PHYS 170 Section 101
Lecture 3
September 10, 2017

SEP 10—ANNOUNCEMENTS

- Tutorials start tomorrow

- Mastering Engineering:
  - Introductory assignment due Friday at 11:59 PM
  - First assignment due next Monday at 11:59 PM

Lecture Outline/Learning Goals

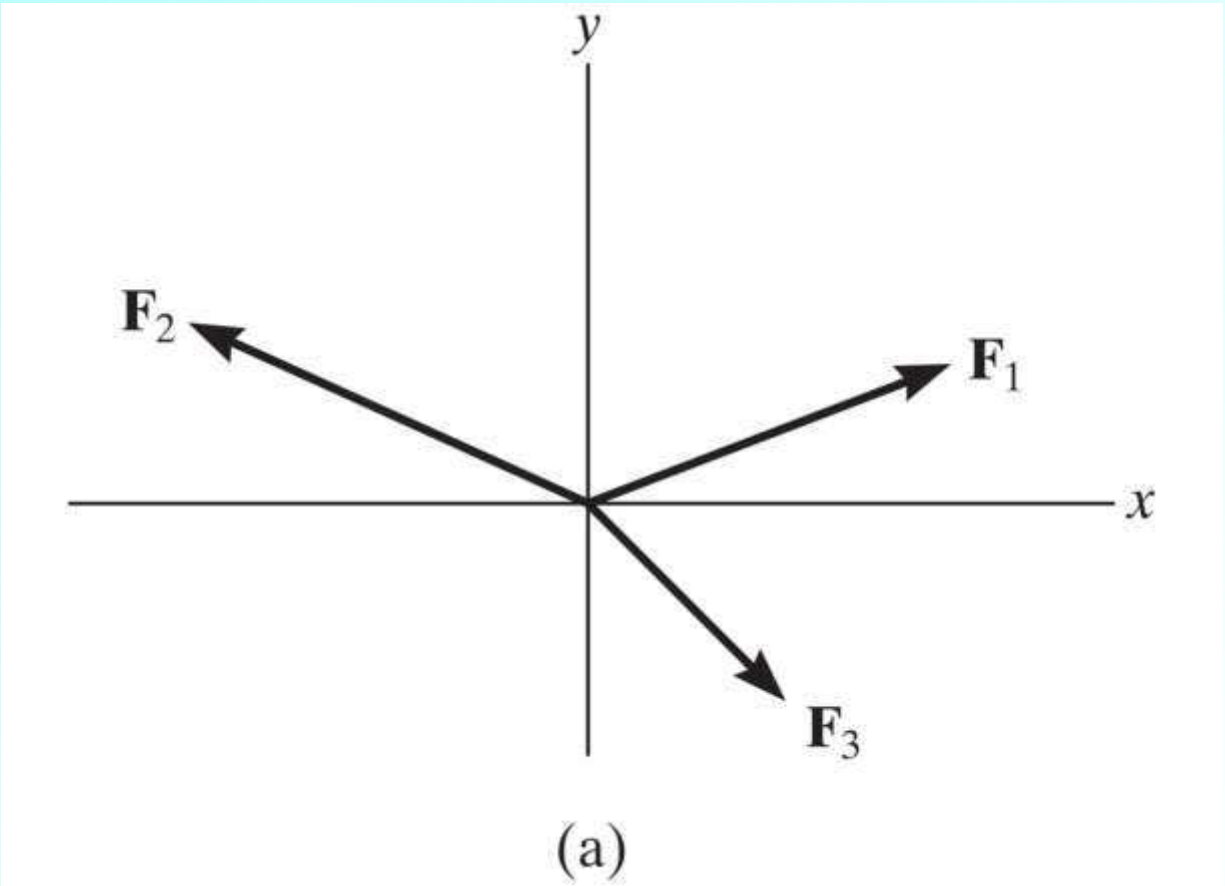
- Sample problems: coplanar force systems

- Cartesian vectors (3 dimensions or 3D)
  - Right handed coordinate systems, rectangular components, unit vectors
  - Cartesian vector representation, magnitude of Cartesian vector
  - Cartesian vector: direction, coordinate direction angles, direction cosines
  - Addition and subtraction of Cartesian vectors

COPLANAR FORCE RESULTANTS

- We now wish to consider summing an arbitrary number of vectors in the *xy* plane. For example:

