

(3) Compute $\vec{F}_1 \cdot \vec{u}$, the projected component of $\vec{F}_1$ in the direction of $\vec{F}_2$

(4) Compute angle between two cables ($\vec{F}_1$ and $\vec{F}_2$) using fundamental definition of dot product



Solution continues in Lecture 7