Problems of calculating wave signals which characterize isolated systems

H. Friedrich

Albert-Einstein-Institut

Golm

April 2005
THE GENERAL SITUATION

time-like cut model ?
asymp. flat model ?

How to calculate the ‘radiation’ ?

we
‘radiation’
collapse

the ‘system’

our cosmos
choose time-like cut $T$ and space-like section $S$ study ibvp, $Ric[g] = 0$ near $T$

technical problems:
- analytical treatment
  initial data: standard
  bdry data: $\chi, \Psi_4$
- sources
- choice of gauge, life time
- numerical treatment

conceptional problems:
- radiation signal = ?
- boundary data = ???
THE ASYMPTOTICALLY FLAT MODEL

↑ time-like infinity

\[ \xrightarrow{\text{light-like infinity } J^+} \]
\[ \xrightarrow{\text{radiation}} \]

\[ \text{the system } S \]

→ space-like infinity \( i^0 \)

space-like ‘extension’ \( S^* \), asymptotically flat

2-dim schematic causal picture:

∃ well defined ‘radiation field’
at ‘null infinity’ \( J^+ \)

\[ \text{conformal representation: } \]
\[ g \rightarrow \tilde{g} = \Omega^2 \, g, \quad \tilde{M} = M \cup J^+, \]
\[ \Omega > 0 \text{ on } M, \ \Omega = 0, \ d\Omega \neq 0 \text{ on } J^+ \]
APPROACHES I

Standard approach:

introduce T to get finite comp. domain

data on $S \cup T$
based on 3+1
- radiation signal = ?
- boundary data = ?
- 'as. flat' helps ?

Characteristic approach:

introduce char. C
perform coordinate compactification

data on $S \cup C$
based on null foliation (and 3 + 1)
calculate radiation field on $J^+$ !
Hyperboloidal approach:

- Introduce $H$
- Use conformal picture and equations

- Data on $S \cup H$
- Based on 3+1 or ...
- Calculate radiation field on $\mathcal{J}^+$!

Global approach:

- Use conformal picture and equations

- Data on $S \cup S^*$
- Based on 3+1 or ...
- Calculate the entire solution and the radiation field on $\mathcal{J}^+$!
- Not done yet!
PROBLEM COMMON TO ALL APPROACHES

How does one choose the data on $T$, $C$, $H$ and on $S'$ so as to calculate signals characterizing the ‘system’ and not the data?

How do the calculated signals depend on the choice of data on $T$, $C$, $H$ and on $S'$?

∃ physical arguments for preferred choices?

Not clear whether ‘the spurious radiation will go out quickly and the basic signal will remain’

Do there exist characteristics of the wave signals which are intrinsic to the ‘system’ and independent of the choice of extensions?

∃ analytical arguments for preferred choices?
Not clear yet, but ...
S. Dain (2004):
With a given asymptotically flat initial data set for Einstein’s field equations can be associated (besides the mass) a geometric invariant which vanishes if and only if the data are stationary.

- Obtained by solving a PDE of order 4.
- In vacuum this invariant can be interpreted as a measure of the radiation content.
- Possible to minimize this invariant in suitable classes of data?

For time-symmetric asymptotically flat initial data the requirement that $\mathcal{I}^+$ be smooth (up to a certain order) appears to imply that the data are asymptotically static (up to a certain order) and vice versa.

- Evidence appears to be irrefutable.
- Possible generalizations under investigation.
- Asymptotic stationarity too strong/enough/too weak?