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$

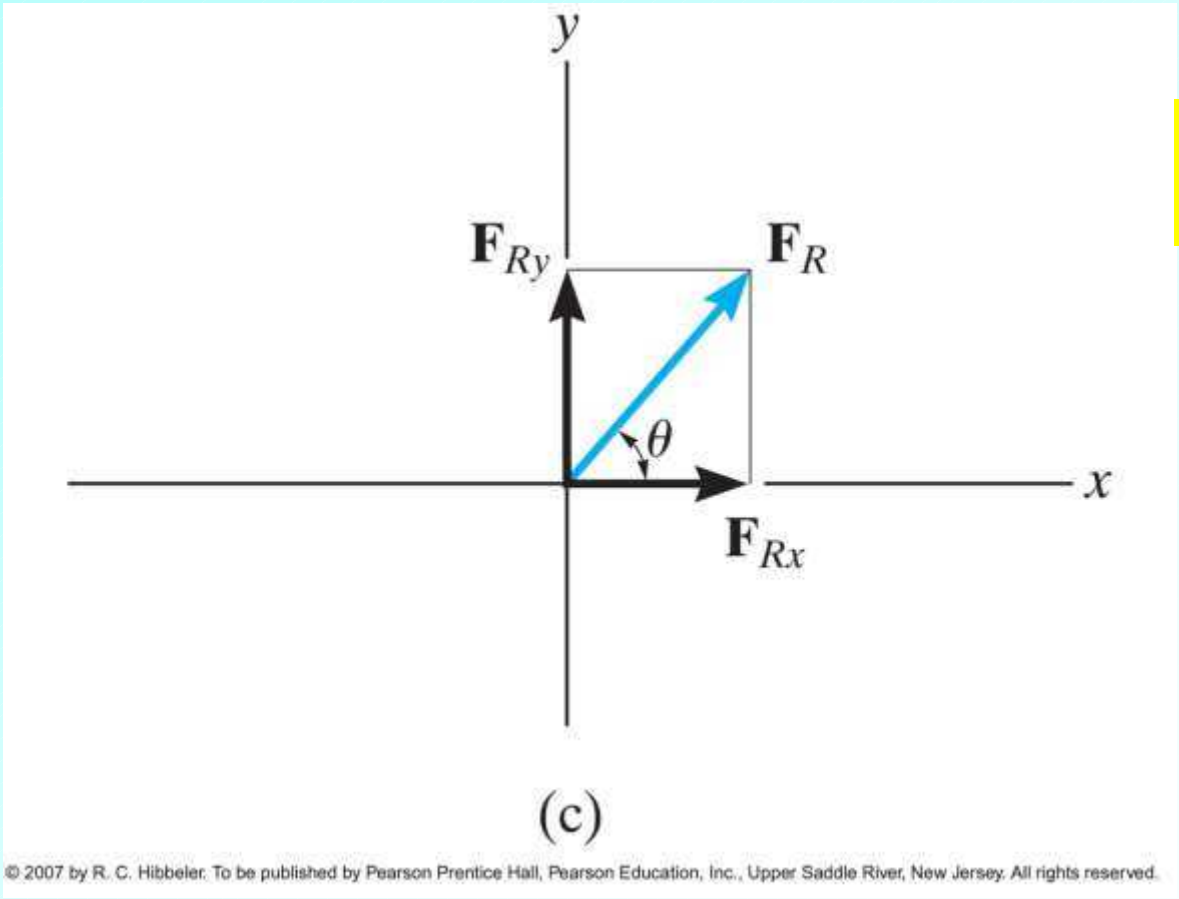


COPLANAR FORCE RESULTANTS

- General case (arbitrary number of forces)

$$F_{Rx} = \Sigma F_x$$
$$F_{Ry} = \Sigma F_y$$

REPRESENTATION AS MAGNITUDE & DIRECTION



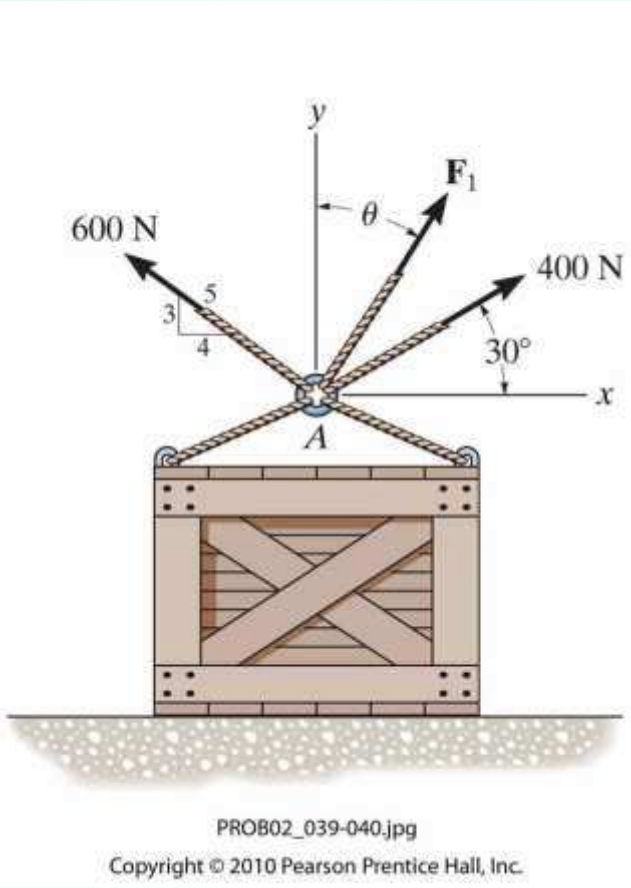
$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2}$$

$$\theta = \tan^{-1}\left|\frac{F_{Ry}}{F_{Rx}}\right|$$

Problem 2-51 (page 41, 13th edition)

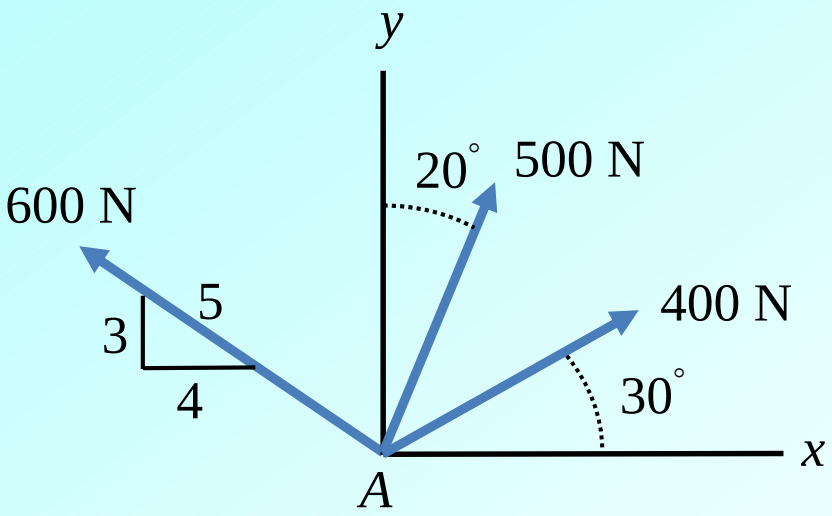
Determine the magnitude and direction measured counterclockwise from the positive *x*-axis of the resultant force of the three forces acting on the ring *A*.

