PHYS 170 Section 101
Lecture 5
September 14, 2018

SEPTEMBER 14—ANNOUNCEMENTS

- Introduction to Mastering Engineering Assignment is due today, Friday, September 14, 11:59 PM (not for marks)
- Assignment 1 due Monday, September 17, 11:59 PM
- Assignment 2 will be available at 6:00 PM this evening, and is due next Friday at 11:59 PM
- Last day to withdraw without a W standing, Tuesday, September 18

SEPTEMBER 14—ANNOUNCEMENTS

Homework 1, Problem 2-81

One of you noticed (thanks, Anonymous!) that if they compute

$$F_{3x} = F_3 \cos(\alpha) \qquad (1)$$

$$F_{3y} = F_3 \cos(\beta) \qquad (2)$$

$$F_{3z} = F_3 \cos(\gamma) \qquad (3)$$

then

$$\sqrt{{F_{3x}}^2 + {F_{3y}}^2 + {F_{3z}}^2} \neq F_3$$

This occurs since the software randomizes the direction angles relative to the original specification of the problem, and doesn't ensure that the sum of the squares of their cosines adds up to 1.

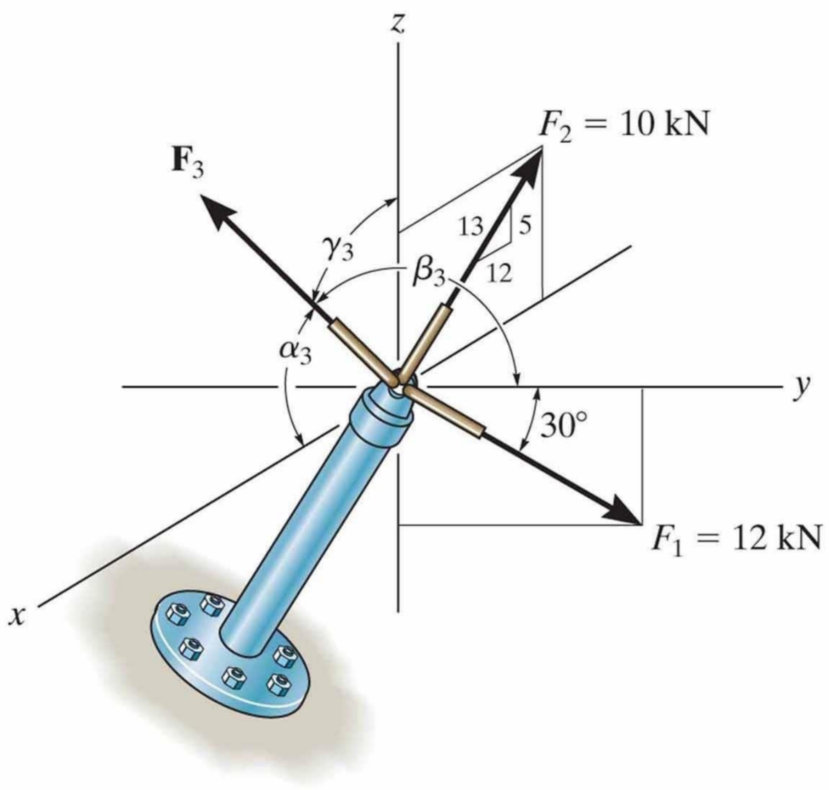
You should ignore this inconsistency and work the problem using equations (1), (2) and (3) above (i.e. pretend the angles are consistent).

Lecture Outline/Learning Goals

- Finish concurrent force system from last day
- Position vectors
- Force vectors directed along a line
- Sample problem using force vectors directed along lines
  - Solution of linear systems using TI graphing calculator

Problem 2-79, (page 54, 12th edition)

Specify the magnitude of $\vec{F}_3$ and its coordinate direction angles α_3, β_3 and γ_3 so that the resultant force is $9\,\vec{j}$ kN



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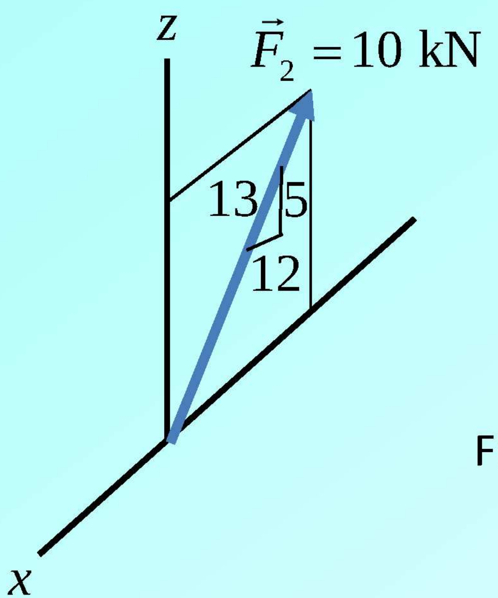


Fig. (a)

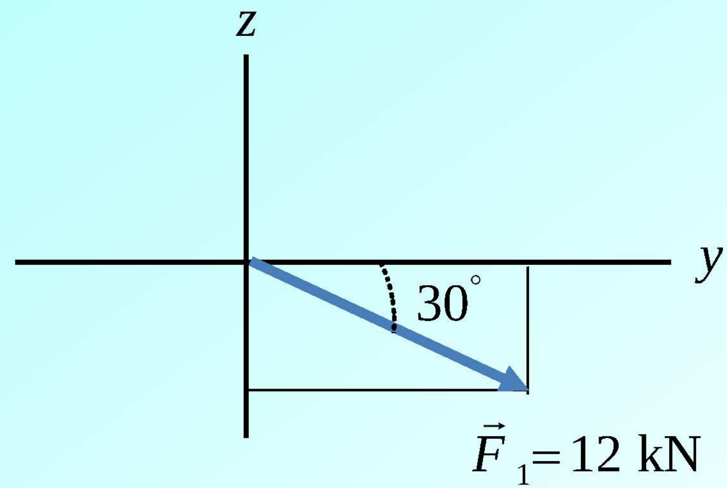


Fig. (b)

