Recent Developments in the 2-Body Problem in Numerical Relativity

Black Holes V
Theory and Mathematical Aspects
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THANKS TO ...

1. THE ORGANIZERS

2. UofA / TPI
   CIAR, CITA, PIMS, PITP

3. Frans Pretorius
   [all simulations shown here]

West of Banff on #1, 0600 May 14 2005
Outline

• Brief history of the dynamical binary black hole problem in numerical relativity

• Pretorius’ new “generalized” harmonic code
  - axisymmetric black hole-boson star collisions
  - fully 3D collisions

• Prognosis
A Brief History of the 2 Black Hole Problem in NR

[DYNAMICS ONLY!; graphic preliminary & subject to correction/modification; apologies for omissions]


2D

Smarr, Eppley ...

NCSA/Wash U/mpi

UNC/Cornell

3D

Masso

BBH GC

NCSA/mpi/LSU

Pittsburgh

UT Austin

Penn State

Brugmann

NASA Goddard

Cornell/Caltech

Unruh suggests black hole excision, c. 1982

Excision used in sph symmetry, Seidel & Suen, 1991

~ 150 PhD theses in NR
Pretorius’s New Code
(in development for about 3 years)

• Key features
  - “ad hoc”; ignored much “conventional wisdom” (often when CW had no empirical basis)
  - Arguably only fundamentals retained from 30 years of cumulative experience in numerical relativity:
    1. Geometrodynamics is a useful concept (Dirac, Wheeler ...)
    2. Pay attention to constraints (Dewitt, ... )
Pretorius’s New Code: Key Features

- **GENERALIZED** harmonic coordinates
- Second-order-in-time formulation and direct discretization thereof
- $O(h^2)$ finite differences with iterative, point-wise, Newton-Gauss-Seidel to solve implicit equations
- Kreiss-Oliger dissipation for damping high frequency solution components (stability)
- Spatial compactification
- Implements black hole excision
- Full Berger and Oliger adaptive mesh refinement
- Highly efficient parallel infrastructure (almost perfect scaling to hundreds of processors, no reason can’t continue to thousands)
- Symbolic manipulation crucial for code generation
Pretorius’ Generalized Harmonic Code


- Adds “source functions” to RHS of harmonic condition

\[
\nabla^\alpha \nabla_\alpha x^\mu \equiv \frac{1}{\sqrt{-g}} \partial_\alpha \left( \sqrt{-g} g^{\alpha\mu} \right) = H^\mu
\]

- Substitute gradient of above into field equations, treat source functions as INDEPENDENT functions: retain key attractive feature (vis a vis solution as a Cauchy problem) of harmonic coordinates

\[
g^{\gamma\delta} g_{\alpha\beta,\gamma\delta} + \ldots = 0
\]

Principal part of continuum evolution equations for metric components is just a wave operator
Pretorius’ Generalized Harmonic Code

• Einstein/harmonic equations (can be essentially arbitrary prescription for source functions)

\[
g^{\gamma\delta} g_{\alpha\beta,\gamma\delta} + 2g^{\gamma\delta} (\alpha g_{\beta})_{\delta,\gamma} + 2H_{(\alpha,\beta)} - 2H_{\delta} \Gamma_{\alpha\beta}^{\delta} + 2\Gamma_{\delta\beta}^{\gamma} \Gamma_{\gamma\alpha}^{\delta} + 8\pi \left( 2T_{\alpha\beta} - g_{\alpha\beta} T \right) = 0
\]

• Solution of above will satisfy Einstein equations if

\[
C_{\mu}^{\mu} \bigg|_{t=0} \equiv \left( H^{\mu} - \nabla^{\alpha} \nabla_{\alpha} x^{\mu} \right) \bigg|_{t=0} = 0
\]

\[
C_{\mu,\nu}^{\mu} \bigg|_{t=0} \equiv \left( H^{\mu} - \nabla^{\alpha} \nabla_{\alpha} x^{\mu} \right)_{,\nu} \bigg|_{t=0} = 0
\]