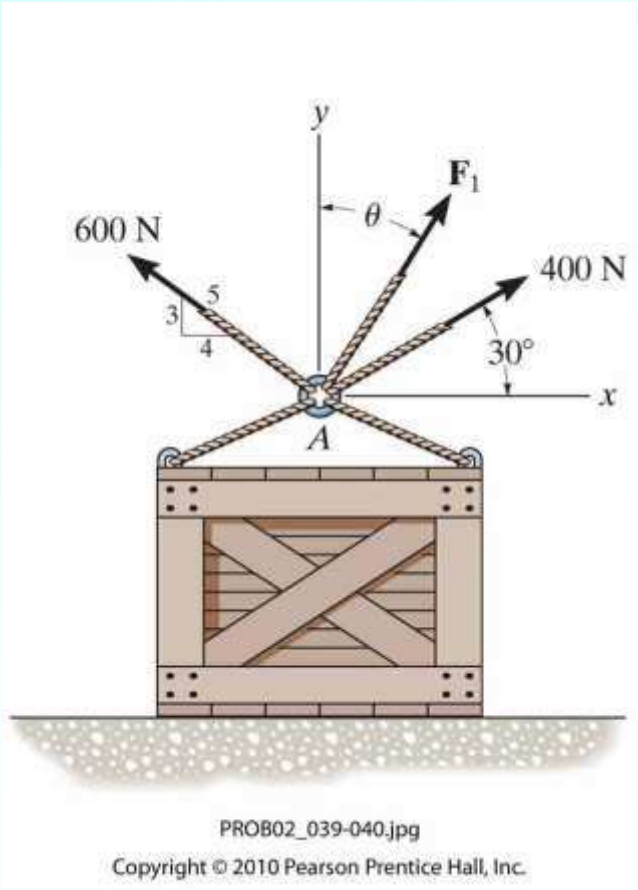
Take $F_1 = 500$ N and $\theta = 20^\circ$.



PROB02_039-040.jpg

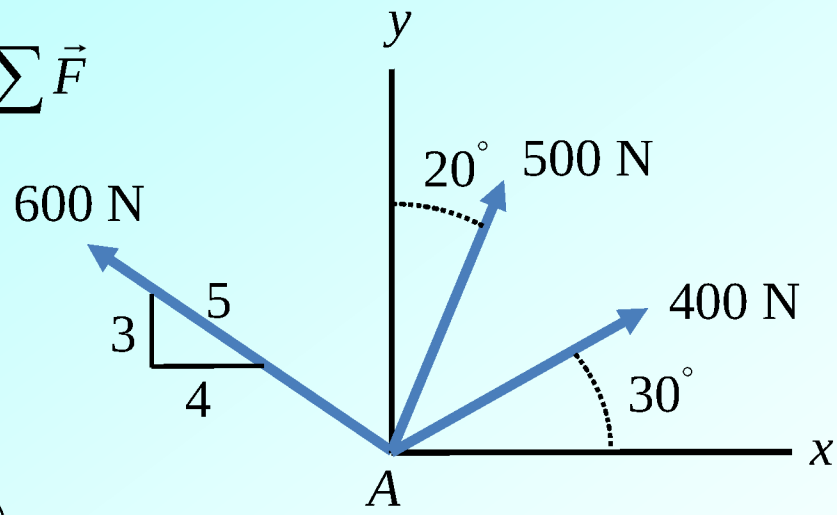
Problem 2-51 (page 41, 13th edition)





Determine the magnitude and direction of $\vec{F}_R = \sum \vec{F}$

Cartesian Vector Method (suppressing units)



$$\vec{F}_R = F_{Rx}\vec{i} + F_{Ry}\vec{j} = F_R(\cos\phi\,\vec{i} + \sin\phi\,\vec{j})$$

$$F_{Rx} = \sum F_x = 400\cos 30^\circ + 500\sin 20^\circ - 600\left(\frac{4}{5}\right) = 37.42 = A$$

Store in calculator memory "A"

$$F_{Ry} = \sum F_y = 400\sin 30^\circ + 500\cos 20^\circ + 600\left(\frac{3}{5}\right) = 1030 = B$$

Store in calculator memory "B"

Answers

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} = \sqrt{A^2 + B^2} = 1.03 \text{ kN}$$

Does this answer make sense?

$$\phi = \tan^{-1}\left(F_{Ry} / F_{Rx}\right) = \tan^{-1}\left(B / A\right) = 87.9^\circ$$

Problem 2-44 (page 41, 13th edition)

The magnitude of the resultant force acting on the bracket is 400 N. Determine the magnitude of $\vec{F}_1$. Take $\phi = 30^\circ$. Disregard the u axis.