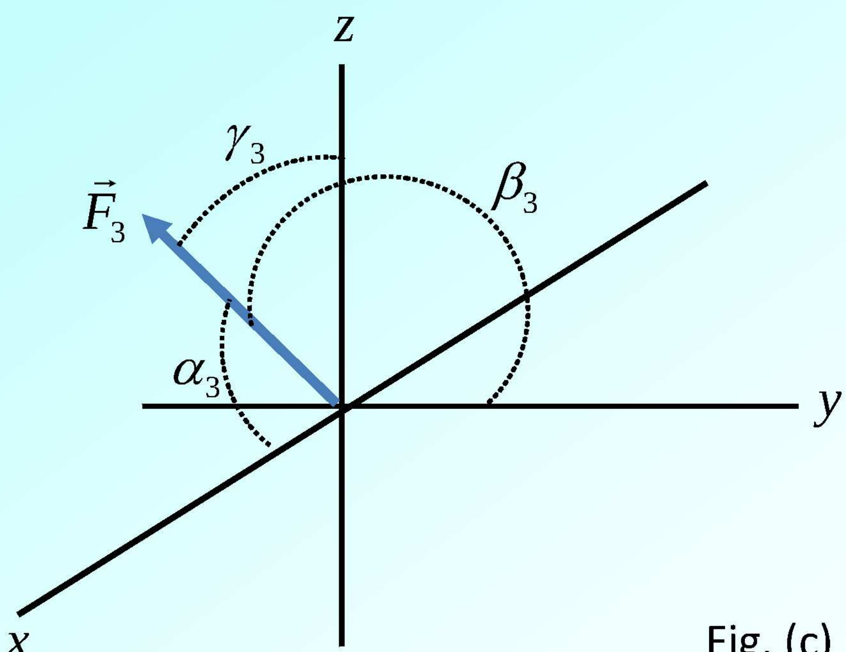
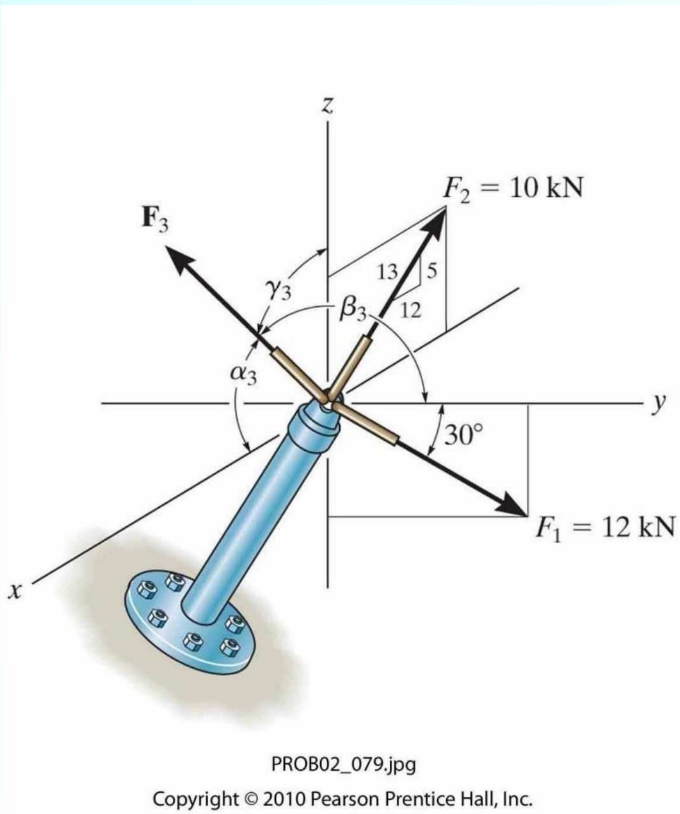


Fig. (c)



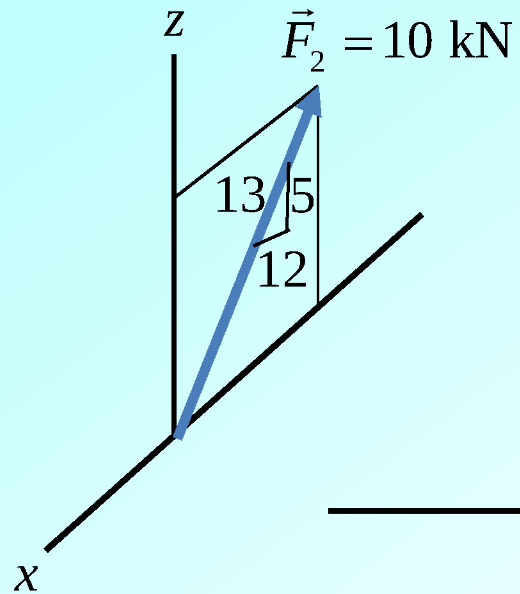
- **Forces** (units suppressed)