Proof: \[
\nabla^{\alpha} \nabla_{\alpha} C^{\mu} = -R_{\nu}^{\mu} C^{\mu}
\]
Choosing source functions from consideration of behaviour of 3+1 kinematical variables

\[ ds^2 = -\alpha^2 dt^2 + h_{ij} \left( dx^i + \beta^i dt \right) \left( dx^j + \beta^j dt \right) \]

\[ H \cdot n \equiv H^\mu n_\mu = -n^\mu \partial_\mu \ln \alpha - K \]

\[ \perp H^i \equiv H_\mu h^{i\mu} = \frac{1}{\alpha} n^\mu \partial_\mu \beta^i + h^{ij} \partial_j \ln \alpha - \Gamma^i_{jk} h^{jk} \]

\[ \partial_t \alpha = -\alpha^2 H \cdot n + \ldots \]

\[ \partial_t \beta^i = \alpha^2 \perp H^i + \ldots \]
Choosing source functions from consideration of behaviour of 3+1 kinematical variables

• Can thus use source functions to drive 3+1 kinematical vbls to desired values

• Example: Pretorius has found that all of the following slicing conditions help counteract the “collapse of the lapse” that generically accompanies strong field evolution in “pure” harmonic coordinates

\[
H_t = \xi \frac{\alpha - 1}{\alpha^n}
\]

\[
\partial_t H_t = \xi \partial_t \left( \frac{\alpha - 1}{\alpha^n} \right)
\]

\[
\nabla^\mu \nabla_\mu H_t = -\xi \frac{\alpha - 1}{\alpha^n} - \xi \partial_t H_t
\]
Constraint Damping
[Brodbeck et al, J Math Phys, 40, 909 (1999); Gundlach et al, gr-qc/0504114]

- Modify Einstein/harmonic equation via

\[ g^{\alpha\beta} g_{\mu\nu,\alpha\beta} + \ldots + \kappa \left( n_\mu C_\nu + n_\nu C_\mu - g_{\mu\nu} n^\alpha C_\alpha \right) = 0 \]

where

\[ C^\mu \equiv H^\mu - \nabla^\alpha \nabla_\alpha X^\mu \]

\[ n_\mu \equiv -\alpha \nabla_\mu t \]

- Gundlach et al have shown that for all positive \( \kappa \), (to be chosen empirically in general), all non-DC constraint-violations are damped for linear perturbations about Minkowski
Effect of constraint damping

- Axisymmetric simulation of single Schwarzschild hole
- Left/right calculations identical except that constraint damping is used in right case
- Note that without constraint damping, code blows up on a few dynamical times
Merger of eccentric binary system

[Pretorius, work in progress!]

• Initial data
  - Generated from prompt collapse of balls of massless scalar field, boosted towards each other
  - Spatial metric and time derivative conformally flat
  - Slice harmonic (gives initial lapse and time derivative of conformal factor)
  - Constraints solved for conformal factor, shift vector components

• Pros and cons to the approach, but point is that it serves to generate orbiting black holes
Merger of eccentric binary system

- Coordinate conditions

\[ \nabla^\mu \nabla_\mu H_t = -\xi \frac{\alpha - 1}{\alpha^n} - \zeta \partial_t H_t \]

\[ H_i = 0 \]

\[ \xi \sim 6 / M, \quad \zeta \sim 1 / M, \quad n = 5 \]

- Strictly speaking, not spatially harmonic, which is defined in terms of “contravariant components” of source fcns

- Constraint damping coefficient: \( \kappa \sim 1 / M \)
Simulation (center of mass) coordinates

\[ t=0 \]
- Equal mass components
- Eccentricity \( \sim 0.25 \)
- Coord. Separation \( \sim 16M \)
- Proper Separation \( \sim 20M \)
- Velocity of each hole \( \sim 0.12 \)
- Spin ang mom of each hole = 0