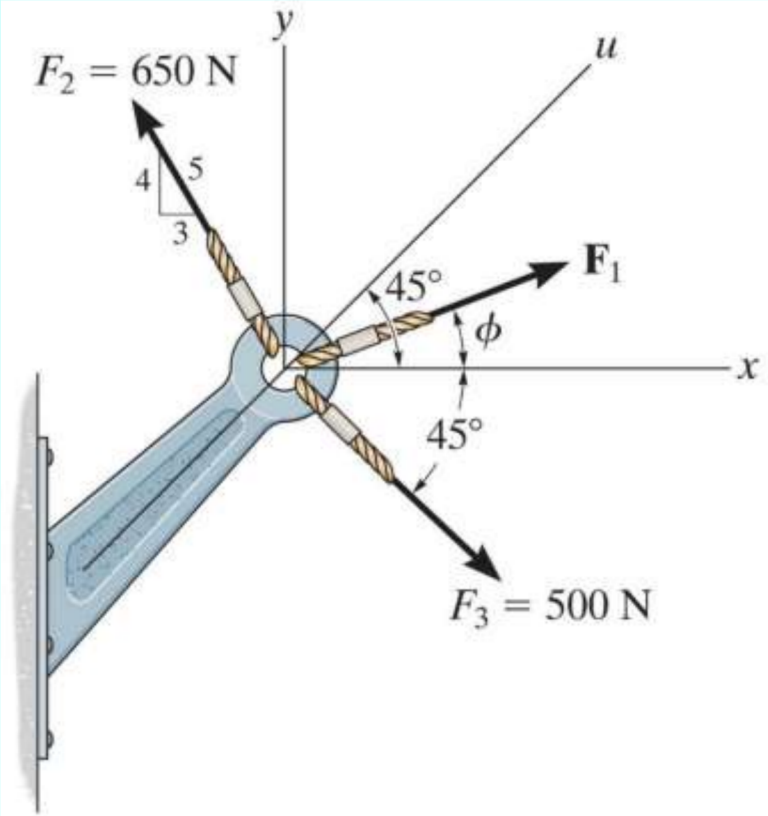
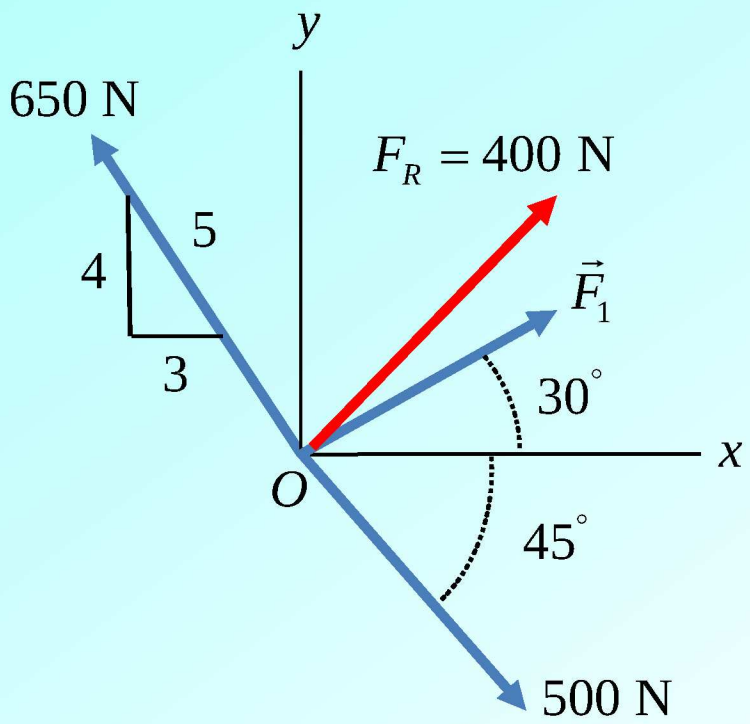
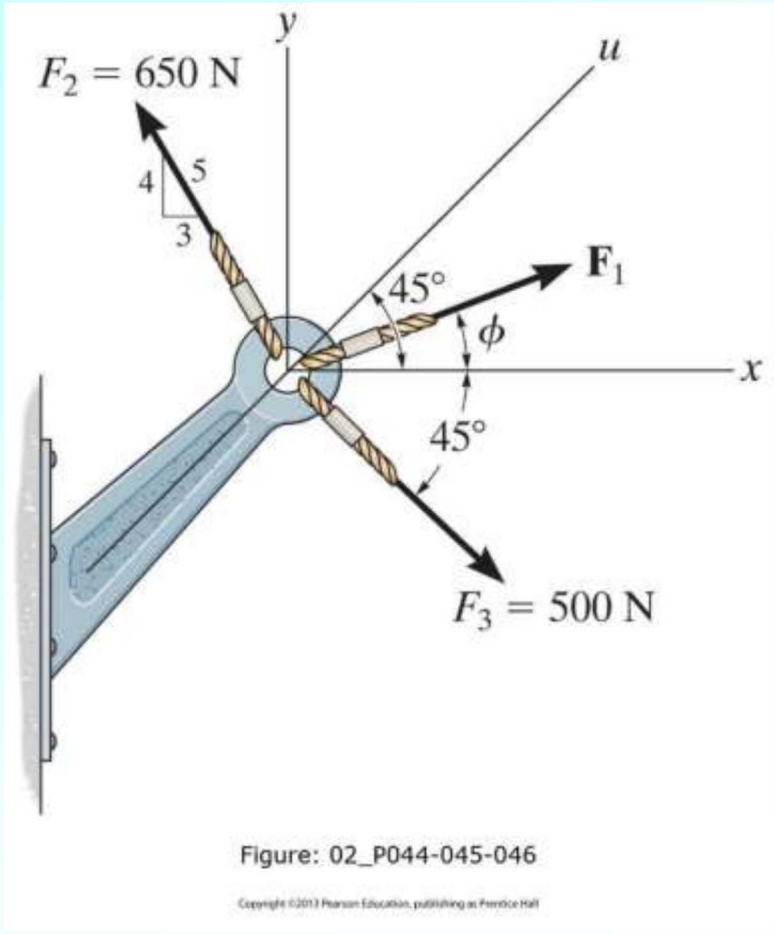