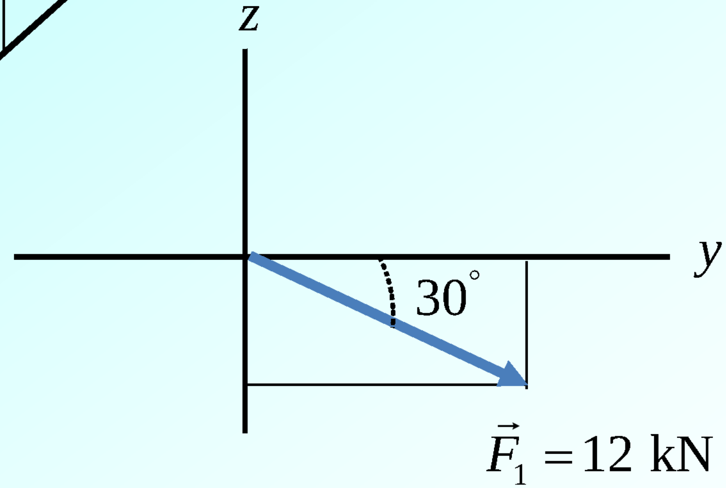
$$\vec{F}_1 = 12\left(\cos 30^\circ\, \vec{j} - \sin 30^\circ\, \vec{k}\right)$$

$$\vec{F}_2 = 10\left(-\frac{12}{13}\vec{i} + \frac{5}{13}\vec{k}\right)$$

$$\vec{F}_3 = F_x\vec{i} + F_y\vec{j} + F_z\vec{k}$$

$$\vec{F}_R = 9\vec{j}$$





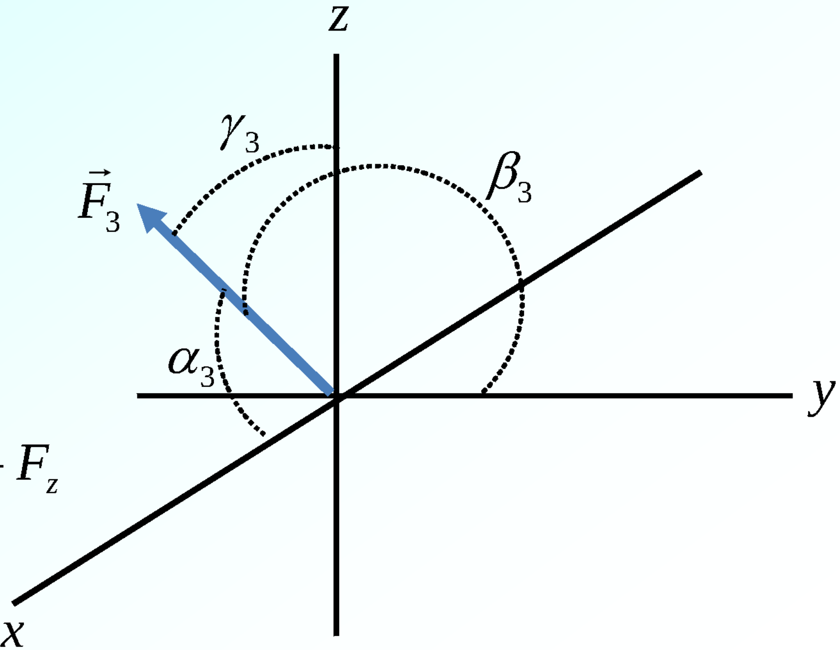
- **Equations for the resultant force**

$$\vec{F}_R = F_{Rx}\vec{i} + F_{Ry}\vec{j} + F_{Rz}\vec{k}$$

$$F_{Rx} = \sum F_x : \qquad 0 = -\frac{120}{13} + F_x$$

$$F_{Ry} = \sum F_y : \qquad 9 = 12\cos 30^\circ + F_y$$

$$F_{Rz} = \sum F_z : \qquad 0 = -12\sin 30^\circ + \frac{50}{13} + F_z$$



- Solve for F_x , F_y , F_z :

$$F_x = \frac{120}{13} = A$$ ← Store in calculator memory A

$$F_y = 9 - 12\cos 30^\circ = B$$ ← Store in calculator memory B

$$F_z = 12\sin 30^\circ - \frac{50}{13} = C$$ ← Store in calculator memory C

- Determine magnitude F_3 and coordinate direction angles

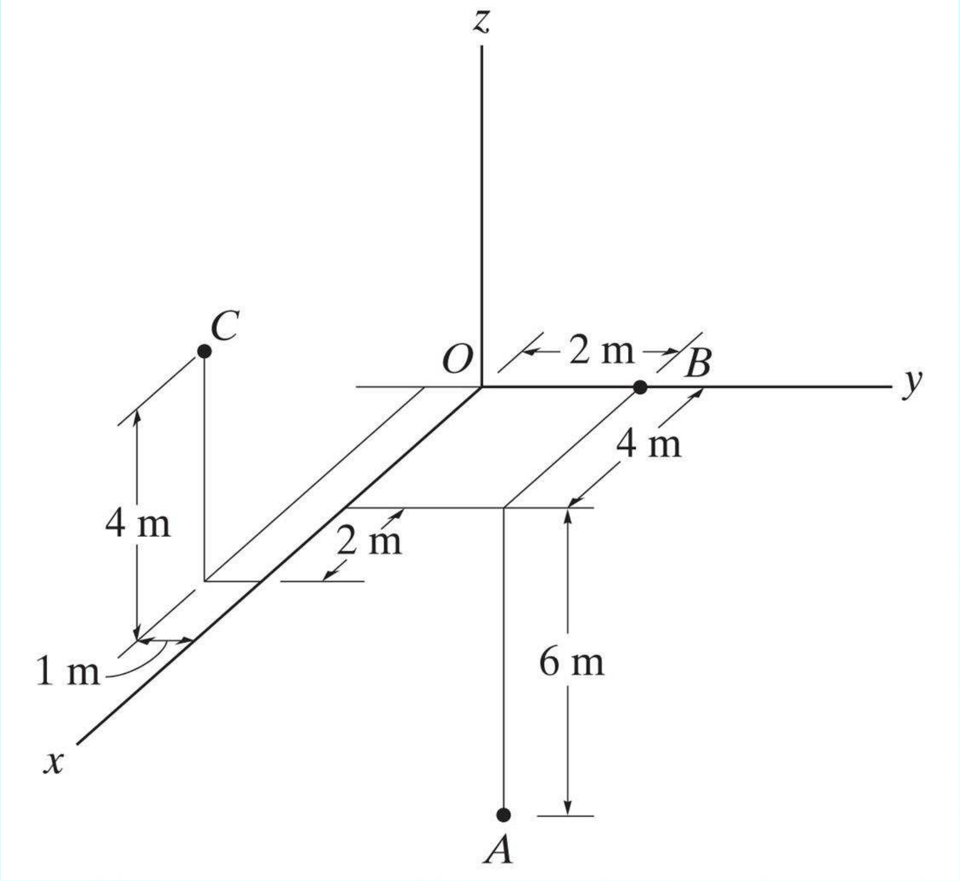
$$F_3 = \sqrt{F_x^2 + F_y^2 + F_z^2} = \sqrt{A^2 + B^2 + C^2} = F = 9.58 \text{ kN}$$