Reduced mass frame; solid black line is position of BH 1 relative to BH 2 (green star); dashed blue line is reference ellipse

\[ t \sim 200 \]
- Final BH mass \( \sim 1.85M \)
- Kerr parameter \( a \sim 0.7 \)
- Estimated error \( \sim 10\% \)
Lapse function
Uncompactified coordinates

- All animations show quantities on the z=0 plane
- Time measured in units of M
Scalar field modulus
Compactified (code) coordinates

\[ \bar{x} = \tan(x\pi / 2), \quad \bar{y} = \tan(y\pi / 2), \quad \bar{z} = \tan(z\pi / 2) \]
Scalar field modulus
Uncompactified coordinates
Gravitational Radiation
Uncompactified coordinates

Real component of the Newman-Penrose scalar: $r\Psi_4$
Computation vital statistics

- **Base grid resolution**: 48 x 48 x 48
  - 9 levels of 2:1 mesh refinement
    - Effective finest grid 12288 x 12288 x 12288

- **Data shown (calculation still running)**
  - ~ 60,000 time steps on finest level
  - CPU time: about 70,000 CPU hours (8 CPU years)
    - Started on 48 processors of our local P4/Myrinet cluster
    - Continues of 128 nodes of WestGrid P4/gig cluster
  - Memory usage: ~ 20 GB total max
  - Disk usage: ~ 0.5 TB with infrequent output!
Hardware
[CFI/ASRA/BCKDF funded HPC infrastructure]

November 1999

vn.physics.ubc.ca
128 x 0.85 GHz PIII, 100 Mbit
Up continuously since 10/98
MTBF of node: 1.9 yrs

March 2005

vn.p4.physics.ubc.ca
110 x 2.4 GHz P4/Xeon, Myrinet
Up continuously since 06/03
MTBF of node: 1.9 yrs

glacier.westgrid.ca
1600 x 3.06 GHz P4, Gigabit
Ranked #54 in Top 500 11/04 (Top in Canada)
Sample Mesh Structure
Boson star – Black hole collisions
[Pretorius, in progress]

- Axisymmetric calculations; uses modified “Cartoon” method originally proposed by J. Thornburg in his UBC PhD thesis
- Work in Cartesian coordinates (rather than polar-spherical or cylindrical); restrict to $z=0$ plane; reexpress $z$-derivatives in terms of $x$ and $y$ (in plane) derivatives using symmetry

- Initial data
  - (Mini) boson-star on the stable branch
  - Again form black hole via prompt collapse of initial massless scalar field configuration, and further boost this configuration towards the black hole
Boson Star - Black Hole Collision: Case 1

- MBS/MBH $\sim 0.75$
- RBS/RBH $\sim 12.5$
- BH initially just outside BH and moving towards it with $v \sim 0.1 \text{ c}$
Boson Star - Black Hole Collision: Case 2

- $M_{BS}/M_{BH} \sim 3.00$
- $R_{BS}/R_{BH} \sim 50.0$
- BH initially just outside BS, and at rest

Mesh spacing $2h$ vs $h$
PROGNOSIS

- The golden age of numerical relativity is nigh, and we can expect continued exciting developments in near term.

- Have scaling issues to deal with, particularly with low-order difference approximations in 3 (or more!) spatial dimensions; but there are obvious things to be tried.
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• **STILL LOTS TO DO AND LEARN IN AXISYMMETRY AND EVEN SPHERICAL SYMMETRY!!**
### APS Metropolis Award Winners
(for best dissertation in computational physics)

<table>
<thead>
<tr>
<th>Year</th>
<th>Winner</th>
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<tbody>
<tr>
<td>1999</td>
<td>LUIS LEHNER</td>
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<tr>
<td>2000</td>
<td>Michael Falk</td>
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<td>2001</td>
<td>John Pask</td>
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<td>Nadia Lapusta</td>
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<td>2003</td>
<td>FRANS PRETORIUS</td>
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<td>Joerg Rottler</td>
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