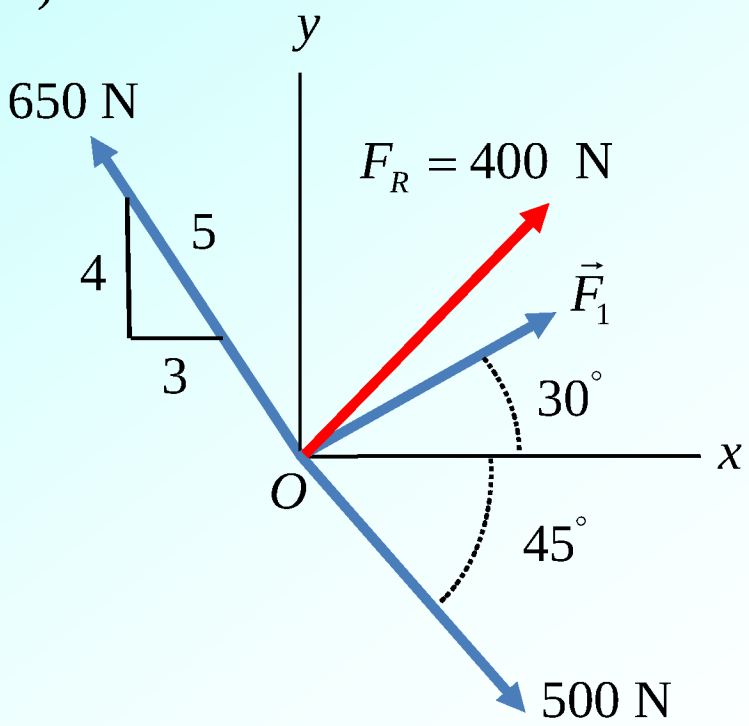
Figure: 02_P044-045-046


650 N
y
$F_R = 400$ N
5
4
3
$\vec{F}_1$
30°
x
O
45°
500 N


y
u
$F_2 = 650$ N
4
5
3
45°
$\mathbf{F}_1$
ϕ
x
45°
$F_3 = 500$ N
Figure: 02_P044-045-046

- Determine F_1 so that $F_R = 400$ N: $\vec{F}_R = \sum \vec{F}$

- **Cartesian Vector Method** (suppressing units)



$$\vec{F}_R = F_{Rx}\vec{i} + F_{Ry}\vec{j}$$

$$F_{Rx} = \sum F_x = -650\left(\frac{3}{5}\right) + F_1 \cos 30^\circ + 500 \cos 45^\circ$$

$$F_{Ry} = \sum F_y = 650\left(\frac{4}{5}\right) + F_1 \sin 30^\circ - 500 \sin 45^\circ$$

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2}$$

$$400 = \sqrt{\left(-390 + F_1 \cos 30^\circ + 500 \cos 45^\circ\right)^2 + \left(520 + F_1 \sin 30^\circ - 500 \sin 45^\circ\right)^2}$$

$$400 = \sqrt{\left(-390 + F_1 \cos 30^\circ + 500 \cos 45^\circ\right)^2 + \left(520 + F_1 \sin 30^\circ - 500 \sin 45^\circ\right)^2}$$

- This is a nonlinear equation in the single unknown F_1 which we could solve by squaring both sides and solving the resulting quadratic equation (leave as an exercise)

- Alternatively, we can use the **solver** function on a TI graphing calculator to get

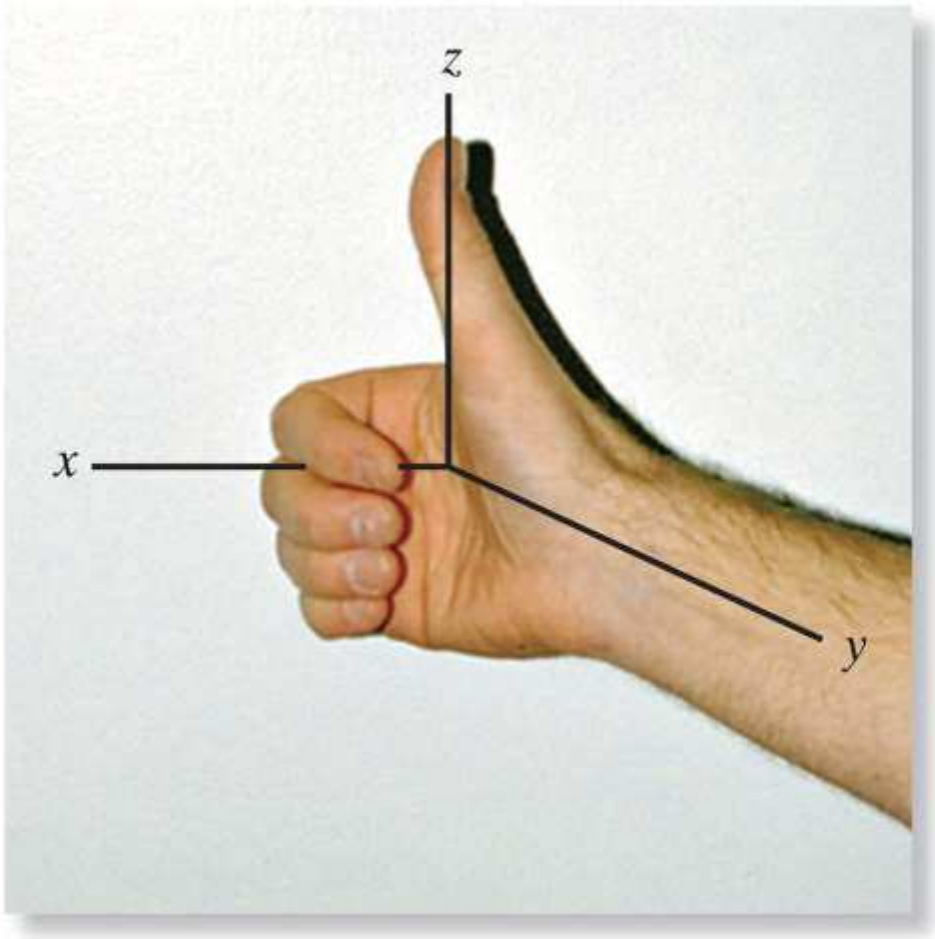
$$F_1 = 314 \text{ N} \qquad \text{or} \qquad F_1 = -417 \text{ N}$$