$$\alpha_3 = \cos^{-1}\left(F_x / F_3\right) = \cos^{-1}\left(A / F\right) = 15.5^\circ$$

$$\beta_3 = \cos^{-1}\left(F_y / F_3\right) = \cos^{-1}\left(B / F\right) = 98.4^\circ$$

$$\gamma_3 = \cos^{-1}\left(F_z / F_3\right) = \cos^{-1}\left(C / F\right) = 77.0^\circ$$

2.7 POSITION VECTORS
x, y, z COORDINATES



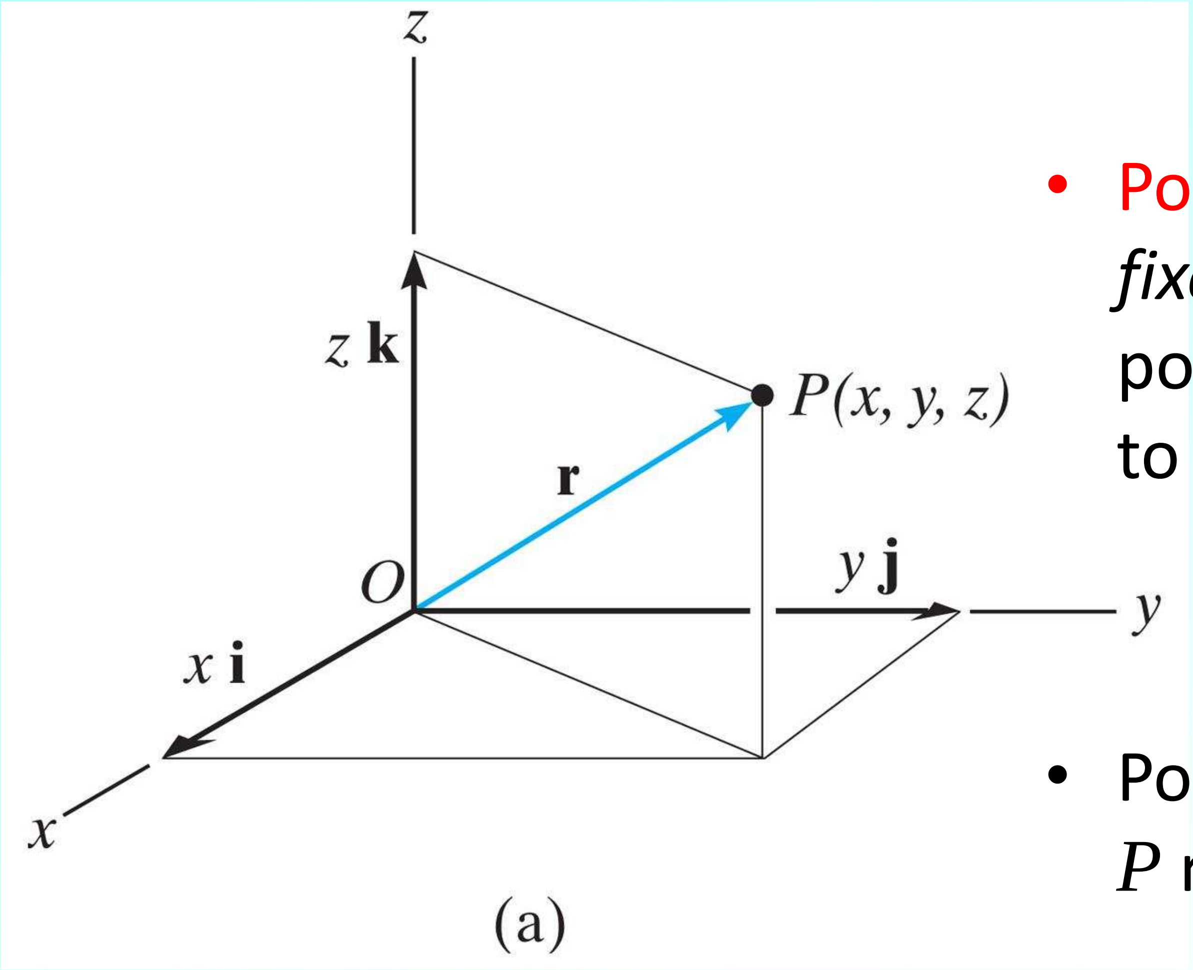
- Use following notation for coordinates of a point

$$P(x, y, z)$$

- Thus have (suppress units)

$$A(x, y, z) = A(4, 2, -6)$$
$$B(x, y, z) = B(0, 2, 0)$$
$$C(x, y, z) = C(6, -1, 4)$$

POSITION VECTOR

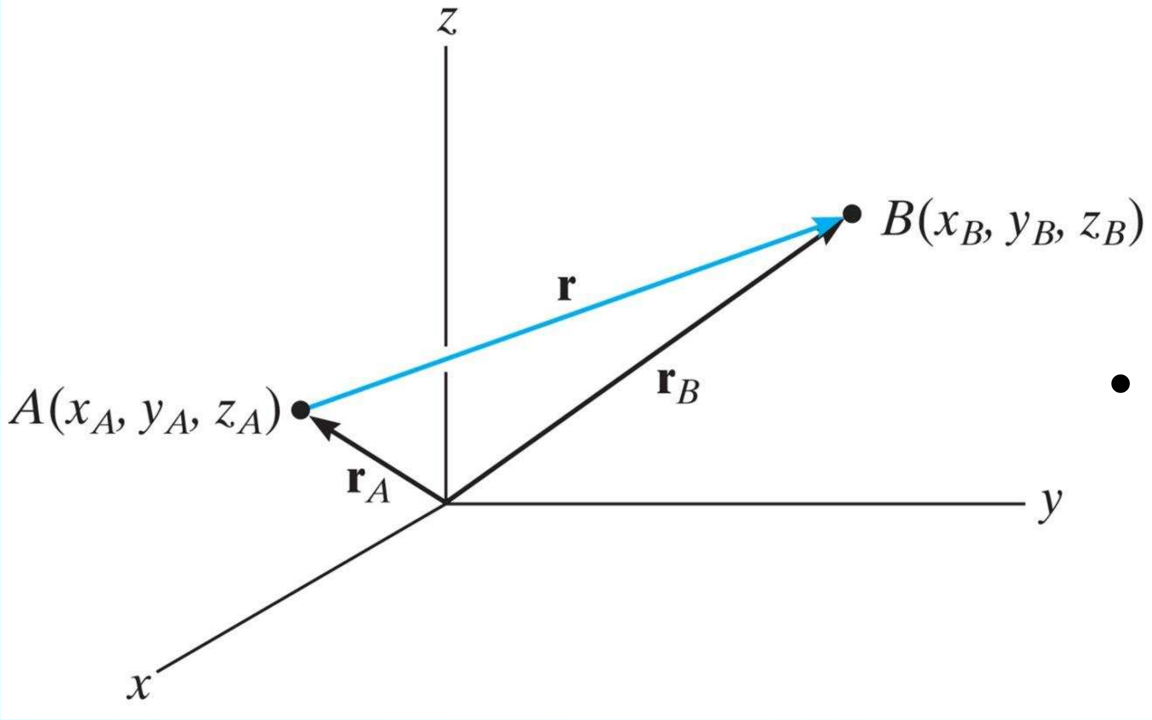


- Position vector, $\mathbf{r}$, is a *fixed* vector that locates point in space relative to another point

- Position vector locating P relative to origin, O, is

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

POSITION VECTOR: GENERAL CASE



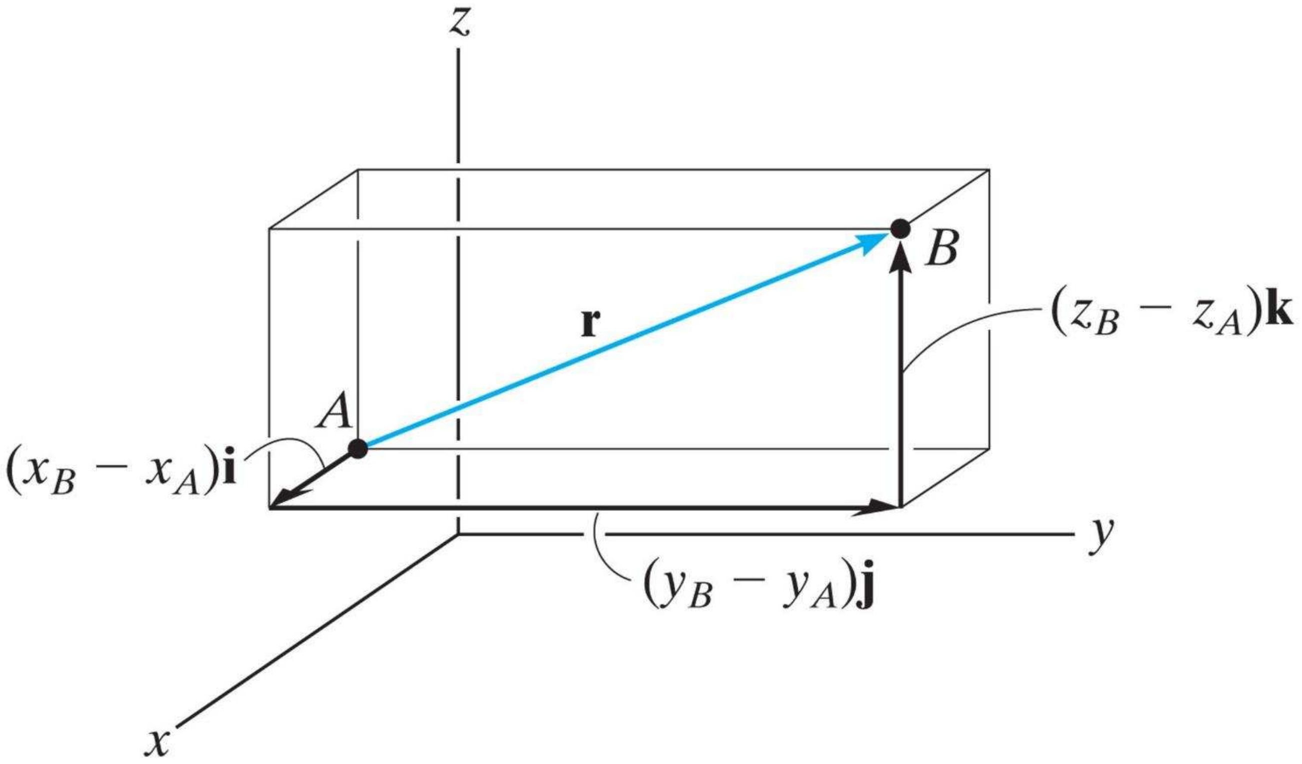
- Consider position vector directed from point A to point B