- Note that the negative sign tells us that for that answer $\vec{F}_1$ must be in the direction opposite to that shown in the figure. Also, to get the two distinct roots from **solver**, I used a large positive number as a guess in the first instance (1000), and a large (in magnitude) negative number in the second (-1000)

2.5-2.6: CARTESIAN VECTORS (3 DIMENSIONS or 3D)

- TRICKY TO MASTER FOR MANY STUDENTS
- **PRACTICE WILL HELP!!**
- Work through examples/problems in text, and additional problems online (Canvas)
- Discussion applies to vectors in general, but will have specific application of force vectors in mind

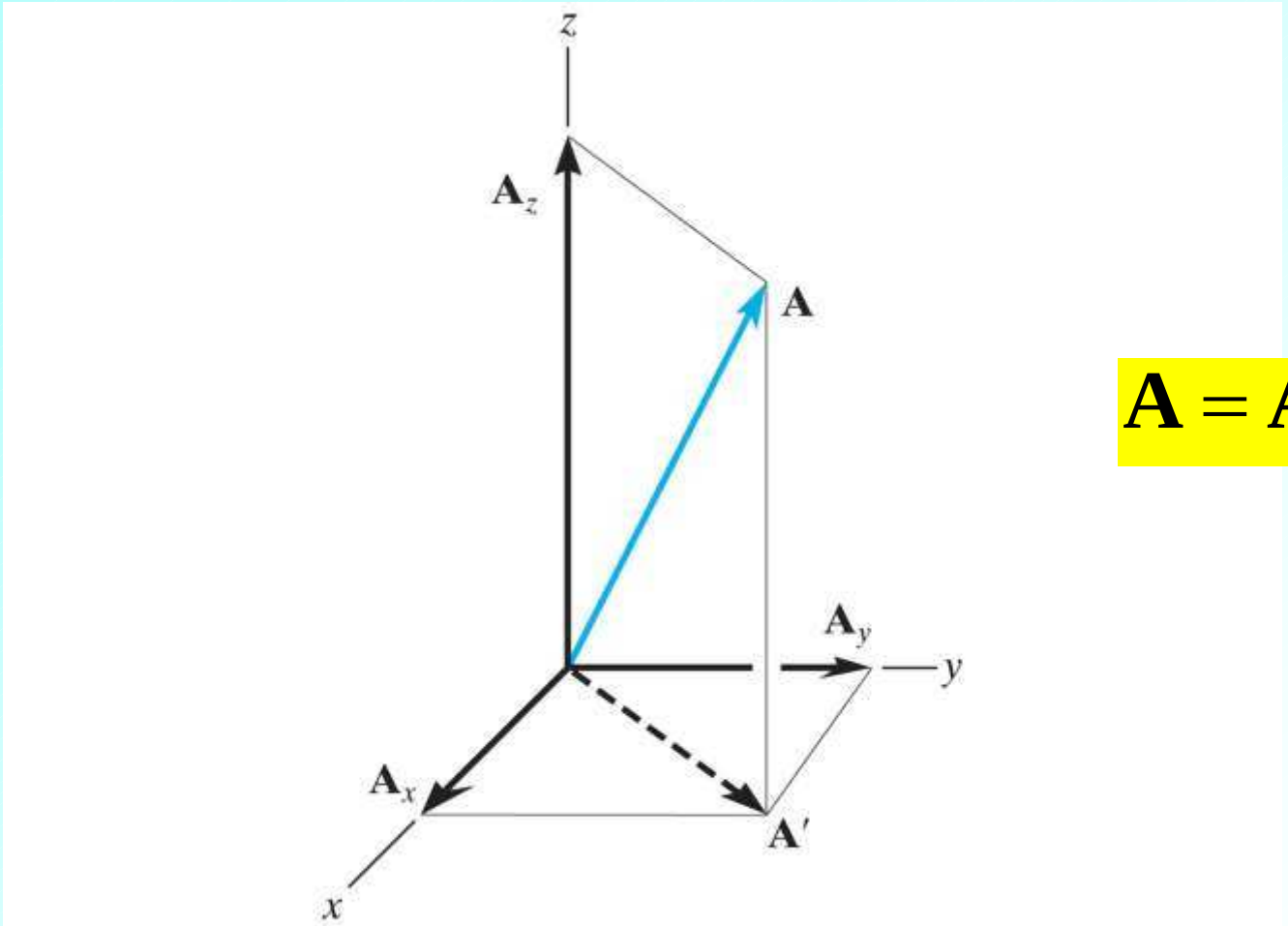
RIGHT HANDED COORDINATE SYSTEM



"Squeeze" (rotate) *x* axis into *y* axis with fingers of right hand—thumb then points in direction of *z* axis