- We have

$$\mathbf{r}_A + \mathbf{r} = \mathbf{r}_B$$

$$\begin{aligned}
\mathbf{r} = \mathbf{r}_{AB} &= \mathbf{r}_B - \mathbf{r}_A \\
&= (x_B\mathbf{i} + y_B\mathbf{j} + z_B\mathbf{k}) - (x_A\mathbf{i} + y_A\mathbf{j} + z_A\mathbf{k}) \\
&= (x_B - x_A)\mathbf{i} + (y_B - y_A)\mathbf{j} + (z_B - z_A)\mathbf{k}
\end{aligned}$$

Forming position vector from coordinates of two points



(b)

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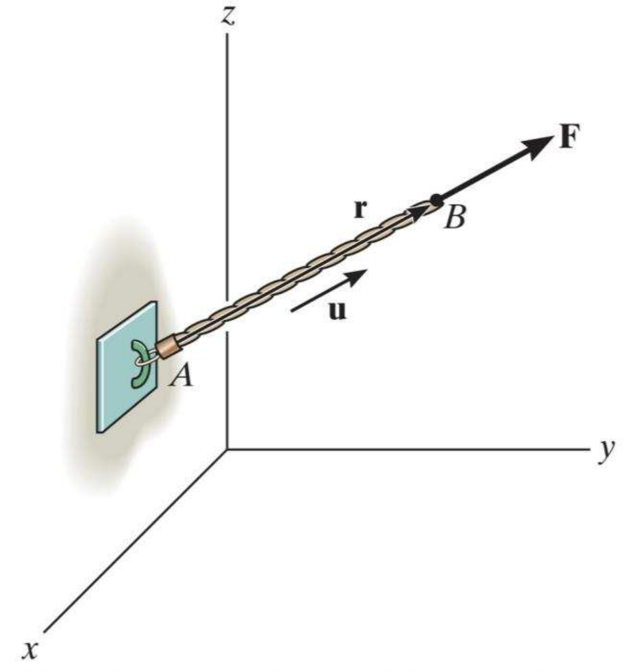
$$\mathbf{r} = \mathbf{r}_{AB} = \mathbf{r}_B - \mathbf{r}_A$$
$$= (x_B - x_A)\mathbf{i} + (y_B - y_A)\mathbf{j} + (z_B - z_A)\mathbf{k}$$

2.8 FORCE VECTOR DIRECTED ALONG A LINE

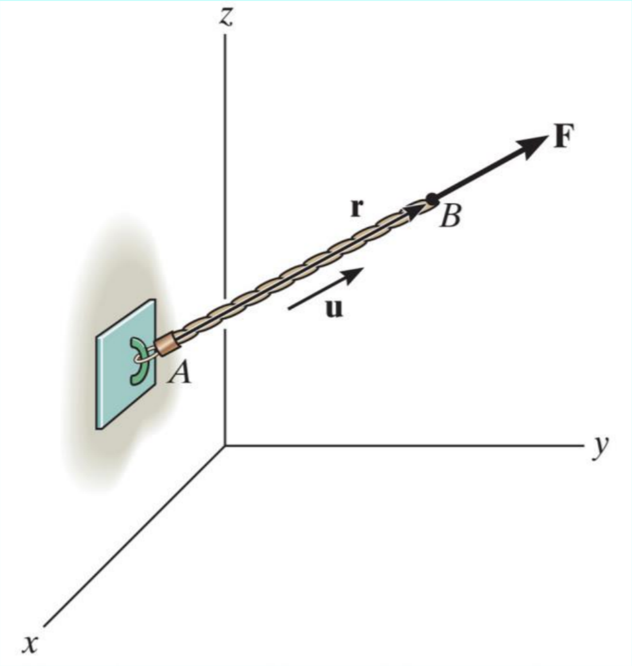


- Will often encounter situation where force vector direction is given by two points lying on line of action, i.e. in direction of position vector defined by two points *A* and *B*

- Force vector **F** has same direction and sense as position vector $\mathbf{r} = \mathbf{r}_{AB}$ that passes through *A* and *B*. Direction is given by unit vector $\mathbf{u} = \mathbf{r} / r$, magnitude is *F*

$$\mathbf{F} = F\mathbf{u} = F\left(\frac{\mathbf{r}}{r}\right)$$

FORCE VECTOR DIRECTED ALONG A LINE



$$\mathbf{F} = F\mathbf{u} = F\left(\frac{\mathbf{r}}{r}\right)$$

$$= F\left(\frac{(x_B - x_A)\vec{i} + (y_B - y_A)\vec{j} + (z_B - z_A)\vec{k}}{\sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}}\right)$$

Calculational trick for use with vectors of unknown magnitude but known direction

- Many of the problems that we'll encounter in this part of the course require the computation of the magnitudes of one or more vectors whose directions can typically be determined from the geometrical specifications of the question.

- Specifically, such a vector will generally be expressible as

$$\vec{F} = F\vec{u} = F\left(\frac{(x_B - x_A)\vec{i} + (y_B - y_A)\vec{j} + (z_B - z_A)\vec{k}}{\sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}}\right)$$

where

$$(x_A, y_A, z_A) \quad \text{and} \quad (x_B, y_B, z_B)$$

are the coordinates of two points *A* and *B* through which the line of action of the vector passes (the case that we just considered)

- Again, we'll assume that the values

$$(x_A, y_A, z_A) \quad \text{and} \quad (x_B, y_B, z_B)$$

are all known (numerically), and that *F* is one of the unknowns of the problem.

- The trick is to introduce a new unknown, which we'll call *X*, and which we will use instead of *F* to formulate and solve equations in the first instance.

- *X* is given by

$$X = \frac{F}{\sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}}$$

and in terms of it, the original vector is

$$\vec{F} = X\,\vec{r}_{AB} = X\,((x_B - x_A)\vec{i} + (y_B - y_A)\vec{j} + (z_B - z_A)\vec{k})$$

- The reason that this is a *useful* trick is that in problem specifications, the coordinates (x_A , x_B , x_C) etc. tend to be integers or rational numbers at worst, so that the coefficients of the linear systems that result from using transformed variables such as *X*, are also integers or rational numbers, which significantly simplifies the mechanics of solving the systems, either by hand, or with a linear solve feature on a calculator. If we instead use the original variables such as *F*, the coefficients we'll have to work with will tend to be quotients with expressions such as

$$\sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}$$

in the denominator which will typically yield irrational values

- One small price that we have to pay for using this transformation is that once we determine *X* and the other transformed variables (if any), we need to determine the original variables via the appropriate inverse transformation, which in the current case is

$$F = X\sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}$$

Problem 2-109 (page 68, 13th edition)

The magnitude of the resultant force is 1300 N and acts along the axis of the strut from *A* towards *O*

Determine the magnitude of each of the three forces acting on the strut.