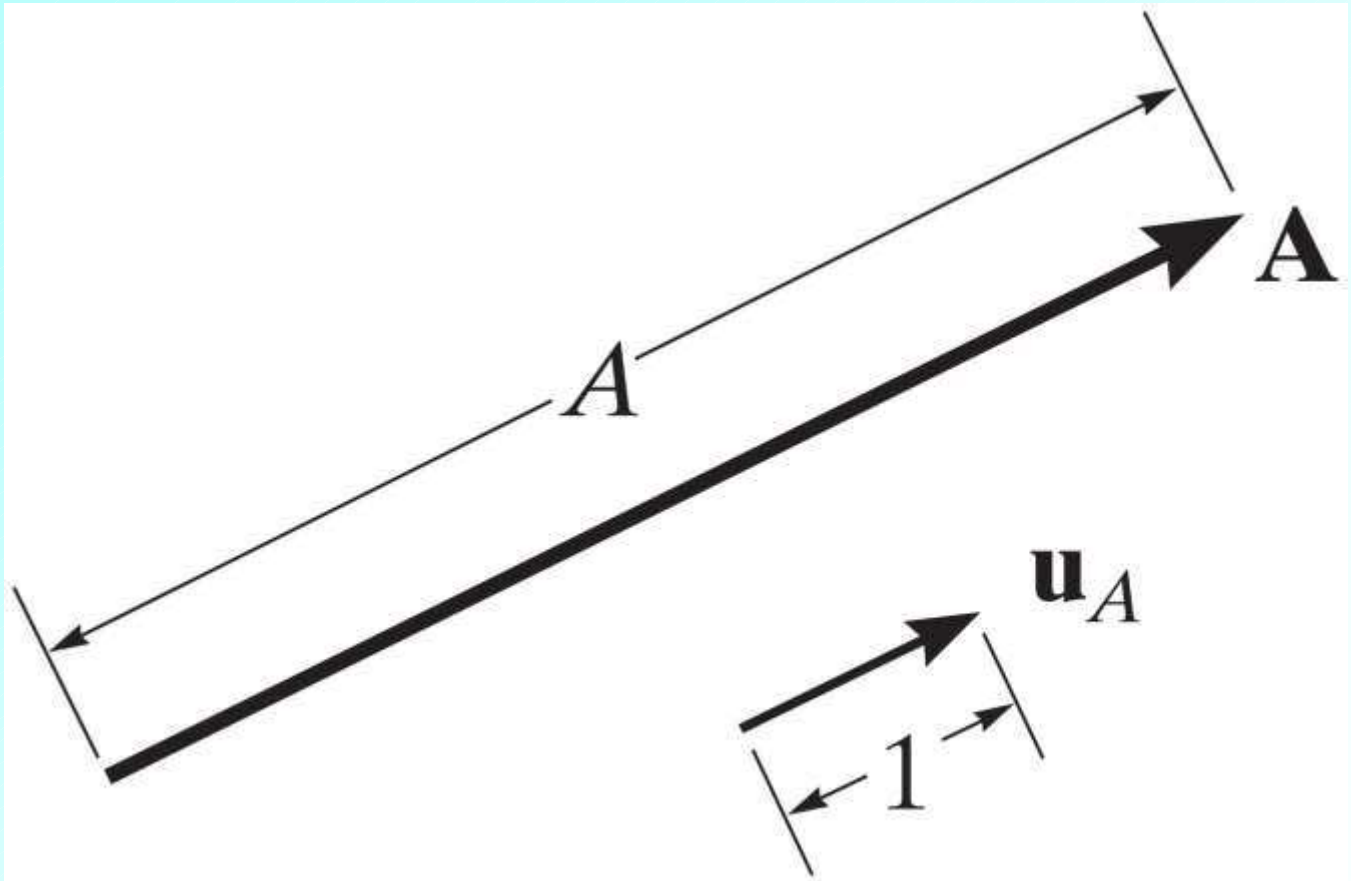
- Also note that by convention will orient axes so that positive direction is upwards