Set $x = 0$ and $z = 5.5$ m

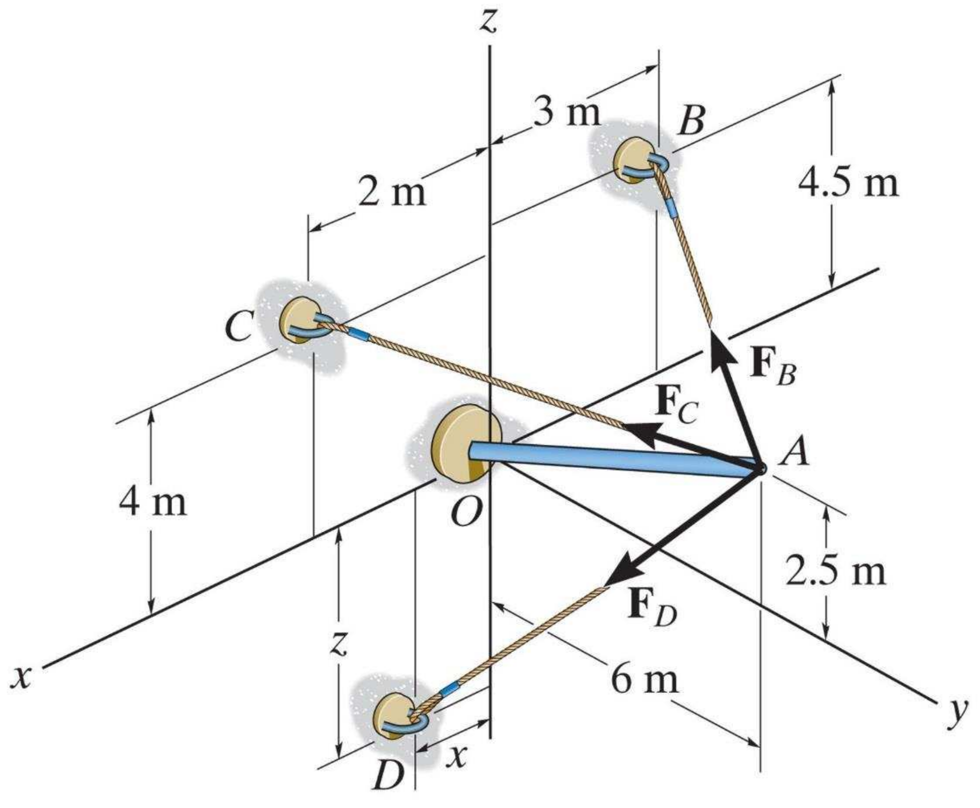


Figure: 02_P108-109

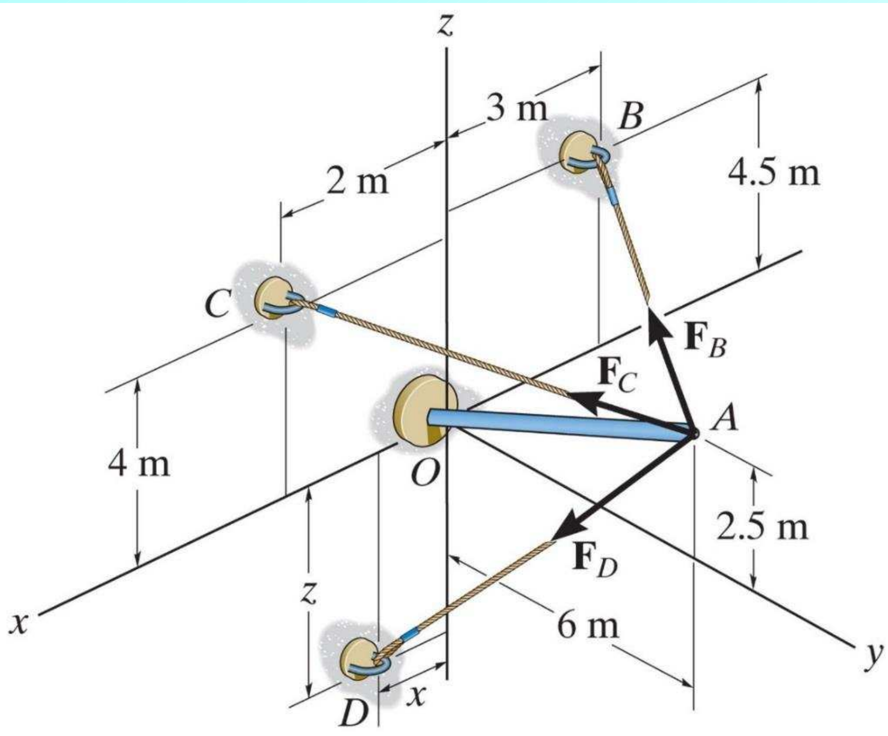
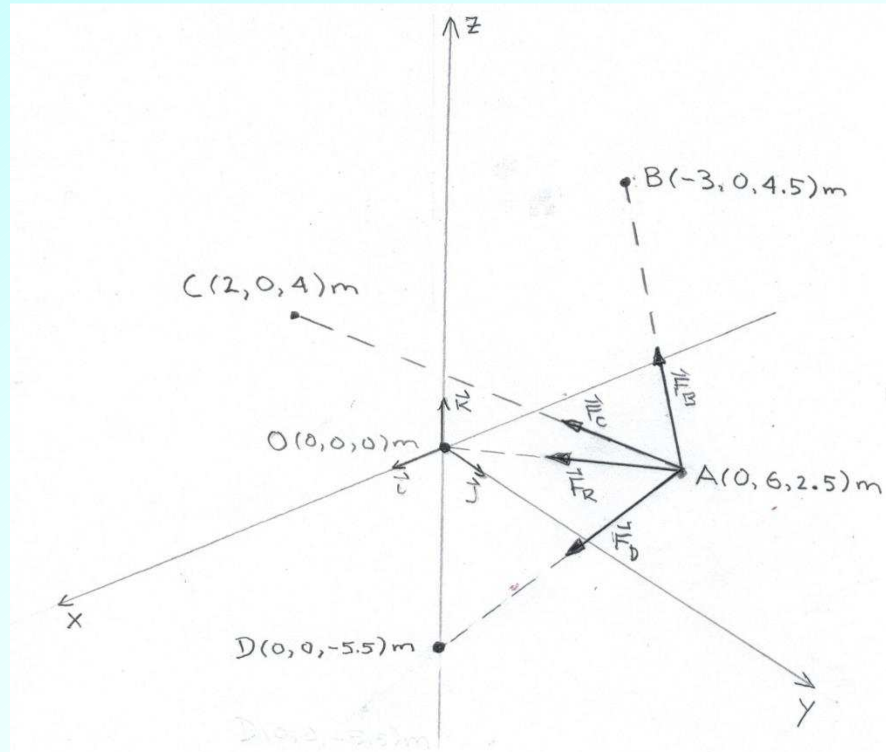


Figure: 02_P108-109



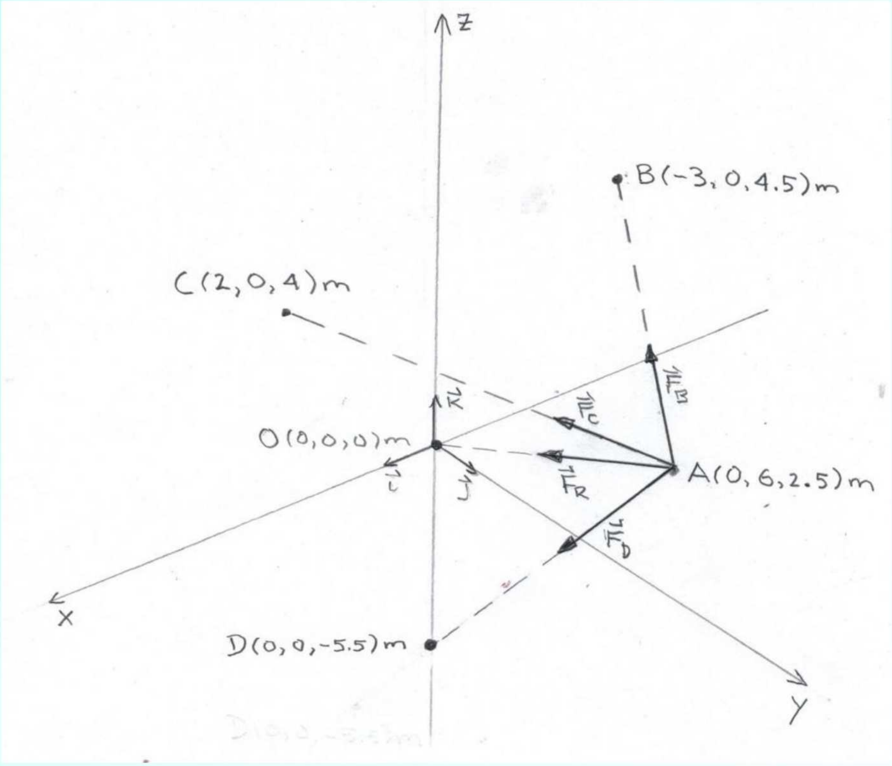
- **Coordinates**

  $A(0,\ 6,\ 2.5)$ m

  $B(-3,\ 0,\ 4.5)$ m

  $C(2,\ 0,\ 4)$ m

  $D(0,\ 0,\ -5.5)$ m



- **Force example** (suppressing units)

$$\vec{F}_B = F_B \vec{u}_{AB} = F_B \left(\frac{\vec{r}_{AB}}{r_{AB}} \right) = \vec{r}_{AB} X \qquad \text{where} \quad X = \frac{F_B}{r_{AB}}$$

$$= \left((-3-0)\,\vec{i} + (0-6)\,\vec{j} + (4.5-2.5)\,\vec{k} \right) X$$

$$= \left(-3\vec{i} - 6\vec{j} + 2\vec{k} \right) X \quad \text{where} \quad X = F_B \,/\, r_{AB} = F_B \,/\, \sqrt{3^2 + 6^2 + 2^2}$$

Solution continues in Lecture 6