RECTANGULAR COMPONENTS OF A VECTOR



$$\mathbf{A} = \mathbf{A}_x + \mathbf{A}_y + \mathbf{A}_z$$

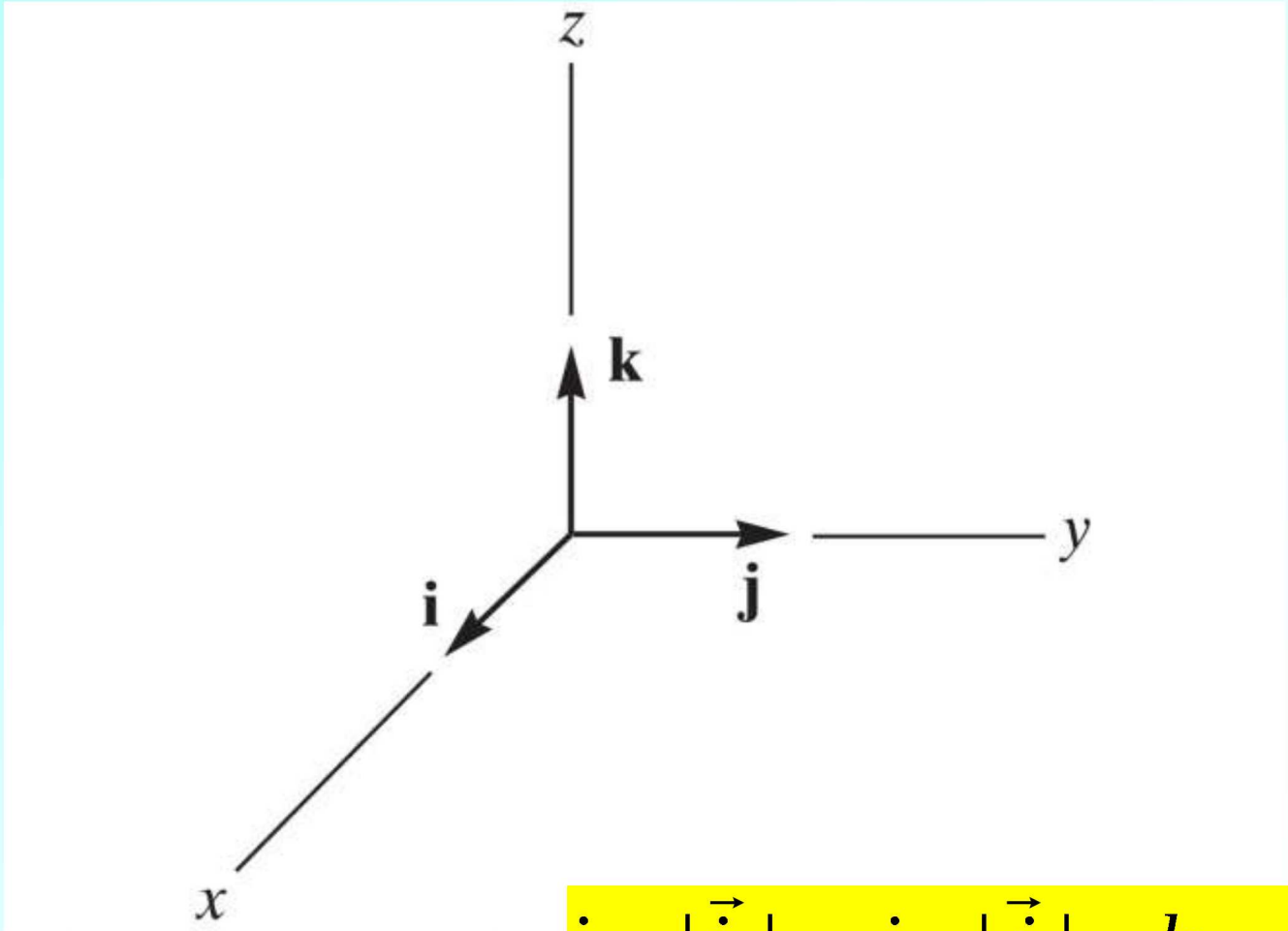
UNIT VECTOR



$$\mathbf{u}_A = \frac{\mathbf{A}}{A}$$

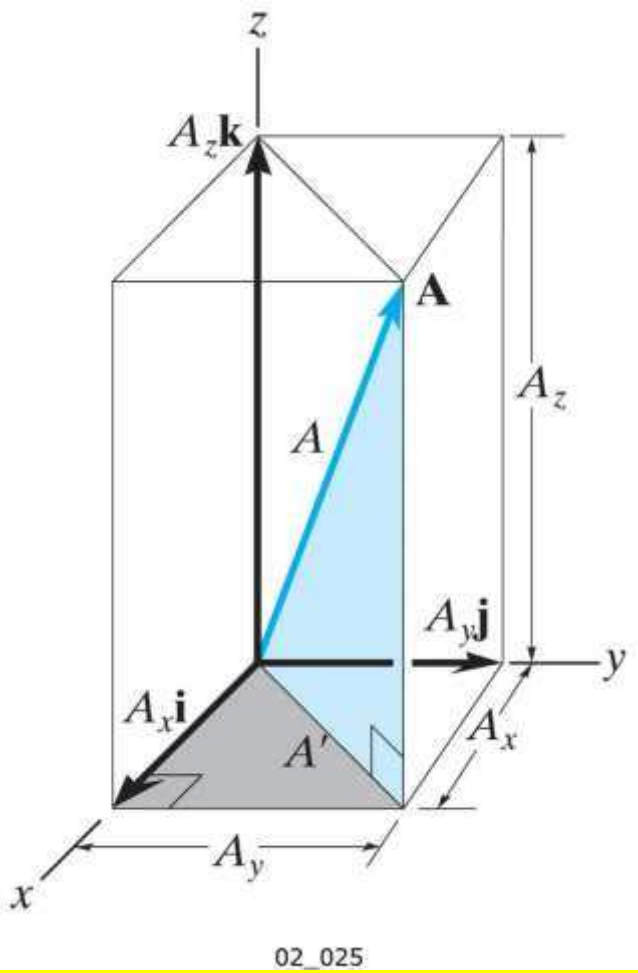
$$u_A = 1$$

CARTESIAN UNIT VECTORS



$$i = |\vec{i}| = j = |\vec{j}| = k = |\vec{k}| = 1$$

CARTESIAN VECTOR REPRESENTATION & MAGNITUDE OF A CARTESIAN VECTOR



$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

Note that despite what the text might imply (if not state explicitly), components A_x, A_y and A_z can have either sign in general.